Econ674

Economics of Natural Resources and the Environment

Session 5

Static Optimization in Natural Resources

Static Optimization

Optimization means the best use of a given stock of resources to achieve a given end. "Best" use generally means the maximization, or most efficient use, of resources, independent of other welfare criteria such as distributive justice.

Static optimization means maximizing some objective function subject to a given set of constraints. The constraints enable us to then redefine the optimization problem using a Lagrangian formulation. By taking the respective partial derivatives of the function and setting them equal to zero, we obtain the optimal solution.

Consider, for example, a Cobb-Douglas production function of the form:

1. $Q = AX_2^{\alpha}X_3^{\beta}$ where:

X_2 and X_3 represent the quantity of inputs,
A represents the level of technology
and α and β represent the respective output elasticities.

Static Optimization- 1

Reformulation of the function in terms of natural logarithms enables one to test directly for the respective output elasticities, providing a general conclusion on the presence or absence of economies of scale, a common question regarding the structure of a given industry in terms of the level of concentration. However, our concern here is: once we have the general parameter estimates from a given Cobb-Douglas production function, how can we derive and validate the optimal input combinations consistent with the underlying principle of technical economic efficiency.

Consider, for example, the following production function:

2. $Q = 1.00(X_2^{.20}\ X_3^{.80})$

subject to: $\$10X_2 + \$10X_3 = \$200$

where \$10 = the unit price of X_2, and

\$10 = the price of X_3, and

\$100 is the firm's variable input budget constraint.

The corresponding Lagrangian function to be maximized is:

3. Max $L = 1.00(X_2^{.20}X_3^{.80})+\Lambda(10X_2+10X_3-200)$.

Static Optimization- 2

Take the first partial derivatives and set them equal to zero:

4. $\delta L/\delta X_2 = .20X_2^{-.80}X_3^{.80}+\Lambda 10=0$

5. $\delta L/\delta X_3 = .80X_2^{.20}\, X_3^{-.20}+ \Lambda 10=0$

6. $\delta L/\delta \Lambda = 10X_2+10X_3 - 200 = 0$

After setting equations 4 and 5 equal to lambda we get the following:

7. $.02(X_2^{-.80}X_3^{.80}) = .08(X_2^{.20}X_3^{-.20})$

We now multiply both sides by a simplifying expression to eliminate one unknown:

8. $(X_2^{.80}\, X_3^{.20}).02(X_2^{-.80}\, X_3^{.80})=(X_2^{.80}\, X_3^{.20})(.08X_2^{.20}\, X_3^{-.20})$

Simplifying, we have:

8.a $.02X_3 = .08X_2$, or $X_3 = 4.00X_2$

Substituting this optimal input ratio into the budget constraint enables us to obtain

9. $10.00X_2 + (10.00)(4.00)X_2 = 200$, thus $50X_2 = 200$,

and $X_2 = 4.00$.

Substituting this value into the budget constraint now enables us to solve for the optimal quantity of $X_3 = 16.00$.

Total expenditures now are verified as:

(10.00)(4.00) = $40.00

(10.00)(16.00) = <u>$160.00</u>

$200.00

Static Optimization - 3

We now derive the corresponding marginal products from the respective input combinations and divide their respective values to verify that the firm has achieved technical efficiency:

10. $\delta L/\delta X_2 = .20(4.00)^{-.80}(16.00)^{.80} = .6063$

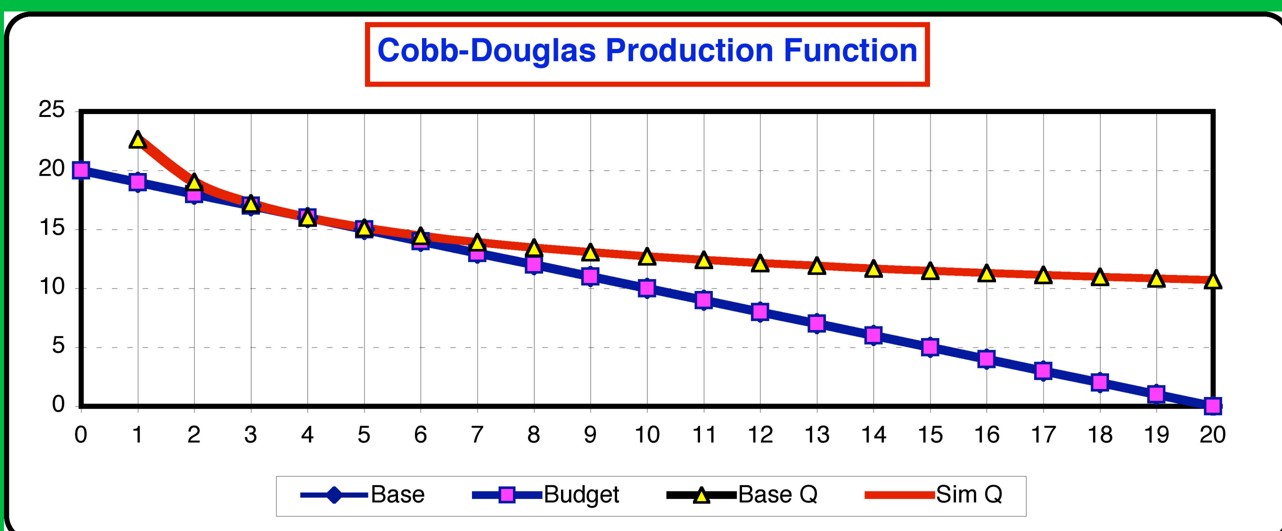
11. $\delta L/\delta X_3 = .80(4.00)^{.20}(16.00)^{-.20} = .6063$

We now divide each marginal product by its corresponding input price to obtain:

12. $(\delta L/\delta X_2)/(PX_2) = (.6063)/(10.00) = .06063$

13. $(\delta L/\delta X_3)/(PX_3) = (.6063)/(10.00) = .06063$

The diagrammatic equilibrium is shown below, where the budget constraint is tangent to the underlying isoquant.



If one of the above inputs represents a natural resource, the corresponding equilibrium only defines the technical efficiency of the firm, not whether the pricing of the natural resource is consistent with a present value optimization, or, in the case of a renewable natural resource, consistent with an environmentally sustainable market equilibrium.

Static Optimization with Natural Resources

Consider the choice of harvesting timber(T) or fish(F) from a given habitat:

1. Maximize $500 T +$800 F

2. subject to: $\left| T^2 + \frac{F^2}{a} - 1800 \right| = 0$

where P(T)= $500 per metric ton
P(F)= $800 per metric ton
L = 1,800.00 hours
a = 2.25

The corresponding Lagrangean is:

3. $L = \$500\,T + \$800\,F - \Lambda \left| T^2 + \frac{F^2}{2.25} - 1800 \right|$

First order conditions are:

4. $\delta L/\delta T = \$500 - 2.00\,\Lambda T = 0.00$
5. $\delta L/\delta F = \$800 - 0.89\,\Lambda F = 0.00$
6. $\delta L/\delta \Lambda = -T^2 - \frac{F^2}{2.25} + 1{,}800.0 = 0.00$

7. $-2.00\,\Lambda T = -500.00$
8. $0.89\,\Lambda F = 800.00$
9.

ΛT =	250.00
ΛF =	1,600.00
F =	6.40 T

Substituting into the constraint yields:

10. $T^2 + 41.0 \frac{T^2}{2.25} = 1{,}800.00$

Re-arranging:

11. $2.25\,T^2 + 41.0\,T^2 = 7{,}200.00$

12. $+43.21\,T^2 = 7{,}200.00$

13. $T^2 = 166.63$

14. **T = 12.91** and by substitution into 2:

15. $166.63 + \frac{F^2}{2.25} = 1{,}800.00$

16. $\frac{F^2}{2.25} = 1{,}633.37$

Static Optimization with Natural Resources - 1

17. $F^2 = 6{,}533.49$

18. **$F = 80.83$**

19. By substituting the solution values of T and F from equations 14 and 18 into equations 7 and 8 we obtain the value of Λ, I. e. :

(7.a) $\Lambda\ 12.91 = 250.00$

$\Lambda = 19.37$ or, equivalently,

(8.a) $\Lambda\ 80.83 = 1{,}600.00$

$\Lambda = 19.79$

In this case, L may be interpreted as the shadow price of labor. Substituting the optimal quantities of T and F in the constraint yields the total number of allocated hours, from which we can obtain the total expenditure on labor:

		Λ	Expenditures
T^2	166.63 hours	19.37 =	$3,227.11
$\frac{F^2}{2.25}$	2,903.77 hours	19.79 =	$57,479.11
	3,070.40 hours		$60,706.22

Thus, the budget constraint embodies the financial constraint of total expenditures being no more than the given level shown above.

In turn, we can derive the total value of the harvest based on the optimal quantity havested:

	Pi	Qi	Revenues
Timber	$500	12.91	$6,454.23
Fish	$800	80.83	$64,663.99
		Total Revenue	$71,118.22

If there are no other costs than labor, the rate of return on sales will be:

Revenues	$71,118.22
Costs	$60,706.22
Π =	$10,412.00
RRS =	14.64%

In this example, the shadow price of labor will vary directly with the market price of timber and fish such that the rate of return of sales is invariant with respect to the quantity harvested. Other formulations would treat labor as a fixed cost per number of hours worked, in which case, ignoring other fixed costs such as boats, nets, and timber harvesting equipment, the rate of return would vary with the selling price of each commodity.

Assumptions and Implications:

1. No production rates are considered
2. No product demand determinants are considered
3. No costs other than labor are considered
4. Production externalities are ignored
5. Property rights are undefined

Comparative Statics - Basic Market Dynamics

Under competitive market conditions, the number of firms is determined by the underlying rate of return, which in the following case, is given as the rate of return on sales (other measures include the rate of return on equity, the rate of return on invested capital, and the rate of return on assets). As long as the economic rate of return on sales (ERRS) is positive, this means that the accounting rate of return exceeds the opportunity cost of capital, and firms will enter the market accordingly.

This process will continue up to the point where all existing firms in a market earn only normal profits, I.e., the economic rate of return on sales is zero, and the accounting rates of return on sales (ARRS) is equal to the opportunity cost of capital. At this point, firms will have achieved a steady-state economic equilibrium, I.e. there is no incentive for firms to enter or leave an industry. This will persist until some perturbation, such as technical change, or a change in product demand, changes the underlying market dynamics.

In such market dynamics, no consideration is given to the effect of the number of firms on the environment. If the economic rate of return is positive, entering firms will accelerate the rate of consumption of natural resources, and this may not be a sustainable steady-state economic and environmental equilibrium. Thus, there is no guarantee that competitive market prices will generate both a steady-state economic and a steady-state environmental equilibrium.

Comparative Statics - Basic Market Dynamics - 1

Basic Market Dynamics

P. LeBel

Consider the following:

A. Market Conditions:

Pd =	60.00	-0.0600 Qd
Ps =	20.00	+ 0.2000 Qs

Opportunity Cost of Capital: 10.00%

B. Individual Firm Conditions:

Cost Function:

TC =	4.00	+41.00Q	+ 0.8000 Q^2

1. **Initial Market equilibrium conditions are:**

Qe=	153.85
Pe=	$50.77
TR=	$7,810.65
Point Own-Price Point Elast. Demand=	5.50

2. **If the firm is in a competitive industry, determine its profit-maximizing output:**

MC =	+41.00	+ 1.6000 Q

Initial Competitive Equilibrium of the Firm

MR = AR = P =	$50.77	
Qe= f(MC=MR) =	6.11	
TR =	$309.99	
TC =	$284.16	
Economic Profit =	$25.82	
Economic Rate of Return on Sales =	8.33%	
Accounting Rate of Return on Sales =	18.33%	
Competitive Firm Market Share =	3.97%	(Based on assumption of isocosts among all firms)
Equiproportional Initial Number of Firms =	25.20	(reciprocal of the competitive firm market share)

Based on competitive market forces, derive the terminal competitive equilibrium of the firm

Q = f(ATC=MC)	2.24	(zero profits are earned where ATC=MC=P=MR of the individual firm)
Pe = f(MC)	$44.58	(solved by setting the zero profit quantity into the marginal cost function)
TR=	$99.68	
TC=	$99.68	
Economic Profit =	$0.00	
Economic Rate of Return on Sales =	0.00%	
Accounting Rate of Return on Sales =	10.00%	
Market Quantity for zero economic profits =	257.04	
New Market Supply Intercept =	-6.83	(Insert the new P and Q and supply coefficient to derive the new supply intercept)
Verify New Market Equilibrium Quantity =	257.04	
New Market Equilibrium Price =	$44.58	
New Market Total Revenue =	$11,458.17	
Competitive Firm Market Share =	0.87%	(Based on assumption of isocosts among all firms)
Equiproportional Initial Number of Firms =	114.95	(reciprocal of the competitive firm market share)

Comparative Statics - Basic Market Dynamics - 2

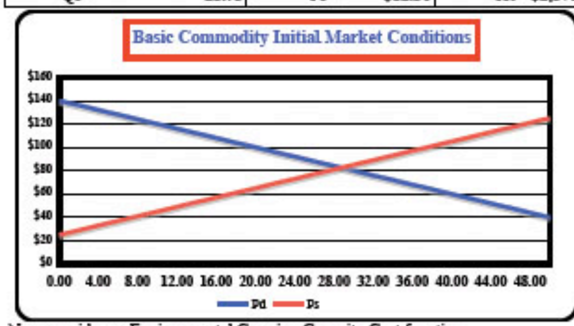
If market prices fail to achieve both an economic and environmentally efficient solution, one alternative is to consider the use of taxes and subsidies. In the example below, we consider the use of tradable permits as a function of the underlying environmental carrying capacity of a given space. Note that the use of taxes and subsidies may not satisfy the conditions for a welfare equilibrium in that no consideration has been given the efficiency, effectiveness, fairness, and simplicity in the use of taxes and subsidies on all agents.

Optimal Pricing of Tradeable Pollution Permits

P. LeBel

Consider the basic market conditions for a commodity:

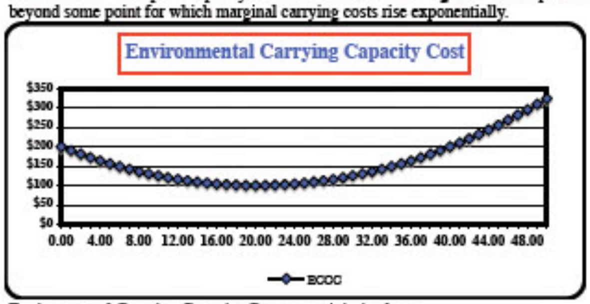
1.		Pd =	140	-2.00 Qd	
2.		Ps =	25	+2.00 Qs	
3.	Qe =	28.75	Pe=	$82.50	TR= $2,371.88



Now consider an Environmental Carrying Capacity Cost function:

4.	ECCC =	200	-10.00 Q	$0.25\ Q^2$

Given that the ECCC function is quadratic in its arguments, there is an initial environmental absorptive capacity that can tolerate diminishing incremental pollution beyond some point for which marginal carrying costs rise exponentially.



Environmental Carrying Capacity Costs are minimized at:

5.	δECCC/δQ=	-10.00	+.500 Q = 0	
6.		Q(ECCC)min =	20.00	
7.	Balance:	**From 3e:**	28.75	
		From 6Q(ECCC):	20.00	
		Balance	8.75	= the environmental surplus quantity

Now consider the pricing of a tradeable pollution permit system that will restore the level of environmental pollution to the minimum carrying capacity cost level Pricing of such a permit is equivalent to an environmental pollution tax that brings the market equilibrium to the minimum environmental carrying capacity cost:

	Pd =	140	-2.00 Qd
	P's =	x	+2.00 Qs
Setting Qe =	20.00	x = 60.00	, which when inserted back into equation 1 yields:
	P'd =	$100.00	and from which the corresponding market supply price is:
	P's =	$65.00	the difference representing the price of the pollution tradeable permit
	Permit price =	$35.00	

Trading environmental pollution via permits requires that in a given environment, some firms will have a surplus and some will have a shortage, exchanges of which will reach the local market "bubble" environmental carrying capacity equilibrium. What modifies this condition is the question of irreversibility in environmental pollution, in which the carrying capacity quantity no longer has a local optimum. Further considerations include engineering estimates of the environmental carrying costs, monitoring emissions, and transactions costs of market permits.