Econ674

Economics of Natural Resources and the Environment

Session 10

Discrete-Time Models of Natural Resource Decisions

Discrete Time Models of Renewable Natural Resources depend on several key relations. They include:

1. A variable growth rate that depends on the intrinsic rate of growth and the carrying capacity of the environment.
2. The rate of discount used in evaluating the net present value of a renewable resource stock.
3. The cost and benefit stream arising from a designated rate of extraction, or harvest.

Together, these relations enable us to specify the following key equations which we will use to derive varying solutions to the management of a renewable natural resource:

1. Basic Logistic Growth Function:

$$x(t) = \frac{K}{1 + ce^{-rt}}, \quad where \quad c = \frac{K - x_0}{x_0}$$

where:

R = the intrinsic rate of growth of the renewable natural resource

K = the carrying capacity of the environment, and

X_o = the initial stock of the renewable resource

Since K, the carrying capacity of the environment, sets an upper limit to the stock of a renewable resource, at any given time, t, the rate of growth will be variable and can be expressed as:

2\. Renewable Stock Growth Rate:

$$\delta X / \delta t = r x_t \left(1 - \frac{x_t}{K}\right)$$

A key relation in renewable natural resource management is the maximum sustainable yield, or MSY. For an undiscounted resource, this can be expressed as:

3\. Logistic Function Maximum Sustainable Yield:

$$\text{MSY} = \frac{K}{1 + c\,(e^{-rt})} = \frac{\text{rK}}{4}$$

In turn, we can further express the number of time periods for an undiscounted renewable natural resource to reach the maximum sustainable yield (MSY):

4\. Time to reach Natural Growth MSY

$$t_{msy} = \frac{\ln\left(\frac{\text{c}}{\text{x}_\text{o}} + 1\right)}{r} \quad \text{where c} = \frac{\text{K - x}_0}{\text{x}_0}$$

In turn, the stock of the renewable natural resource at the maximum sustainable yield (MSY) level can be expressed as:

5.
$$\text{Stock of Undiscounted RNR at MSY :}$$
$$\text{Stock } X_{msy} = rK$$

Harvesting of a renewable natural resource will alter the time path of equation 1 as follows:

6.
$$\text{Net Growth of a Renewable Natural Resource :}$$
$$X_{i+1} = X_i + r * X_i\left(1 - \frac{X_i}{K}\right) - Y_i$$

where Y_i is the level of harvest in period I.

Adoption of a positive rate of discount to a renewable natural resource will alter the optimal harvest level as well as the optimal stock level. We can refer to the first notion as the Present Value Optimal Harvest Rate (PVOHR), and the second as the Present Value Optimal Stock Level (PVOSL), defined below, respectively, as:

7.
$$\text{Present Value Optimal Harvest Level :}$$
$$Y^* = K\left(r^2 - \delta^2\right)/4r$$

8.
$$\text{Present Value Optimal Stock Level :}$$
$$X^* = K(r - \delta)/2r$$

where δ = the rate of discount

As long as the rate of discount is positive, the discounted optimal harvest and stock levels will be less than implied by the maximum sustainable yield. Sustainability of a renewable natural resource stock can be defined in terms of whether Y* is greater or less than a critical value Y_c* As long as the discounted optimal harvest rate exceeds this critical value, the renewable resource stock will permit a replication over time without leading to extinction. Whether the Present Value Optimal Harvest Rate falls below the critical level depends on the discount rate, δ, the intrinsic rate of growth, r, the carrying capacity of the environment, K, and the time horizon of the resource, t.

Assuming that the discounted optimal harvest rate does not result in the extinction of a resource, we can make several propositions:

1. The higher is the intrinsic rate of growth, the shorter will be the time to reach the MSY level; A constant positive discount rate will generate a lower Present Value of the Optimal Harvest Rate (PVOHR) than an Undiscounted Maximum Sustainable Yield (UMSY) solution, just as it will lead to a lower Present Value Optimal Stock Level (PVOSL) in comparison to the Undiscounted Maximum Stock Level (UMSL), but all will be larger the greater is the intrinsic rate of growth.

Proposition 1	A.	B.	C.	D.	E.	F.	G.
r =	0.20	0.25	0.30	0.35	0.40	0.45	0.50
c =	39.00	39.00	39.00	39.00	39.00	39.00	39.00
Carrying Cap. K =	20.00	20.00	20.00	20.00	20.00	20.00	20.00
x_o =	0.50	0.50	0.50	0.50	0.50	0.50	0.50
Y_o =	0.01	0.01	0.01	0.01	0.01	0.01	0.01
t_{UMSY} =	21.85	17.48	14.56	12.48	10.92	9.71	8.74
(UMSL) X_T =	4.00	5.00	6.00	7.00	8.00	9.00	10.00
(PVOSL) X* =	0.30	0.50	0.75	1.05	1.40	1.80	2.25
(UMSY) Y_T =	1.00	1.25	1.50	1.75	2.00	2.25	2.50
(PVOHR) Y* =	0.94	1.20	1.46	1.71	1.97	2.22	2.48
δ, Disc. Rate =	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%

2. The higher is K, the carrying capacity of the environment, the longer will be the time to reach the undiscounted maximum sustainable yield (UMSY), X_T; Under a constant and invariant positive rate of discount, the Present Value of Optimal Stock (PVOSL), X*, will be less than the Undiscounted Maximum Stock level (UMSL), X_T, just as the Present Value of Optimal Harvest Rate (PVOHR), Y*, will be less than the Undiscounted Maximum Sustainable Yield (UNMSY), Y_T; X*, XT, Y*, and YT will be greater the higher is the level of K.

Proposition 2	A.	B.	C.	D.	E.	F.	G.
r =	0.20	0.20	0.20	0.20	0.20	0.20	0.20
c =	39.00	49.00	59.00	69.00	79.00	89.00	99.00
Carrying Cap. K =	20.00	25.00	30.00	35.00	40.00	45.00	50.00
x_o =	0.50	0.50	0.50	0.50	0.50	0.50	0.50
Y_o =	0.01	0.01	0.01	0.01	0.01	0.01	0.01
t_{UMSY} =	21.85	22.98	23.90	24.67	25.34	25.94	26.47
(UMSL) X_T =	4.00	5.00	6.00	7.00	8.00	9.00	10.00
(PVOSL) X* =	0.30	0.38	0.45	0.53	0.60	0.68	0.75
(UMSY) Y_T =	1.00	1.25	1.50	1.75	2.00	2.25	2.50
(PVOHR) Y* =	0.94	1.17	1.41	1.64	1.88	2.11	2.34
δ, Disc. Rate =	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%	5.00%

3. A higher discount rate has no effect on the Undiscounted Maximum Stock Level (UMSL), X_T, or on the Undiscounted Maximum Sustainable Yield (UMSY), Y_T. However, positive and increasing discount rates will have an effect on the difference between the undiscounted and discounted values of optimal stocks and harvesting, and will be lower the higher is the rate of discount, δ.

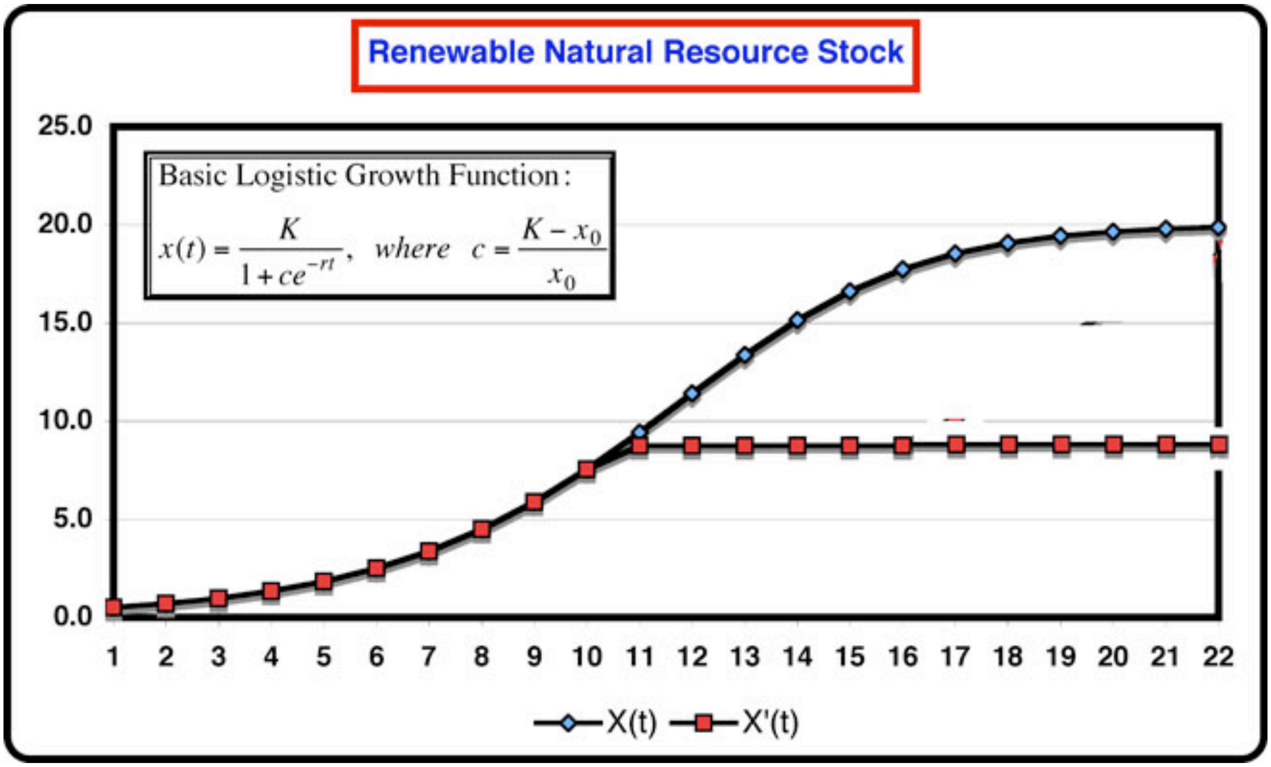
Proposition 3	A.	B.	C.	D.	E.	F.	G.
r =	0.20	0.20	0.20	0.20	0.20	0.20	0.20
c =	39.00	39.00	39.00	39.00	39.00	39.00	39.00
Carrying Cap. K =	20.00	20.00	20.00	20.00	20.00	20.00	20.00
x_o =	0.50	0.50	0.50	0.50	0.50	0.50	0.50
Y_o =	0.01	0.01	0.01	0.01	0.01	0.01	0.01
t_{UMSY} =	21.85	21.85	21.85	21.85	21.85	21.85	21.85
(UMSL) X_T =	4.00	4.00	4.00	4.00	4.00	4.00	4.00
(PVOSL) X* =	0.36	0.34	0.32	0.30	0.28	0.26	0.24
(UMSY) Y_T =	1.00	1.00	1.00	1.00	1.00	1.00	1.00
(PVOHR) Y* =	0.99	0.98	0.96	0.94	0.91	0.88	0.84
δ, Disc. Rate =	2.00%	3.00%	4.00%	5.00%	6.00%	7.00%	8.00%

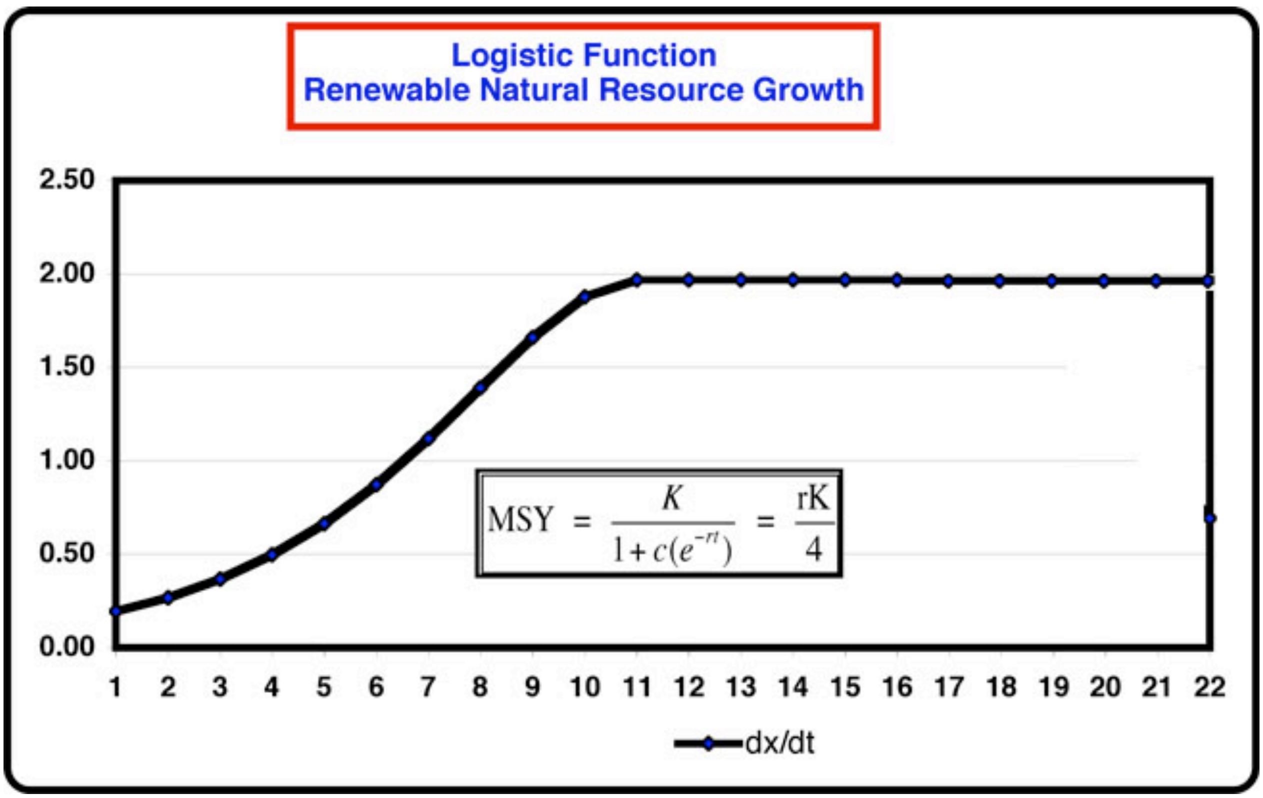
Harvesting a renewable natural resource is based on the discounted present value of extraction for a given time horizon. Under positive rates of discount, it may be optimal to allow a renewable resource to expand without harvesting initially, after which one can achieve a steady-state constant rate of harvesting for a given steady-state stock whose growth is just equal to the level of the harvest for each subsequent time period.

Various methods can be used to achieve the maximum level of the net present value of a renewable resource harvest. We use here the Newton-Raphson method of nonlinear programming to illustrate the solution to an optimization problem, whose parameters are given in the table below, followed by the graphical solutions:

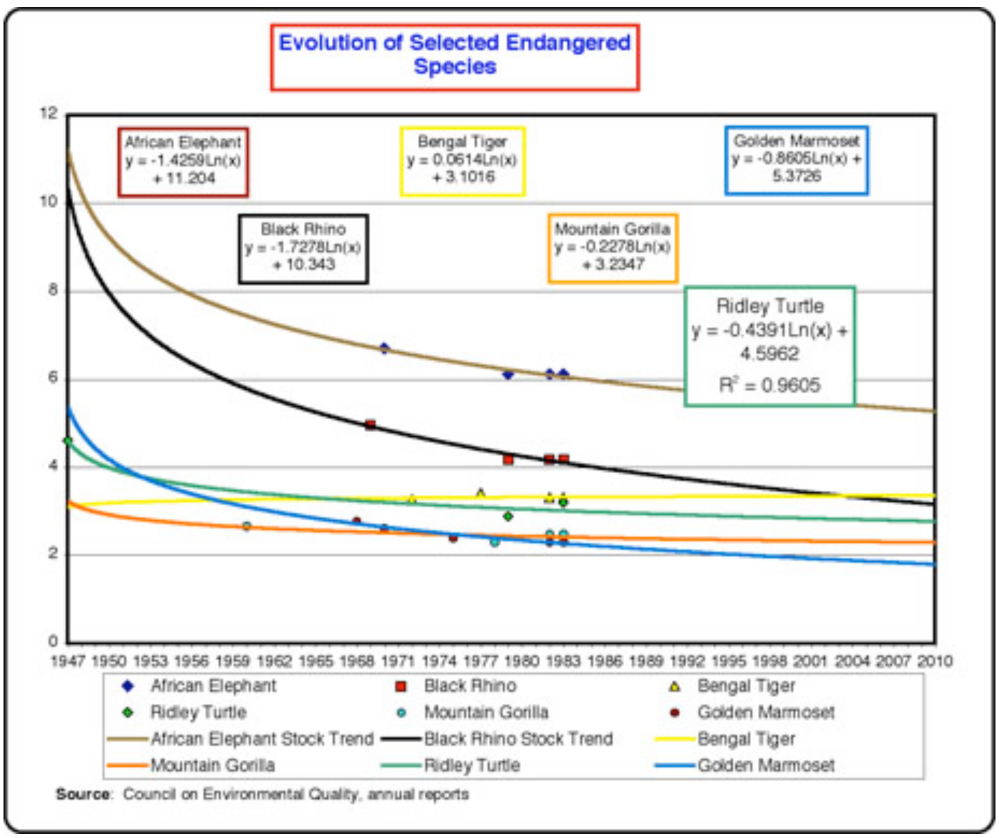
Renewable Natural Resource Parameters		t(MSY) =	10.92
r =	0.40	(UMSL), X_T =	10.00
c =	39.00	(PVOSL) X* =	8.75
Carrying Cap. K =	20.00	(UMSY), Y_T =	2.00
Initial RNR Stock, X_o =	0.50	(PVOHR), Y* =	1.97
Initial Harvest Level, Y_o =	0.01	δ =	0.0500
Current Unit Price, P_o =	$2.00	Unit Cost =	$0.0200
		PVNB =	$108.05

The solution calls for no harvesting to begin until period 9, which then rises to the optimal level, Y*, in period 13, and continues thereafter.


Renewable Natural Resource Stock
Basic Logistic Growth Function:
$x(t) = \frac{K}{1+ce^{-rt}}, \quad where \quad c = \frac{K - x_0}{x_0}$
25.0
20.0
15.0
10.0
5.0
0.0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22
X(t) X'(t)


Logistic Function
Renewable Natural Resource Growth
$MSY = \frac{K}{1+c(e^{-rt})} = \frac{rK}{4}$
2.50
2.00
1.50
1.00
0.50
0.00
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22
dx/dt

Our discussion thus far has proceeded on the basis of a competitive market structure with well-defined property rights. Under imperfectly competitive conditions, a monopolist would tend to behave in a similar fashion as we have seen in the case on exhaustible resources, one result being that instead of competitive prices, marginal revenue-marginal cost rules are used to generate a higher rate of harvesting than would the case under competitive conditions. This implies, other things equal, that a monopolist would bring the harvesting level closer to a critical level of sustainability than would be the case for a competitive solution. However, neither the competitive solution nor the monopoly solution guarantees that a biologically sustainable solution will be found.



One factor that complicates the pricing and production of renewable natural resources is the question of property rights. Where property rights are weak or absent, as is often the case under a common property resource regime, market prices may not reflect the relative scarcity of a resource, in which case, a regulatory regime that allocates property rights may be necessary. This question was first taken up by Gordon (1954) in the case of fishing. His solution was that a sustainable outcome could be achieved as long as one adopted a zero rate of discount, something that does not obtain in any given realistic situation. Moreover, Gordon's solution is based on an optimal solution for a given species. What is needed is to take into consideration the optimal pricing of biodiverse renewable natural resources.

Optimal Pricing of Biodiverse Renewable Natural Resources

Most approaches to renewable natural resource management proceed on the basis of single species rules designed to limit harvesting to some level below a critical biologically sustainable number. The problem with this approach is that it ignores inter-species symbiosis, with the result that setting a regulatory or tax limit on one species may adversely affect the population of another species on which it depends. This often occurs in the case of predator-prey relations. In the case of fish, the food chain may start with bottom feeders and range all the way to sub-surface species in which changes in the population of one species can have significant effects on the level and distribution of another.

This applies as well to plant species, to animal-species, as well as to the classic fruit orchard-bee production example noted by Coase (1960).

We examine here the question of multiple-species interdependence and how this affects optimal harvesting rates, and thus the pricing of biodiverse renewable natural resources.

Optimal Pricing of Biodiverse Renewable Natural Resources								
0.8889 =X1o/OptX1			Scenario: Scenario:	Sustainable Stocks All Species Optimal Harvest				
Logistical Growth Function: $X(t)=\frac{K}{1+ce^{-rt}}$		Grass Resource X1	Net Benefit function: $\pi(Y_t)=aY_t-(b/2)Y_t^2$		Herbivore Resource X2			Carnivore Resource X3
a =	70.00	Net benefits 1st term	a =	70.00	Net benefits 1st term	a =	70.00	Net benefits 1st term
b =	1.0000	Net benefits 2nd term	b =	1.0000	Net benefits 2nd term	b =	1.0000	Net benefits 2nd term
r =	0.0800	Intrinsic growth rate	r =	0.0800	Intrinsic growth rate	r =	0.0800	Intrinsic growth rate
K=	300.0000	Carrying capacity	K=	99.9900	Carrying capacity, f(s)	K=	9.9990	Carrying capacity, f(h)
δ =	0.0200	Discount rate	δ =	0.0200	Discount rate	δ =	0.0200	Discount rate
ρ =	0.9804	Initial PWFactor	ρ =	0.9804	Initial PWFactor	ρ =	0.9804	Initial PWFactor
β =	0.0200	herbivore grass cons.rate				$\lambda=(1+\delta)(a-bY^*)=(1+r(1-2X/K))(a-bY^*)$		
γ =	0.0100	predation rate per carnivore				$\lambda^*=(1+\delta)\left[a-bK\frac{(r^2-\delta^2)}{4r}\right]$		
η =	0.1000	predator/herb.support ratio						
ε =	0.3333	herbivore to grass ratio						
X1o(min) =	125.0000		X2o(min) =	41.6625		X3o(min) =	4.1663	
X1o =	100.00	Initial Production rate	X2o =	33.33	Initial Production rate	X3o =	3.33	Initial Production rate
Y1o =	0.0000	Initial Harvest rate	Y2o =	0.0000	Initial Harvest rate	Y3o =	0.0000	Initial Harvest rate
Opt 'X1	125.0000		Opt 'X2	41.6625		Opt 'X3	4.1663	
Opt Y1								
Opt X1	112.5000	X* = K(r-d)/2r	Opt X2	37.4963	X* = K(r-d)/2r	Opt X3	3.7496	X* = K(r-d)/2r
Opt Y1	5.6250	Y* = K(r2-d2)/4r	Opt Y2	1.8748	Y* = K(r2-d2)/4r	Opt Y3	0.1875	Y* = K(r2-d2)/4r
	65.66	λ+ = (1+δ)[α−βK(ρ2−δ2)/(4ρ)]		69.49	λ+ = (1+δ)[α−βK(ρ2−δ2)/(4ρ)]		71.21	λ+ = (1+δ)[α−βK(ρ2−δ2)/(4ρ)]
λ1*o =	65.66	= optimal shadow price	λ2*o =	69.49	= optimal shadow price	λ3*o =	71.21	= optimal shadow price
λ1o =	65.23	= shadow price today	λ2o =	52.86	= shadow price today	λ3o =	43.00	= shadow price today
X1 min.sus =	100.00	Resource X1			Resource X2			Resource X3

The table above illustrates the technical conditions and initial parameters for a three-species renewable natural resource problem. First is a natural resource stock that serves as a food source for an herbivore population. In turn, the herbivore population serves as a food source for a predator species population. The problem is how to derive an optimal harvest rate across the species that preserves a given level of biodiversity.

Comparative Effects of Non-Harvesting and Sequential Optimal Harvesting under Base Case Scenarios

	PVNB	IRB	λ1	λ2	λ3	λ+1	λ+2	λ+3
Base Case								
biomass alone, no harvesting	$5,923.60	1.0000	60.89	73.58	43.00	65.66	69.49	71.21
biomass, herbivore no harvesting	$15,201.13	0.0670	64.48	73.58	43.00	65.66	69.49	71.21
all species no harvesting	$15,507.53	0.1817	64.26	73.58	43.00	65.66	69.49	71.21
biomass alone optimal harvest	$18,436.49	1.0000	65.66	50.47	43.00	65.66	69.49	71.21
biomass with herbivore optimal harvest	$17,530.98	0.1082	63.96	73.58	43.00	65.66	69.49	71.21
carnivore optimal harvest	$15,818.48	0.1859	64.35	73.58	43.00	65.66	69.49	71.21
biomass-herbivore optimal harvest	$19,705.43	0.0689	65.83	73.58	43.00	65.66	69.49	71.21
biomass-carnivore optimal harvest	$17,496.48	0.1715	65.93	73.58	43.00	65.66	69.49	71.21
herbivore-carnivore optimal harvest	$16,263.24	0.2024	63.67	73.58	43.00	65.66	69.49	71.21
all species optimal harvest	$18,026.86	0.1701	66.09	66.27	43.00	65.66	69.49	71.21

Base Case Parameters		X1	X2	X3
Net benefits 1st parameter	a =	70.00	70.00	70.00
Net benefits 2nd parameter	b =	1.00	1.00	1.00
Intrinsic growth rate	r =	0.0800	0.0800	0.0800
Carrying capacity	K=	300.00	99.99	10.00
Discount rate	δ =	0.0200	0.0200	0.0200
	$X^* = K(r-\delta)/2r$	112.50	37.50	3.75
	$Y^* = K(r^2-\delta^2)/4r$	5.63	1.87	0.19
herbivore grass cons.rate, β =	0.0200			
predation rate per carnivore, γ =	0.0100			
herbivore to grass ratio, σ =	0.3333			
predator/herb.support ratio, ħ =	0.1000			
n =	30.00			

Under base case assumptions, the optimal stocks of X*1, X*2, and X*3, respectively, are 112.5, 37.5, and 3.75, with the corresponding optimal harvest rates set at 5.63, 1.87, and .19, all over a given 30 year time horizon and a discount rate of 2 percent. The optimal pricing of a unit for each species is 65.55, 69.49, and 71.21, respectively, leading to an index of relative biodiversity at .1701.

It may be the case that the initial stocks of a set of renewable natural resources do not satisfy a biological steady-state. Random effects can result in disequilibrium initial conditions, which in turn would affect the choice of optimal harvest rates. We consider this possibility in terms of initial deficit or surplus stocks of the three renewable resources, and then derive adjusted optimal levels and their corresponding prices.

Comparative Effects of Initial Excess and Deficient Sustainable Stocks

	PVNB	IRB	λ1	λ2	λ3	λ+1	λ+2	λ+3
Excess Minimum Initial Stocks								
biomass alone, no harvesting	$11,009.66	1.0000	63.71	73.58	43.00	65.45	69.42	71.20
biomass, herbivore no harvesting	$12,516.33	0.0670	66.64	73.58	43.00	65.45	69.42	71.20
all species no harvesting	$12,838.25	0.1701	66.64	73.58	43.00	65.45	69.42	71.20
biomass alone optimal harvest	$17,021.86	1.0000	65.67	50.66	43.00	65.45	69.42	71.20
biomass with herbivore optimal harvest	$14,450.32	0.1518	61.96	73.58	43.00	65.45	69.42	71.20
carnivore optimal harvest	$13,155.52	0.2088	62.00	73.58	43.00	65.45	69.42	71.20
biomass-herbivore optimal harvest	$21,814.80	0.0690	65.78	61.54	43.00	65.45	69.42	71.20
biomass-carnivore optimal harvest	$20,218.29	0.1222	65.78	61.54	43.00	65.45	69.42	71.20
herbivore-carnivore optimal harvest	$13,483.51	0.1595	61.85	73.58	43.00	65.45	69.42	71.20
all species optimal harvest	$21,261.96	0.1811	65.63	61.14	43.00	65.45	69.42	71.20
Deficient Minimum Initial Stocks								
biomass alone, no harvesting	$5,923.60	1.0000	60.89	73.58	43.00	66.61	64.8	71.24
biomass, herbivore no harvesting	$10,879.03	0.0670	61.69	73.58	43.00	66.61	64.8	71.24
all species no harvesting	$11,678.15	0.2094	61.44	73.58	43.00	66.61	64.8	71.24
biomass alone optimal harvest	$18,243.26	1.0000	65.23	49.55	43.00	66.61	64.8	71.24
biomass with herbivore optimal harvest	$13,657.37	0.1440	61.46	73.58	43.00	66.61	64.8	71.24
carnivore optimal harvest	$11,985.14	0.2101	61.45	73.58	43.00	66.61	64.8	71.24
biomass-herbivore optimal harvest	$22,928.16	0.0670	65.23	52.86	43.00	66.61	64.8	71.24
biomass-carnivore optimal harvest	$21,328.78	0.0670	65.23	52.86	43.00	66.61	64.8	71.24
herbivore-carnivore optimal harvest	$11,803.45	0.2118	61.44	73.58	43.00	66.61	64.8	71.24
all species optimal harvest	$21,401.01	0.1707	65.19	52.86	43.00	66.61	64.8	71.24

Excess and Deficiency Stock Parameters		X1	X2	X3
Net benefits 1st parameter	a =	70.00	70.00	70.00
Net benefits 2nd parameter	b =	1.00	1.00	1.00
Intrinsic growth rate	r =	0.0800	0.0800	0.0800
Carrying capacity	K=	300.00	99.99	10.00
Discount rate	δ =	0.0200	0.0200	0.0200
Base Case $X^* = K(r-\delta)/2r$		112.50	37.50	3.75
Base Case $Y^* = K(r^2-\delta^2)/4r$		5.63	1.87	0.19
Excess Initial Stock, $X^* = K(r-\delta)/2r$		80.00	26.66	2.67
Excess Initial Harvest, $Y^* = K(r^2-\delta^2)/4r$		5.63	1.87	0.19
Deficient Initial Stock, $X^* = K(r-\delta)/2r$		125.00	41.66	4.17
Deficient Initial Harvest, $Y^* = K(r^2-\delta^2)/4r$		5.63	1.87	0.19
herbivore grass cons.rate, β =	0.0200			
predation rate per carnivore, γ =	0.0100			
herbivore to grass ratio, σ =	0.3333			
predator/herb support ratio, h =	0.1000			
n =	30.00			

Another question is whether a re-balanced steady-state solution would be sufficient in the presence of population growth. Classical economists predicted the rising population growth would set a limit to the stock of natural resources, thus leading to what they characterized as the steady-state. Here we consider the possibilities of embodied and disembodied technical change. Embodied technical change can be seen in terms of genetically modified renewable natural resources whose growth rates are superior to existing rates. Disembodied technical change reflects adaptive characteristics of species to live in a more crowded habitat. While the former may be subject to human engineering, the latter is less predictable. Nevertheless, both cases illustrate the possibility of a neoclassical solution to natural resource scarcity instead of the classical diminishing returns scenario.

Comparative Effects under Technical Change

	PVNB	IRB	λ1	λ2	λ3	λ+1	λ+2	λ+3
Embodied Technical Change (r=.10 versus .08 base case)								
biomass alone, no harvesting	$4,588.87	1.0000	57.71	74.36	36.27	64.06	68.95	71.16
biomass, herbivore no harvesting	$10,084.86	0.0670	58.57	74.36	36.27	64.06	68.95	71.16
all species no harvesting	$11,936.57	0.2072	58.29	74.36	36.27	64.06	68.95	71.16
biomass alone optimal harvest	$23,051.76	1.0000	64.06	38.07	36.27	64.06	68.95	71.16
biomass with herbivore optimal harvest	$14,327.71	0.1611	58.22	74.36	36.27	64.06	68.95	71.16
carnivore optimal harvest	$12,315.00	0.2107	58.31	74.36	36.27	64.06	68.95	71.16
biomass-herbivore optimal harvest	$29,916.24	0.0678	64.12	50.37	36.27	64.06	68.95	71.16
biomass-carnivore optimal harvest	$27,396.27	0.1717	64.18	50.37	36.27	64.06	68.95	71.16
herbivore-carnivore optimal harvest	$13,075.24	0.2239	58.15	74.36	36.27	64.06	68.95	71.16
all species optimal harvest	$28,978.52	0.1748	64.09	50.37	36.27	64.06	68.95	71.16
Disembodied Technical Change (K=10% over base; r = .08)								
biomass alone, no harvesting	$5,321.02	1.0000	60.69	73.58	43.00	65.66	69.49	71.21
biomass, herbivore no harvesting	$10,255.64	0.0670	61.49	73.58	43.00	65.66	69.49	71.21
all species no harvesting	$11,148.01	0.2103	61.23	73.58	43.00	65.66	69.49	71.21
biomass alone optimal harvest	$19,109.58	1.0000	65.66	50.39	43.00	65.66	69.49	71.21
biomass with herbivore optimal harvest	$13,329.18	0.1613	61.17	73.58	43.00	65.66	69.49	71.21
carnivore optimal harvest	$11,530.39	0.2143	61.25	73.58	43.00	65.66	69.49	71.21
biomass-herbivore optimal harvest	$21,669.99	0.0670	65.78	61.31	43.00	65.66	69.49	71.21
biomass-carnivore optimal harvest	$14,081.04	0.2091	61.13	71.15	51.01	65.09	69.09	71.15
herbivore-carnivore optimal harvest	$14,381.01	0.2212	61.02	73.15	51.01	65.09	69.09	71.15
all species optimal harvest	$24,357.64	0.1744	65.14	61.86	51.01	65.09	69.09	71.15

Technical Change Parameters		X1	X2	X3
Net benefits 1st parameter	a =	70.00	70.00	70.00
Net benefits 2nd parameter	b =	1.00	1.00	1.00
Intrinsic growth rate	r =	0.0800	0.0800	0.0800
Embodied Technical Change growth rate	r' =	0.1000	0.1000	0.1000
Carrying capacity	K=	300.00	99.99	10.00
Disembodied Technical carrying capacity	K'=	330.00	109.99	11.00
Discount rate	δ =	0.0200	0.0200	0.0200
	$X^* = K(r-\delta)/2r$	112.50	37.50	3.75
	$Y^* = K(r^2-\delta^2)/4r$	5.63	1.87	0.19
	Disembodied Optimal Stock, $X^* = K(r-\delta)/2r$	123.75	45.37	4.99
	Disembodied Optimal Harvest, $Y^* = K(r^2-\delta^2)/4r$	6.19	2.27	0.25
herbivore grass cons.rate, β =	0.0200			
predation rate per carnivore, γ =	0.0100			
herbivore to grass ratio, σ =	0.3333			
predator/herb support ratio, h =	0.1000			
n =	30.00			

Next,we consider the impact of alternative discount rates. While we saw the effects of alternative discount rates for a single species, in the case of multiple species, a uniformly different discount rate does not necessarily result in a biologically sustainable equilibrium in that intrinsic growth rates of species may differ, and thus higher discount rates that generate higher rates of harvesting may produce a biological disequilibrium. There is no simple solution to this question, except to note that setting a discount rate sufficient to offset the intrinsic growth rates of the slowest growing species may provide one scenario that is consistent with an underlying standard of environmental sustainability.

Comparative Effects under Alternative Discount Rates

	PVNB	IRB	λ1	λ2	λ3	λ+1	λ+2	λ+3
Increase in discount rate (δ = 5% vs. 2% base)								
biomass alone, no harvesting	$1,022.26	1.0000	62.75	74.28	43.04	69.66	72.22	73.37
biomass, herbivore no harvesting	$1,877.44	0.0670	63.58	74.28	43.04	69.66	72.22	73.37
all species no harvesting	$2,015.34	0.2094	63.32	74.28	43.04	69.66	72.22	73.37
biomass alone optimal harvest	$8,061.23	1.0000	69.65	67.41	43.04	69.66	72.22	73.37
biomass with herbivore optimal harvest	$3,946.90	0.2438	63.04	74.28	43.04	69.66	72.22	73.37
carnivore optimal harvest	$2,260.54	0.2322	63.46	74.28	43.04	69.66	72.22	73.37
biomass-herbivore optimal harvest	$8,902.54	0.0670	68.11	65.00	43.04	69.66	72.22	73.37
biomass-carnivore optimal harvest	$7,499.27	0.1701	68.11	65.84	43.04	69.66	72.22	73.37
herbivore-carnivore optimal harvest	$2,841.92	0.2257	63.22	74.28	43.04	69.66	72.22	73.37
all species optimal harvest	$6,927.46	0.2043	65.78	71.34	43.04	69.66	72.22	73.37
Decrease in discount rate (δ = 0% vs. 2% base)								
biomass alone, no harvesting	$204,377.12	1.0000	60.54	73.44	42.99	64.06	68.07	69.87
biomass, herbivore no harvesting	$375,350.45	0.0670	61.33	73.44	42.99	64.06	68.07	69.87
all species no harvesting	$402,921.75	0.2094	61.08	73.44	42.99	64.06	68.07	69.87
biomass alone optimal harvest	$399,004.89	1.0000	64.06	3.13	42.99	64.06	68.07	69.87
biomass with herbivore optimal harvest	$401,733.37	0.1217	61.16	73.44	42.99	64.06	68.07	69.87
carnivore optimal harvest	$407,601.18	0.2050	61.11	73.44	42.99	64.06	68.07	69.87
biomass-herbivore optimal harvest	$534,197.95	0.0678	64.11	52.31	42.99	64.06	68.07	69.87
biomass-carnivore optimal harvest	$544,730.73	0.1734	64.11	52.31	42.99	64.06	68.07	69.87
herbivore-carnivore optimal harvest	$410,327.70	0.2018	61.13	73.44	42.99	64.06	68.07	69.87
all species optimal harvest	$410,342.71	0.2018	61.13	73.44	42.99	64.06	68.07	69.87

Alternative Discount Parameters		X1	X2	X3
Net benefits 1st parameter	a =	70.00	70.00	70.00
Net benefits 2nd parameter	b =	1.00	1.00	1.00
Intrinsic growth rate	r =	0.0800	0.0800	0.0800
Carrying capacity	K=	300.00	99.99	10.00
Base case discount rate	δ =	0.0200	0.0200	0.0200
Alternative discount rate 1	δ' =	0.0500	0.0500	0.0500
Alternative discount rate 2	δ' =	0.0000	0.0000	0.0000
	$X^* = K(r-\delta)/2r$	112.50	37.50	3.75
	$Y^* = K(r^2-\delta^2)/4r$	5.63	1.87	0.19
	Optimal Stock, $X^* = K(r-\delta)/2r$	123.75	45.37	4.99
	Optimal Harvest, $Y^* = K(r^2-\delta^2)/4r$	6.19	2.27	0.25

herbivore grass cons.rate, β =	0.0200
predation rate per carnivore, γ =	0.0100
herbivore to grass ratio, σ =	0.3333
predator/herb support ratio, h =	0.1000
n =	30.00

Next, we consider the impact of <u>random behavior</u> on the choice of an optimal solution. Depending on the relative magnitude and time-dependent uniformity of random behavior involving renewable natural resources, it is possible to derive adjusted optimal solutions, based on expected values of the respective outcomes. However, if there is trend random behavior, then it is less obvious that a unique solution may be found. The possibility of such patterns is what lies behind the precautionary approach to renewable natural resource and environmental sustainability policies. We do not propose to answer here whether this is an acceptable standard, partly because we have not yet considered an explicit intertemporal welfare function from which to arrive at a consistent conclusion.

Comparative Effects under Stochastic Behavior

	PVNB	IRB	λ1	λ2	λ3	λ+1	λ+2	λ+3
Increase in discount rate (δ = 5% vs. 2% base)								
biomass alone, no harvesting	$1,022.26	1.0000	62.75	74.28	43.04	69.66	72.22	73.37
biomass, herbivore no harvesting	$1,877.44	0.0670	63.58	74.28	43.04	69.66	72.22	73.37
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Carrying capacity	K=	300.00	99.99	10.00
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