



Financial Analysis of Development Projects

July 2011

Phillip LeBel, Ph.D., Director
Center for Economic Research on Africa
Montclair State University
(translated and updated from the French original version, ©2010,2009,2004,2001,1999)
lebelp@mail.montclair.edu
<http://msuweb.montclair.edu/~lebelp/professionalactivity.html>

Acknowledgements

This module was prepared originally in French by Alan Johnson, Deputy Administrator of the U.S. AID project of the ENEA School in Dakar, Senegal, and by Richard Vengroff, Dean, Division of International Affairs at the University of Connecticut at Storrs, with an update and English translation by Phillip LeBel, Director of the Center for Economic Research on Africa (CERAF) at Montclair State University in Upper Montclair, New Jersey. Recent versions reflect revisions and extensions by the author.

Overview

The following exercises have been developed to introduce basic notions of financial analysis as used in the evaluation of development projects. They have been used in management training seminars in Africa and in the U.S. for the past several years, primarily to French-speaking participants, and are now being made available to English speaking audiences. While many project administrators often have a good working knowledge and experience with development issues, they often may lack training in the tools of financial analysis in the context of project management. This module is designed as an introduction to the most commonly used tools and draws on examples within an applied developed setting.

The module is divided into three parts. Part one concentrates on a self-contained set of exercises on present value calculations. While based on individual work by participants, there are several ways that these materials can be structured for group participation and presentations. Wherever appropriate, participants should also ask questions directly to instructors to ensure a good mastery of materials. Part two is based on calculating the net present value of a project, its interpretation and its application within a development project setting. Part three uses a case study that draws on concepts and techniques developed throughout the preceding sections. Participants first work individually on the case study, then are organized into groups to share their findings among each other. Then, using discount rates which the instructor assigns to each group, participants develop a project analysis evaluation tableau on flip charts, or, where available, on computer spreadsheets, to evaluate the acceptability of the case study project.

A. On the Range and Scope of Financial Markets

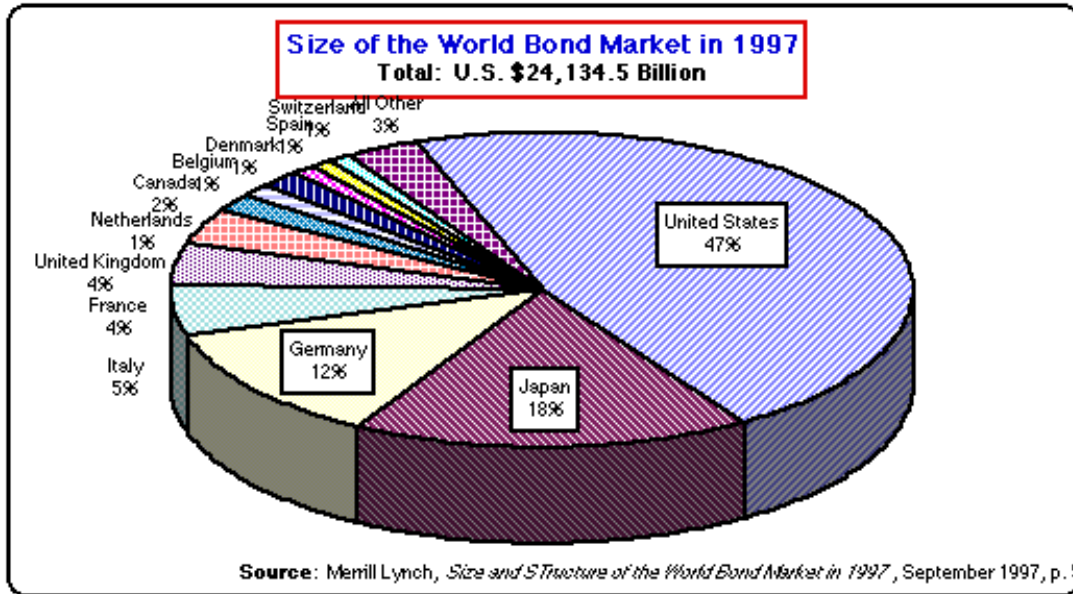
Financial markets play a crucial role in the allocation of investment resources. They do so not just in terms of decisions that take place over time. They also manage the implicit and explicit risks that are inherent in any time-based decision. This carries particular significance for developing economies, and efforts to develop local financial institutions is an important step in achieving sustainable economic growth. As such, we first take a look at the range and scope of financial markets.

If production decisions were made in the absence of durable capital equipment, there would be little need for financial markets. Yet because modern economic growth takes place within a broad spectrum of the production and use of capital goods that there is a need for a corresponding set of financial institutions and contracts to allocate these resources in an efficient manner. Financial markets allow for the scheduling of financial flows of an investment project in such a way that the costs and benefits may be evaluated in a consistent fashion. Such markets involve a broad range of instruments that cover debt and equity finance, along with various insurance tools to manage the corresponding levels of risk.

To have some idea of the scope of these instruments, let us look first at the scale of the global bond market. Based on statistics from the World Bank, the 1995 value of the global capital market stood at \$U.S. 27.59 trillion, and as of 2010 stood at over \$U.S. 45 trillion. In 1997, the assets of the various financial institutions participating in this market were valued at \$U.S. 24.13 trillion, of which some \$U.S.23.69 trillion represented the current value of equity markets, and with bank loans operating at a level of \$U.S. 26.49 trillion. When compared to the global value of production, the capitalized value of financial markets represented some three times the value of global economic production.

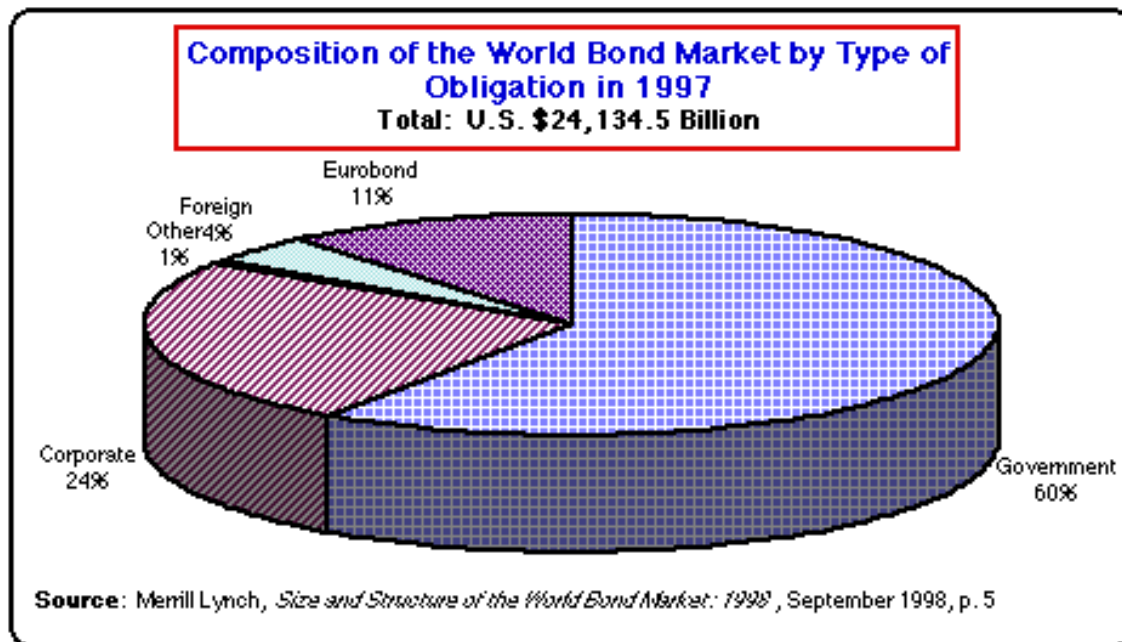
Figure 1 below provides a breakdown of the size of the global bond market in 1997. As can be seen, The United States accounted for just under one-half of all bonds issued, followed by Japan at 18 percent, and Germany at 12 percent.

Figure 1



Bonds are issued in various denominations with various maturities over differing interest rates. They can be issued by government agencies as well as by private firms. Although firms can and do issue bonds, for the most part it is government agencies that dominate this type of market. As can be seen in Figure 2 below, governments account for more than half of all outstanding bonds, a proportion that has varied little over the years.

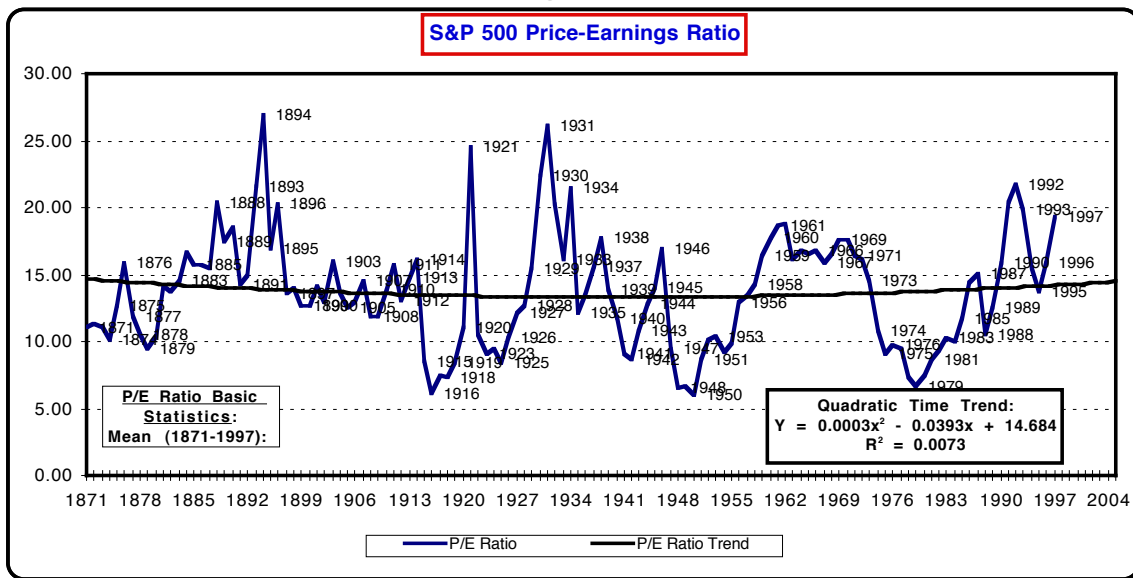
Figure 2



The choice of financing for capital investments is driven by several considerations. One is the primacy of claims over the borrowing institution. In the case of bonds, no ownership is conferred, and bonds carry a priority claim over equities. For this reason, bonds generally carry lower levels of interest than the underlying rates of interest associated with equity markets, as will be noted below.

The pricing of financial assets depends on several measures, or ratios, and for which whose prices adjust in response to changes in the level and distribution of information. In developed country markets at least, financial economists have contended that within all of the financial fluctuations that we observe over time, the pricing of these markets is efficient in that asset values reflect current and future economic conditions surrounding any particular asset.

Figure 3

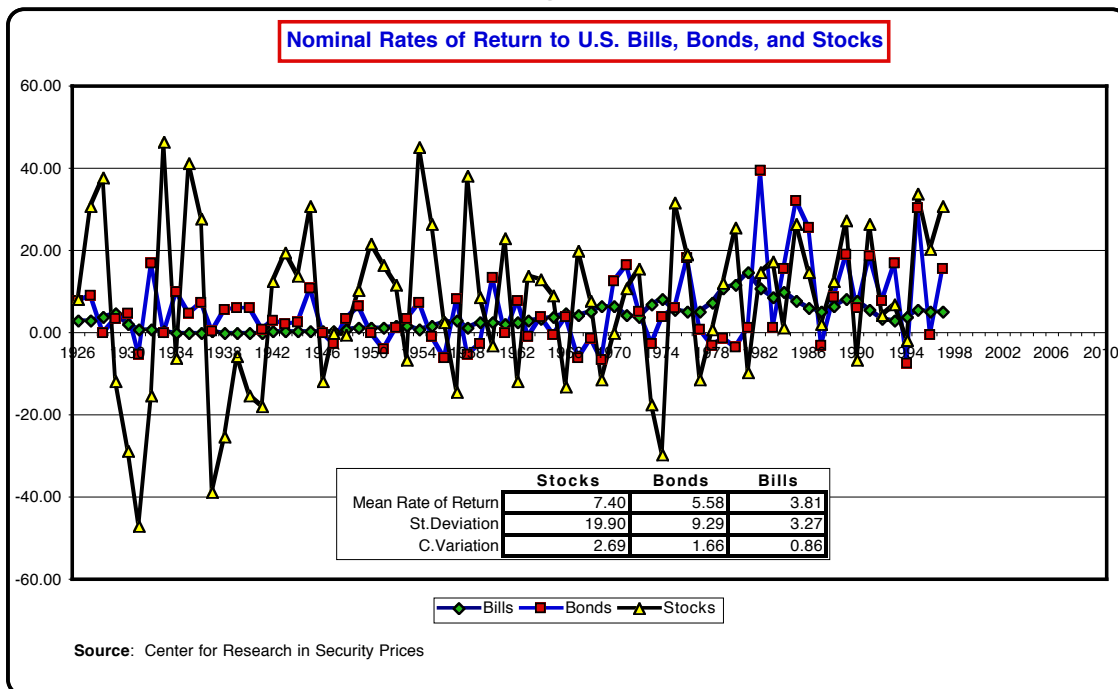


In deciding among various types of financing instruments, one way to establish a common denominator is to derive a corresponding yield, or rate of return. For bonds, this is a straightforward calculation in which the absolute level of interest earned over time is divided by the corresponding price of the bond. For stocks, the yield is reflected in the ratio of the earnings plus the appreciation in the value of a share of stock divided by its price. While financial markets are considered to be relatively efficient, there are several measures that are used as benchmarks to determine if prices are either over or undervalued.

One such measure is the price-earnings ratio, or the reciprocal of the rate of return. Financial evaluations are undertaken by any number of firms to determine if bonds or stock prices are mis-specified by some historical reference. One firm, Standard and Poors, has tracked a basket of 500 stocks over time to provide a reference on stock pricing efficiency. In figure 3 above, we show the S&P 500 price-earnings ratio for the 1871-1997 time period, and plot the corresponding long term trend. The higher is the P-E ratio for any given time period relative to a long-term trend, the more the stock is over-priced, and vice-versa. While this benchmark is not an absolute guarantee for the determination of an efficient equity price level, it does serve as a first-order approximation.

One principle guides the pricing of financial assets and their associated rates of return: the price should correspond to the pure time-value of money plus a given risk premium. The more volatile is an asset, the higher the level of risk, and for which the average rate of return should be higher than a risk-free asset's rate of return. In Figure 4 below, we show a comparison of the annual rate of return on stocks, bonds, and treasury bills for the 1926-1995 time period. A quick inspection suggests that stocks are more volatile than bonds, and in turn more volatile than treasury bills. Not surprisingly, there is a direct relationship between the mean rate of return and the corresponding level of volatility.

Figure 4

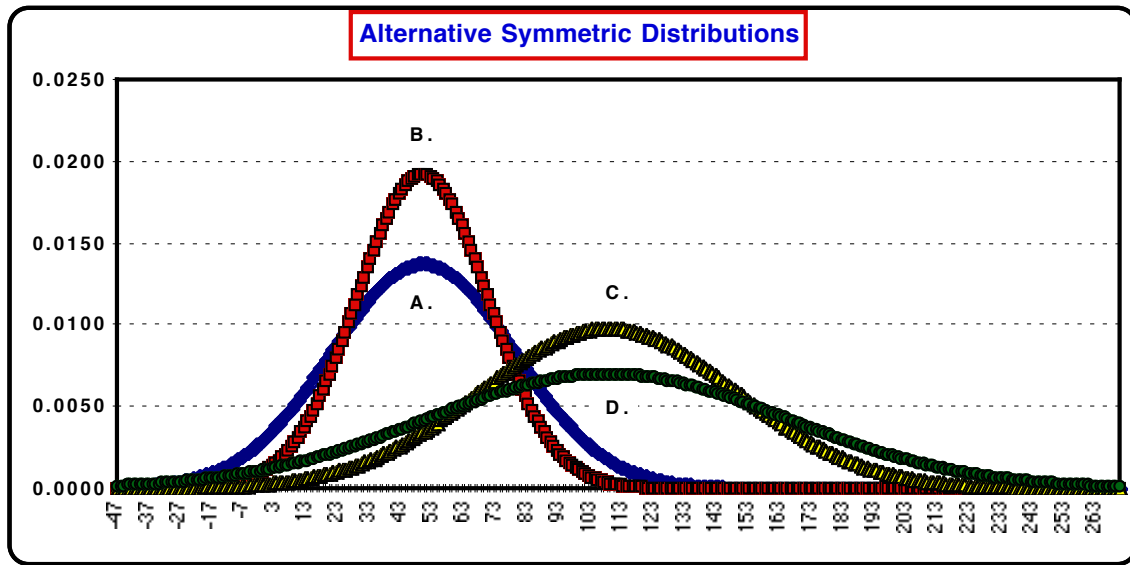


If the yields on stocks, bonds, and bills differ in proportion to the underlying level of risk, we have a reasonable solution to the problem of their prices, i.e, the prices are efficient in that they reflect all underlying information regarding the present and future state of events surrounding the asset. This said, we can not state at this point that once we have a given level of accuracy in the measurement of risk we have a clear choice as to how to select among the various alternatives.

To illustrate the question of risk selection, consider the various distributions listed below in Figure 5. Each distribution is symmetric in that the the area to the left of each peak is identical to the area to its right. On the X-axis we have the various possible outcomes from a given choice, and as can be seen, this includes some negative ones as well as positive ones. On the Y-axis we have the various probabilities that are associated with each possible outcome. The expected value is the sum of the products of each probability multiplied by its corresponding outcome.

Let us now look at two of the symmetric distributions, namely, distribution A and B. Which of these would you prefer? They each have the same expected outcome as can be seen by the peaks corresponding to the same outcome on the X-axis. To answer this question, consider what a riskless choice would look like – it would be a perfectly vertical line, with no distribution tails on either side. With this in mind, it is clear that distribution B has less of a spread, and thus less risk, than distribution A. If these were the only choices you would confront, if you are normally risk averse, you would prefer distribution B to distribution A. Now consider distribution C and D. they both have a higher expected outcome than either distribution A or B. If the choice were between distribution C and D alone, the risk averse logical choice would be C as it has a smaller level of risk than distribution D.

Figure 5



We have now considered the easier choices. But now consider which choice to make is it only between distribution C or B. Distribution C has a higher expected value than distribution B, but it also has a higher level of risk. The choice in this case is not obvious. In fact, it depends entirely on the risk tolerance, or preference, of the individual making the decision. For some individuals, the higher expected value of distribution C is sufficiently higher than for distribution B, in which case C is adopted. However, for others, the risk premium is not sufficient to overcome their level of risk aversion. In this case, they would select distribution B over distribution C. Either choice is acceptable and consistent with the notion of efficient markets in that markets allow for and recognize differences in individuals' perceptions and attitudes toward risk. In short, in order to enjoy a higher outcome, one has to undertake some additional level of risk.

Although we have now provided a mechanism for the measurement of risk, thus far we have looked only at financial risk. Yet financial risk is inter-related to other factors, namely, political, economic, and environmental risks. For this reason, particularly in the context of developing countries that do not issue sovereign debt, there is a need to provide some measure of the overall risk climate in a given country. The World Bank periodically publishes such an index, as prepared by the International Country Risk Group, or ICRG. This index, which ranges from 0 to 100, with higher values inversely related to the level of aggregate country risk, provides a proxy measure of the level of risk as countries engage in the continuing process of economic growth and development.

Table 1 provides a profile of the ICRG index across several regions in relation to GDP and per capita GNP. As can be seen, there is a rough inverse relationship between the level of per capita GNP or per capita GDP of a country and the aggregate level of country risk.

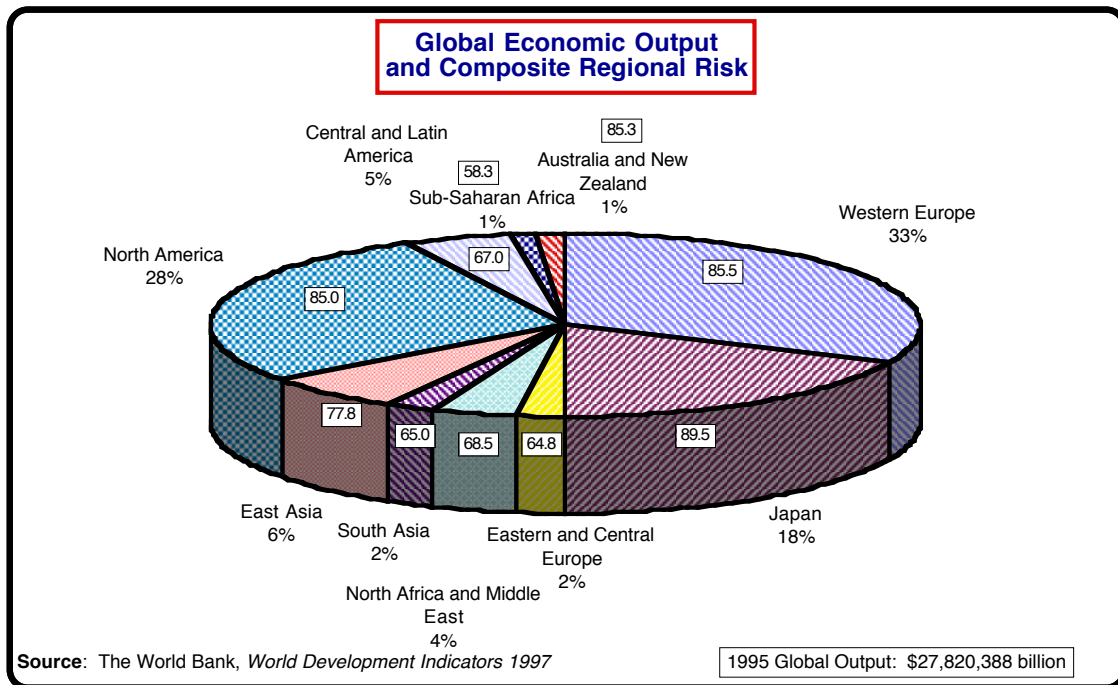
Table 1

Global Economic Output and Composite Regional Risk				
	Total GDP	Median C.Risk, 1996	Median C.Risk, 1991	Median PPP PCGNP
Western Europe	\$8,828,092	85.5	81.5	\$19,950
Japan	\$5,108,540	89.5	84.5	\$22,110
Eastern and Central Europe	\$619,087	64.8	56.5	\$4,480
North Africa and Middle East	\$1,061,724	68.5	55.5	\$5,320
South Asia	\$624,835	65.0	44.0	\$2,230
East Asia	\$1,733,147	77.8	66.0	\$5,230
North America	\$7,770,986	85.0	83.0	\$21,130
Central and Latin America	\$1,365,961	67.0	58.0	\$3,870
Sub-Saharan Africa	\$302,164	58.3	51.3	\$1,175
Australia and New Zealand	\$405,852	85.3	79.3	\$17,650
Total	\$27,820,388	68.5	56.5	\$4,360

Source: *The Wall Street Journal*

What is crucial in this context is that to the extent that risk is a determinant of economic growth, then countries need to take the question of risk management into consideration in the framing and execution of development growth strategies. That this is relevant to the development context can be appreciated in reference to Figure 6, which translates the data from Table 1 into graphical form.

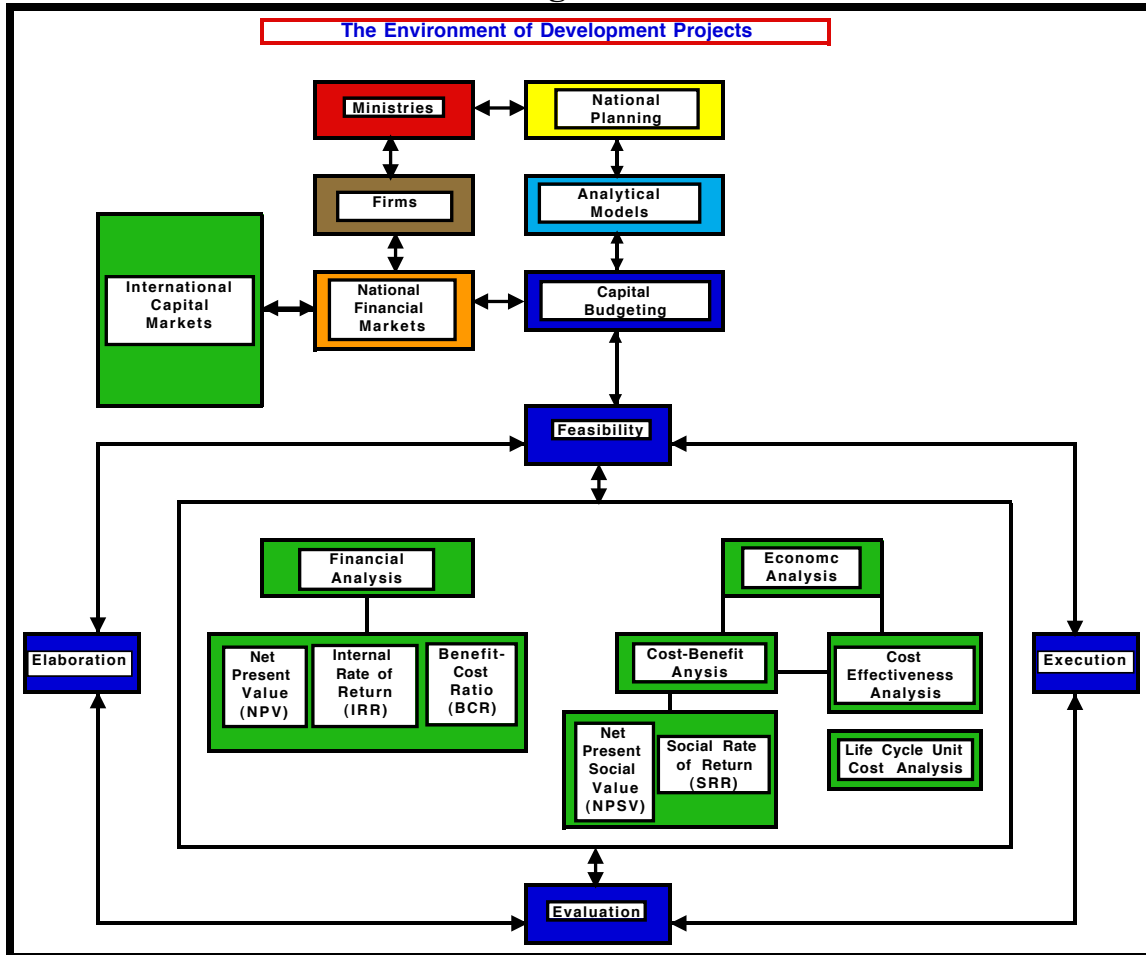
Figure 6



Given the measure and distribution of risk in a given economy, we now link its role to that of the various institutions charged with making resource allocation decisions. In the context of development projects, this means that we must integrate the framework of project identification, implementation, evaluation, and follow-up with explicit tools to measure and measure the associated levels of risk.

Developing countries often are faced not just with the challenge of raising per capita incomes. They also must work with and foster the development of financial institutions that can allocate investment resources to their most efficient uses. Figure 7 below illustrates the sometimes complex interactions that define the context of development projects. In addition to a series of public sector institutions through which decisions must pass, international institutions also interact with locally based financial institutions and firms to define the context and rationale of a given project. To do so, one utilizes the tools of project management, namely, financial analysis, project management tools, as well as operations analysis to define the structure of a project. To be successful, such a project must also gain the commitment of local stakeholders, for without their commitment any project will fall far short of its goals.

Figure 7



As we have already noted, comparability of projects is determined in reference to a set of common evaluation standards. As can be seen in Figure 7, several such measures are used. For a purely financial analysis, the key tools are the derivation of a project's net present value, or NPV, the corresponding internal rate of return, or IRR, and the corresponding Benefit-Cost ratio. If we extract the internal rate of return as our decision variable, the decision to accept or reject a project is conditioned by the opportunity cost of capital. This refers to the cost of funds made available to a project in terms of the next best use of those resources. In aggregate, financial markets interact toward a continuing equilibrium through the intersection of the opportunity cost of capital in relation to the underlying rate of return to a given project. As long as a series of projects have underlying expected rates of return in excess of the opportunity cost of capital, this forms the basis for their inclusion in a given decision environment.

Figure 8

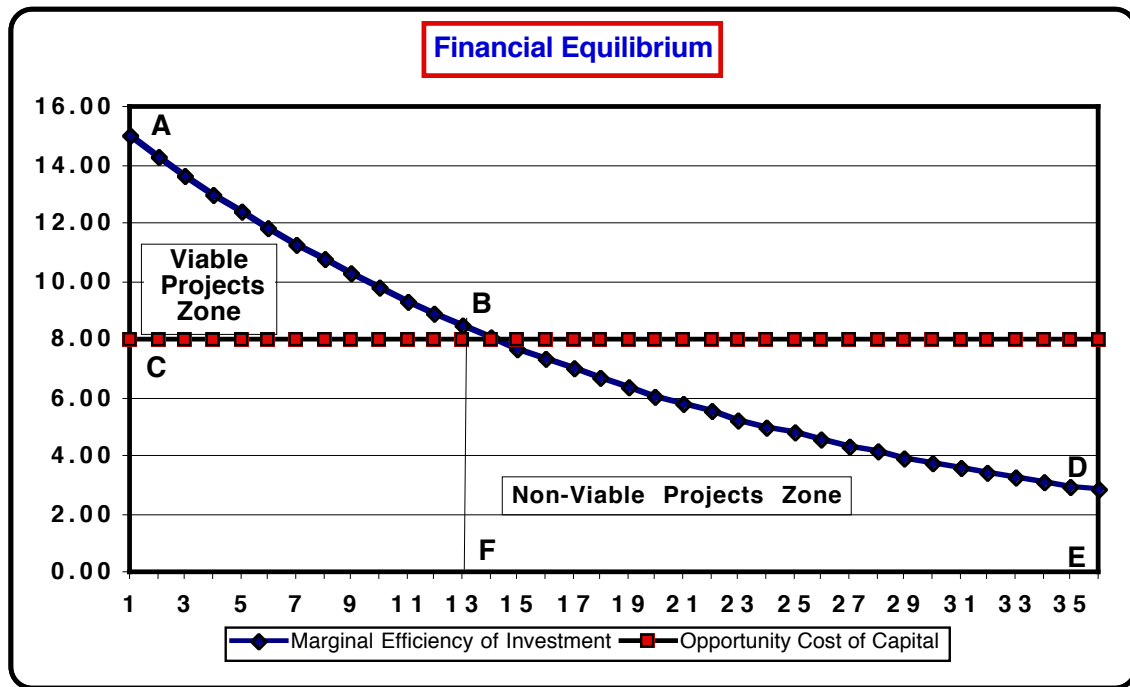


Figure 8 illustrates the relationship between a ranking of various investments and the opportunity cost of capital. To the left of the intersection is the zone of acceptable projects. They have rate of return that are in excess of the opportunity cost. To the right are projects whose rate of return is less than the opportunity cost of capital. The determination of the opportunity cost of capital thus is an important step in deciding whether to accept or reject a given project. Since this is governed by a country's underlying set of interest rates, how the central bank of a country adopts its rate of discount will in turn determine the underlying level of investment at a given moment in time.

Now that we have in place the determinative role of interest rates, let us turn to the role of interest rates in financial analysis. We do so first in terms of the time value of money, followed by a series of exercises that progressively lead to the formulation of an investment project evaluation.

B. The Time Value of Money

To undertake a thorough financial and economic analysis of projects, development planners have had to address the fundamental problem of evaluating costs and benefits - a problem based on what economists call the time value of money.

The notion of time value, however it may seem, is basically quite simple. It is first of all essential to draw a distinction between the present value and the future value of resources. These two values are not equivalent. To understand why, let us consider a concrete example.

Suppose your brother comes to you with a strange proposition. As he steps into your house he makes you an offer between two basic alternatives:

- a. to give you 100,000 CFA francs on the spot which you can spend as you see fit.
- b. to give you 100,000 CFA francs in five years, under the same conditions.

Which of the two would you choose?

Without thinking in terms of economic reasoning, most people would take the money on the spot and be off with it without any further discussion. After all, who could tell what might happen to you in the meantime, what could happen to your brother, or what could take place financially during the next five years.

You have undoubtedly heard of the famous proverb: “a bird in the hand is worth two in the bush”. Apart from the basic common sense of this proverb, there are also economic reasons that justify accepting the money on the spot. These reasons are based on the time value of money. As we will see, the example put forth by your brother will have an important role to play in the analysis of economic benefits of a project.

B.1. The Time Value of Future Revenues

Suppose that you were called upon to undertake a financial analysis of a cattle vaccination project. This project is expected to provide you with a total of 100,000 CFA francs in revenue during the next five years, as is summarized in Table 1

	Annual Revenue
Year 0	100,000
Year 1	100,000
Year 2	100,000
Year 3	100,000
Year 4	100,000
Total:	<hr/> ?

How would you evaluate the value of these annual revenues? Would you add the revenue of each year to calculate the total? Unfortunately, things are not quite that simple. The future revenues of the project, that is, those received in years 1, 2, 3, 4, and 5, do not have the same worth to us today as those which are given in the table.

Think about the numbers in Table 1. Why did you decide to accept the 100,000 CFA francs from your brother today rather than five years from now? You reasoned that to accept the money today was worth more than to wait five years, and you were right. In the same fashion, the benefits of our livestock project today are not equivalent to those to be received in the future.

Unfortunately, contrary to the choices offered by your brother, we often do not have a choice as to when we can receive the benefits from a project. We must wait for a certain period of time in order for the project to generate any benefits. We have to wait for a period of 5 years during which time we receive an annual sum of 100,000 CFA francs. Nevertheless, we have to make a decision today as to whether we should accept or refuse this project. For this reason, we need to calculate the present value of future benefits of our project.

B.2 Simple Interest

We can calculate the present value of benefits from a project in applying a present value formula. Before we do so, let us first look at simple interest, which is the financial corollary and exact opposite of present value calculations. If you lend money to someone, you give up the right to use this money yourself during the period of the loan. In making your money temporarily available to someone else, you are providing a service to the borrower. You thus should be compensated for this service. This compensation takes the form of interest on the loan.

Simple interest is expressed in terms of a percentage in relation to the initial loan amount, known as the principal. Suppose you lent 100,000 CFA francs to someone at an interest rate of 10 percent. At the end of the year, the borrower should repay you as follows:

$$\begin{array}{l} 100,000 + (0.10 \times 100,000) = 110,000 \text{ CFA francs} \\ \text{(principal)} \times \text{(interest)} \quad \quad \quad = \text{(total)} \end{array}$$

If this same borrower were to use the 100,000 CFA francs for a period of 3 years, the calculation would be based on the same principle.

Table 2
Calculation of Simple Interest

Year	Principal	Interest (10%)
0	100,000	10,000
1	100,000	10,000
2	100,000	10,000
<hr/>		
	Total Interest	30,000
	Original Loan	<u>100,000</u>
	Total Due	130,000

Each year, the borrower must pay 10 percent on the principal in order to compensate the lender the right to use the money. After 3 years, the borrower owes the original amount of the loan (100,000) plus 30,000 CFA francs in interest (10,000 CFA francs each year during the 3 year life of the loan).

Problem 1

Using the preceding framework, calculate the simple interest on a loan of 500,000 CFA francs at 13 percent interest for a period of 10 years in the space provided in Table 3.

Table 3
Calculation of Simple Interest

Year	Principal	Interest (13 %)
0	500,000	65,000
1		"
2		"
3		"
4		"
5		"
6		"
7		"
8		"
9		"

	Total Interest	<u> </u>
	Face Value of Loan	500,000
	Total Amount Owed	<u> </u>

4. Compound Interest

Simple interest is rarely used by banks and other types of lending institutions because it poses a disadvantage to the lender. In the preceding example, the borrower is accountable for repayment only at the time of maturity of the loan, or ten years later. Let us look at the following table for a borrower at the end of the first year.

Table 4
Compound Interest

Year	Principal	Interest (10%)	Real Value of the Loan
0	100,000	10,000	(110,000)
1	100,000	10,000	(120,000)
2	100,000	<u>10,000</u>	(130,000)
	Total Interest	30,000	

After one year, the borrower owes the lender simultaneously the 100,000 CFA francs of the initial loan along with 10,000 CFA francs in interest. The real value of the initial loan has accumulated. The same thing takes place at the end of year 2. The borrower owes the lender in effect 120,000 francs (100,000 francs of the principal plus 20,000 francs in interest). It is important to note that the interest here is calculated on the basis of the initial loan of 100,000 CFA francs.

Lenders have eliminated this disadvantage by adopting the system of compound interest. Calculating compound interest is as follows: If you lend me 100,000 CFA francs for a period of 3 years at 10 percent interest, the 10,000 francs in interest that I owe you at the end of the first year of the loan is added to the principal, thus leaving a new principal of 110,000 CFA francs. Next, interest in the following year is calculated on the basis of the 110,000 CFA francs accrued by the loan. To see how this is done, look at the summary calculations for this loan in Table 5.

Table 5
Calculation of Compound Interest

Year	Principal	Interest (10%)	Principal in the Following Year
0	100,000	10,000	100,000
1	110,000	11,000	121,000
2	121,000	12,100	133,100
	Total Interest	33,100	
	Loan Face Value	<u>100,000</u>	
	Total Owed	133,100	

If you compare the total owed in interest based on the simple interest calculations in Table 2 with the compound interest in Table 5, you can see why the latter is more advantageous to the lender and that it reflects much more accurately the time value of the funds lent. Why? Because the value of the initial loan increases over time: the original loan face value of 100,000 CFA francs increases regularly in reaching a value of 133,000 CFA francs at the end of 3 years. Now at this point, consider again the initial proposition made to you by your brother: why did you accept the 100,000 CFA francs today instead of waiting 5 years to do so? Why was it more advantageous for you to accept the money today?

To practice the calculation of compound interest, complete the following table below:

<i>Problem 2</i>			
Year	Principal	Interest (10%)	Principal in the Following Year
0	100,000	10,000	110,000
1	110,000	11,000	121,000
2	121,000	_____	_____
3	_____	_____	_____
4	_____	_____	_____
	Total Interest	_____	
	Face Value of Loan	100,000	
	Total Owed	_____	

Now you can see why you would accept the money today instead of waiting for 5 years. The 100,000 CFA francs that you received today are worth 161,051 CFA francs at the end of 5 years with an interest rate of 10 percent. You made a good choice.

Problem 3

To make sure that you have understood the difference between simple and compound interest, calculate the compound interest on a loan of 175,000 CFA francs at 5 percent for a period of 6 years, using the table shown below.

Year	Principal	Interest (5%)	Principal in the Following Year
0	_____	_____	_____
1	_____	_____	_____
2	_____	_____	_____
3	_____	_____	_____
4	_____	_____	_____
5	_____	_____	_____
	Total Interest	_____	
	Face Value of Loan	175,000	
	Total Owed	_____	

5. Financial Interest in the Real World

Let us now see how the concepts we have introduced are applied in the real world. What do these concepts mean for a development project manager or a private investor who finds him or herself in the middle of an isolated rural community many kilometers from the nearest bank or financial lending institution?

Even if the villagers had available the sum of 100,000 CFA francs, it would be extremely difficult for them, if not impossible, to deposit this money in a savings account with a sufficiently high enough interest rate. Thus, you may be tempted to conclude that all of this discussion on interest rates, on present and future values have absolutely nothing to do with the real world. But if you did, you would be wrong.

Suppose one day that a farmer in a small isolated village miraculously finds an envelope stuffed with CFA bank notes. After the initial excitement of discovery, he discovers that he is indeed in possession of 100,000 CFA francs. What should he do with this money?

If he decides to spend all of the money immediately on household provisions, on clothing, on a radio cassette player and a few other consumer items, we could stop our inquiry right here: there simply would be no need to be concerned about the future value of the 100,000 CFA francs, since the present value of 100,000 CFA francs would have been disposed of forthwith by the purchase of all those goods.

Let us consider, instead, that the harvest this past year was a good one, that our farmer's house is in relatively good condition, that there really isn't much

need to buy any new clothing, and that since no one seems to be aware that he has found this money, he can keep the whole amount for himself. What should he do with the money? If he hides the money under his mattress for 5 years, there certainly is not going to be any increase in its future value. Worse yet, if there is any inflation in prices, the purchasing power of this money is simply going to go down with each successive year that he holds on to it. In any case, it is fairly unlikely that he would simply decide to hide the money under the mattress, not only because he has an intuitive sense of the time value of resources, but also because he saw what happened when mice attacked his larder during the last dry season.

Think for a moment about the ways in which farmers generally make savings decisions in a developing country environment. They rarely have access to banking facilities and, with the exception of *tontines* and a few community based banking institutions such as the Grameen Bank of Bangladesh, the lending of money to local third parties for profit is relatively rare. Now since hiding the money under a mattress or burying it makes little sense, what do farmers generally do with their money? They usually buy physical assets such as livestock, as is typically the case with pastoral farming communities in Africa. The social factors that shape this behavior also make good economic sense.

Suppose that our lucky farmer decides to take the 100,000 CFA francs to buy a few head of cattle. On the day of the purchase, the cattle acquired turn out to be worth exactly 100,000 CFA francs. Thus, our farmer makes the purchase. What is the future value of this purchase? How can the cattle he purchases compensate him over time for the money he has just spent?

If the farmer took the precaution of buying both bulls and cows, then as the cattle reproduce, he could increase the size of his herd and thus the value of his investment. In turn, these cattle also provide meat for household consumption, dairy milk, leather, as well as fertilizer for his fields. To the extent that the calves gain weight, they also add to his market value. Our farmer could even accelerate this process by starting up a livestock breeding operation. Given all of these options, if he manages his investment well, he will have assets worth well in excess of the initial 100,000 CFA purchase at the end of five years.

This is exactly the same kind of transformation that took place in the value of the loans that you calculated earlier. The process may look different but the result is the same. In the latter case, the capital yield (which is

equivalent to the interest rate) depends on the tastes and skills of the farmer, on his choice of production technology, on the health of his animals that he buys, as well as other factors. What is clear is that there is a significant difference between the present and future value of money for our farmer, whether or not the money is lent directly to someone else or it is invested in livestock.

Sometimes it seems difficult to see how these tools, which have been developed and applied to developing countries by developed country financial institutions, make sense. Yet even in places where the formal concept of interest is either alien or prohibited, there is an equivalent valuation of compound interest, and thus of the time value of money, in almost all operations.

Our farmer example illustrates clearly how the time value of money can be seen in a real world setting. What the livestock purchase shows is that there is an equivalent to the time value of money in most real world settings. In our present context, we abstract from some of the real world complexities so that we may define and calculate precisely how the time value of money is measured. To evaluate further the yield of such investments as our livestock purchase in terms of their profitability is obviously a bit more complicated, as we shall see, and yet one which needs to be addressed if resources are to be used efficiently.

6. Present Value

Let us return to the problem of compound interest, but this time in reverse. We just saw that for an initial investment of 100,000 CFA francs at a 10 percent rate of interest over 5 years would result in a future value of 161,051 CFA francs. What would be the present value of this same 161,051 CFA francs in the following 5 years? Obviously, it would be worth 100,000 CFA francs.

Consider the following logic: if your brother modified his offer and said instead that you could have 100,000 CFA francs immediately or, if you were willing to wait for 5 years, you could have 161,051 CFA francs. Which would you now choose? Your reply no longer depends on the real value of the two propositions since they are precisely equivalent in value (with a 10 percent rate of interest being understood as given). Thus it is no longer a question of calculating the real value of your sum of money, and turns instead on the question of when you prefer to receive the money.

As another illustration, let us return to our farmer who has taken the 100,000 CFA francs and decided to buy a herd of cattle. Let us analyze five years down the road to see what has become of the different products his herd has produced.

Table 6

Estimated Value of the Farmer's Livestock

A.	Value of livestock 5 years after purchase	150,000
B.	Derivative products	
	Milk	25,000
	Sour Milk	50,000
	Meat	<u>30,000</u>
C.	Future Value	255,000

We can thus see that the livestock and associated products are worth 255,000 CFA francs at the end of 5 years, and which is equivalent to their future value. In terms of our interest calculations, to increase the value of a 100,000 CFA investment to 255,000 CFA over a 5 year period would require a interest rate of over 20 percent compounded annually. Thus, our farmer's investment, if it produced the items listed in Table 6, would indeed have been a good one.

Why should we be so preoccupied with present and future value calculations? In the analysis of development projects, we need to be able to assess decisions which project manager must undertake today in terms of their future worth. As in the case of our livestock versus pure financial loan example, which decision one makes can make an enormous difference over time. We thus need to take into consideration the present value of future revenues generated by investments made today.

7. Future Worth Coefficients (Factors)

You have already calculated the future value of loans with initial face values of 100,000 and 175,000 CFA francs in preceding sections. You calculated interest for the initial year, found the new principal for the following year, recalculated interest for the following year, and repeated the process several times in order to calculate the total amount of interest accrued. Finally, you have added the accumulated amount of interest to the face value of a loan to determine the future value of the loan.

Because the numbers we have been using have been rounded off, because the time period involved has been reasonably short, and because you have also had ready-to-calculate tables, the time needed to complete these

calculations has been fairly limited. Unfortunately, there are many real-world situations far more complicated than the ones we have been using, and we need to find a way to do the calculations without generating ever more complex tables.

Try to imagine how difficult it would be if someone asked you to calculate the future value of 65,123,984 CFA francs at 14.876 percent interest for a period of 36 years. If you had used only the tools that we have thus far applied, you would take far too long to complete the calculation. Fortunately, there is a more simple approach.

Calculating the future value depends on three variables:

1. the initial investment
2. the interest rate
3. the number of time periods

Go back to Table 5. If you take each amount under the column labeled "Principal" and you multiply it by (1+ the interest rate), the result gives the figure under the column "Principal for the following year". Try this out with a calculator to verify the results. Notice also that the last entry under "Principal of the Following Year" is the same as the amount listed for "Total Interest" at the bottom of the table.

What we have been doing is to multiply the principal by a multiplier coefficient to derive the future value for each period. The formula for this compound interest calculation can be summarized as follows:

Formula A:

$$\text{Future Value} = (1 + \text{Interest Rate})^{\text{(Number of Years)}} \times \text{(Present Value)}$$

This formula provides for a more rapid calculation of the future value of whatever sum of money we wish to evaluate. First you raise the compound interest formula to the number of years of the loan and then multiply it times the present value to obtain the future value of the initial amount.

To return to the example of the brother and the 100,000 CFA franc proposition, what is the future value of the initial amount of money after 5 years, based on a given interest rate of 10 percent?

$$\text{Future Value} = (1.10)^5 \times 100,000 = 161,051.$$

Problem 4

For practice, use Formula A from above to calculate the future value of the following sums of money:

	Present Value	Interest Rate	Periods	Future Value
1.	120,000	12%	2	_____
2.	139,876	9%	3	_____
3.	1,345,908	14%	10	_____
4.	4,908,433	11%	25	_____

Problem 4.a

	Present Value	Interest Rate	Periods	Future Value
1.	500,000	12.9%	10	_____
2.	1,234,678	9.075%	5	_____
3.	10,000	15.3%	35	_____

Now that we know how the calculation works, let us redefine the formula with letter definitions:

$$\begin{aligned} \text{Future Value} &= \text{FV} \\ \text{Present Value} &= \text{PV} \\ \text{Interest Rate} &= i \\ \text{Number of Years} &= n \end{aligned}$$

This converts our Formula A into Formula B as follows:

Formula B:

$$\text{FV} = (1+i)^n \times (\text{PV})$$

Since "A", the present value amount, is already known, the most important part of the formula is the compound interest expression.

The compound interest part of Formula B is known as the future worth factor. Once you know the corresponding interest rate and the number of time periods you can calculate the future worth factor. In multiplying the future worth factor by the present value, you derive the future value of a financial sum of money.

Problem 5

Given the following examples for interest rates and time periods, calculate the corresponding future worth factors.

	Period (n)	Interest Rate (i)	Future Worth Factor (FWF)
1.	3	10%	_____
2.	5	9%	_____
3.	7	16%	_____
4.	15	12%	_____

Problem 5.a

	Period (n)	Interest Rate (i)	Future Worth Factor (FWF)
1.	4	13.4%	_____
2.	12	12.8%	_____
3.	3	0%	_____

8. Present Worth Coefficients (Factors)

Notice that present value is the mirror image of compound interest. In other words, rather than calculate the future value of "A", we find the present value of "A" from a future sum of money "F" in the future. By manipulating the compound interest formula B, we can thus derive the present worth of a future sum of money.

Formula C:

$$A = \frac{1}{(1 + i)^n} \times F$$

The important part of this formula is the present worth coefficient. Given that present value is the inverse of compound interest, the present worth coefficient is thus the reciprocal of the future worth coefficient.

Problem 6

Use formula C to find the present value of the following future values:

	Future Value (F)	Interest Rate (i)	Time periods (n)	Present Value (A)
1.	100,000	10%	5	_____
2.	161,051	10%	5	_____
3.	2,000,000	14%	3	_____
4.	1,238,407	8%	20	_____

9. Present Valuation of a Stream of Future Revenues

The present worth coefficient, as with its future worth corollary, varies according to the interest rate and the number of time periods. Even if the interest rate remains invariant, the present worth coefficient will change as the number of time periods, "n", increases. This becomes especially important when you have to estimate the present value of a stream of future revenues.

Suppose we wanted to invest in an egg production operation expected to generate 1,000 CFA francs of net income annually for a period of 4 years. To evaluate the present worth of this investment, we would have to undertake the kind of calculations as are given in Problem 7 below.

Problem 7

Present Worth Valuation of an Investment

Profits (or Net Benefits)	Present Worth Coefficient
Year 0 1,000	$\frac{1}{(1.10)^0} = 1.0000$
Year 1 1,000	$\frac{1}{(1.10)^1} = \underline{\hspace{2cm}}$
Year 2 1,000	$\frac{1}{(1.10)^2} = \underline{\hspace{2cm}}$
Year 3 1,000	$\frac{1}{(1.10)^3} = \underline{\hspace{2cm}}$

The present worth coefficient decreases as the number of time periods becomes greater. This explains why the present value of future revenues is less than its nominal value. The more we project into the future, the less something is worth.

If you are uncomfortable with mathematics you probably are thinking that there is too much calculation involved in these formulas. Certainly they become more involved the greater the number of time periods and the large the amount to be evaluated. While many of today's calculators make these formulas easy to derive, there are also tables of present and future worth

coefficients to give a short-hand overview. A sample table of present worth coefficients is given below.

The advantage of a table of present worth coefficients is that one can readily look up the value corresponding to a given interest rate and a stated number of time periods. The only difficulty with such a table is that it may not contain all of the range of values that one may need, in which case one has to either interpolate for a rough equivalent or use a calculator to obtain a more precise estimate.

Table 7.a
Present Worth Coefficients

	8%	9%	10%	11%	12%	13%	14%
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	0.9259	0.9174	0.9091	0.9009	0.8929	0.8850	0.8772
2	0.8573	0.8417	0.8264	0.8116	0.7972	0.7831	0.7695
3	0.7938	0.7722	0.7513	0.7312	0.7118	0.6931	0.6750
4	0.7350	0.7084	0.6830	0.6587	0.6355	0.6133	0.5921
5	0.6806	0.6499	0.6209	0.5935	0.5674	0.5428	0.5194
6	0.6302	0.5963	0.5645	0.5346	0.5066	0.4803	0.4556
7	0.5835	0.5470	0.5132	0.4817	0.4523	0.4251	0.3996
8	0.5403	0.5019	0.4665	0.4339	0.4039	0.3762	0.3506
9	0.5002	0.4604	0.4241	0.3909	0.3606	0.3329	0.3075
10	0.4632	0.4224	0.3855	0.3522	0.3220	0.2946	0.2697
11	0.4289	0.3875	0.3505	0.3173	0.2875	0.2607	0.2366
12	0.3971	0.3555	0.3186	0.2858	0.2567	0.2307	0.2076
13	0.3677	0.3262	0.2897	0.2575	0.2292	0.2042	0.1821
14	0.3405	0.2992	0.2633	0.2320	0.2046	0.1807	0.1597
15	0.3152	0.2745	0.2394	0.2090	0.1827	0.1599	0.1401
16	0.2919	0.2519	0.2176	0.1883	0.1631	0.1415	0.1229
17	0.2703	0.2311	0.1978	0.1696	0.1456	0.1252	0.1078
18	0.2502	0.2120	0.1799	0.1528	0.1300	0.1108	0.0946
19	0.2317	0.1945	0.1635	0.1377	0.1161	0.0981	0.0829
20	0.2145	0.1784	0.1486	0.1240	0.1037	0.0868	0.0728
21	0.1987	0.1637	0.1351	0.1117	0.0926	0.0768	0.0638
22	0.1839	0.1502	0.1228	0.1007	0.0826	0.0680	0.0560
23	0.1703	0.1378	0.1117	0.0907	0.0738	0.0601	0.0491
24	0.1577	0.1264	0.1015	0.0817	0.0659	0.0532	0.0431
25	0.1460	0.1160	0.0923	0.0736	0.0588	0.0471	0.0378
26	0.1352	0.1064	0.0839	0.0663	0.0525	0.0417	0.0331
27	0.1252	0.0976	0.0763	0.0597	0.0469	0.0369	0.0291
28	0.1159	0.0895	0.0693	0.0538	0.0419	0.0326	0.0255
29	0.1073	0.0822	0.0630	0.0485	0.0374	0.0289	0.0224
30	0.0994	0.0754	0.0573	0.0437	0.0334	0.0256	0.0196
35	0.0676	0.0490	0.0356	0.0259	0.0189	0.0139	0.0102
40	0.0460	0.0318	0.0221	0.0154	0.0107	0.0075	0.0053
45	0.0313	0.0207	0.0137	0.0091	0.0061	0.0041	0.0027
50	0.0213	0.0134	0.0085	0.0054	0.0035	0.0022	0.0014

Problem 8

Present worth coefficients are based on the number of time periods and a given interest rate. In present valuation problems, the interest rate is known as the discount rate. Using Table 7.a, you should now check the results of your work in Table 7 to see if you have the correct answers. Once you have made this check, derive the corresponding present worth factors (PWF) for the revenue streams given in problems 8, 8.a, and 8.b, based on the given discount rates for each.

		Discount Rate:	12%	
	Revenues		PWF	Present Value (PV)
Year 0	5,000		_____	_____
Year 1	5,000		_____	_____
Year 2	5,000		_____	_____
Year 3	5,000		_____	_____
Year 4	5,000		_____	_____
			Total	_____

		Discount Rate:	8%	
	Revenues		PWF	Present Value (PV)
Year 0	15,000		_____	_____
Year 1	15,000		_____	_____
Year 2	15,000		_____	_____
Year 3	15,000		_____	_____
Year 4	15,000		_____	_____
			Total	_____

		Discount Rate:	15%	
	Revenues		PWF	Present Value (PV)
Year 0	5,000,000		_____	_____
Year 1	5,000,000		_____	_____
Year 2	5,000,000		_____	_____
Year 3	5,000,000		_____	_____
Year 4	5,000,000		_____	_____
			Total	_____

Problem 9

If you have not already done so, calculate the present value of revenues for each year by multiplying the nominal values times their present worth

factors. Compare the totals for each column of revenues with the total for each column of present value sums. This gives you a clear idea of why present valuation is so important.

10. Deriving the Net Present Value of an Investment

As we have seen, one of the reasons why calculating the present value of an asset is that it allows us to compare the value of our current investments with their future stream of profits. The simplest and most widely used measure of present valuation of an investment is the net present value, or NPV. The first step in arriving at the net present value of an asset is the derivation of its corresponding cash flow.

Each year, the implementation of a project requires that one allocates financial resources. These resources are allocated in order to realize the expected profits from the investment. The simple accounting difference between costs and revenues of a project for each year of the project life cycle is known as the cash flow. To see how this is derived, let us look at the costs and revenues for an irrigation project shown in the following table.

Table 8
The Lake Chad Irrigation Project

Year	Capital Expenditures	Operation and Maintenance Costs	Production Costs	Total Costs	Total Benefits
0	7,500	0	0	7,500	0
1	6,000	0	0	6,000	0
2	0	600	700	1,300	6,000
3	0	600	700	1,300	6,000
4	0	600	700	1,300	6,000
5	0	600	700	1,300	6,000
6	0	600	700	1,300	6,000
	13,500	3,000	3,500	20,000	30,000

Project evaluation tables, which are often prepared on electronic spreadsheet computer programs, can be done in one of two ways. Table 8 shows each object category by column, while each project year is shown by row. Depending on the size of the table, i.e., whether the number of years is greater or less than the number of object categories, one should adopt either the format given in Table 8, or the reverse, as is shown in Table 9.a.

In terms of the object categories, total costs represent the sum of capital expenditures, operations and maintenance costs, and production costs. In terms of this example, the Lake Chad irrigation project shows a relatively large expenditure of capital during the first two years of the project, with nothing thereafter, while operation, maintenance, and production costs begin only in the third year of the project. Since no production begins until the third year, there are no benefits to be generated by the project until the third year.

The next step in arriving at the net present value of a project is to calculate its cash flow. As already defined, the cash flow is the difference between total benefits and total costs for each year of the project. It is important to note that since the cash flow is realized over the 7 years of the project life cycle, the total of 10,000 does not represent the present value of the project, and is only an accounting entry. By itself, the cash flow total does not carry any operational meaning in terms of accepting or rejecting a project.

Table 9
The Lake Chad Irrigation Project

Year	Capital Expenditures	Operation and Maintenance Costs	Production Costs	Total Costs	Total Benefits	Cash Flow
0	7,500	0	0	7,500	0	-7,500
1	6,000	0	0	6,000	0	-6,000
2	0	600	700	1,300	6,000	4,700
3	0	600	700	1,300	6,000	4,700
4	0	600	700	1,300	6,000	4,700
5	0	600	700	1,300	6,000	4,700
6	0	600	700	1,300	6,000	4,700
	13,500	3,000	3,500	20,000	30,000	10,000

In order to make a decision as to whether a project is acceptable or not, the next step is to choose an appropriate discount rate, then calculate the present worth coefficient for each time period, and multiply the present worth coefficient times the corresponding cash flow value. The sum of the discounted cash flow values is called the net present value. In terms of our irrigation project, let us suppose that the appropriate discount rate to use is 12 percent. The corresponding present worth factors, discounted cash flows, and the net present value are given in Table 9.a. Because the number of objects is now greater than the number of years, the table years and object entries have been reversed in terms of column and row entries.

Table 9a:
Lake Chad Irrigation Project
Discount Rate: 12.00%

Year	0	1	2	3	4	5	6
A. Costs							
1. Capital Expenditures	7,500	6,000	0	0	0	0	0
2. Operation and Maintenance Costs	0	0	600	600	600	600	600
3. Production Costs	0	0	700	700	700	700	700
4. Total Costs	7,500	6,000	1,300	1,300	1,300	1,300	1,300
B. Total Benefits	0	0	6,000	6,000	6,000	6,000	6,000
C. Cash Flow (Undiscounted Net Benefits)	-7,500	-6,000	4,700	4,700	4,700	4,700	4,700
D. Present Worth Factor (PWF)	1.0000	0.8929	0.7972	0.7118	0.6355	0.5674	0.5066
F. Annual Present Values (ADCF)	-7,500	-5,357	3,747	3,345	2,987	2,667	2,381
G. Net Present Value (NPV)	2,270						

Notice carefully the present worth factor coefficients. They have been derived using the present worth coefficient from formula C in section 8. In terms of the choice of a specific discount rate, while we have used a rate of 12 percent in this example, how and why a particular rate is chosen is an important consideration in evaluating the net present value of a project. For the moment, let us assume that our choice of 12 percent is the most appropriate one.

To sum up where we are, the net present value is the sum of the discounted cash flow amounts of a project. Once the cash flow, or net benefit stream has been discounted using the corresponding present worth coefficients, one adds up these discounted amounts to arrive at the net present value. With each of these steps in mind, try out your skills on a sample project, the Timbuktu Millet Mill Project

Problem 10

Using the data given below, calculate the net present value of the Timbuktu Millet Mill project.

1. Purchase of the millet mill 1,525,000 CFA francs
2. Construction of mill shelter 140,000 CFA francs
3. Production costs 200,000 CFA francs
4. Operating and maintenance costs 75,000 CFA francs
5. First year expected revenues 750,000 CFA francs
6. Each subsequent year revenues 1,000,000 CFA francs
7. Operating and maintenance costs are estimated to be the same for each of the 5 years of the project
8. The discount rate given by the Ministry of Finance is 10 percent.

Bibliography

- Benninga, Simon (1996). *Numerical Techniques in Finance*. (Cambridge, Mass.: MIT Press)
- Bodie, Zvi, and Robert C. Merton (1998). *Finance*. (Upper Saddle River, N.J.: Prentice-Hall Publishing Company).
- Day, Alastair L. (2001). *Mastering Financial Modeling*. (New York: Prentice-Hall Financial Times).
- Eatwell, John, Murray Milgate, and Peter Newman (1989, 1987). *The New Palgrave Finance*. (New York: W.W. Norton and Company).
- Hull, John C. (1997). *Options, Futures, and Other Derivatives*, 4th edition. (Upper Saddle River, N.J.: Prentice-Hall).
- Jorion, Philippe (1997). *Value at Risk: The New Benchmark for Controlling Derivatives Risk*. (New York: McGraw-Hill).
- Leahigh, David J. (1996). *A Pocket Guide to Finance*. (New York: The Dryden Press).
- Luenberger, David G. (1998). *Investment Science*. (New York: Oxford University Press).
- Marrison, Chris (2002). *The Fundamentals of Risk Management*. (New York: McGraw-Hill).
- Rose, Peter S., and Milton H. Marquis (2008). *Money and Capital Markets*, 10th edition. (New York: McGraw-Hill).
- Shiller, Robert J. (2003). *The New Financial Order: Risk in the 21st Century*. (Princeton, N.J.: Princeton University Press).
- Shiller, Robert J. (2000). *Irrational Exuberance*. (Princeton, N.J.: Princeton University Press)
- Squire, Lyn, and Herman Van der Tak (1975). *Economic Analysis of Investment Projects*. (Baltimore, Md.: Johns Hopkins University Press for the World Bank).
- Van Horne, James C. (1998). *Financial Market Rates and Flows*, 5th, ed. (Upper Saddle River, N.J.: Prentice Hall Publishing Company).
- Van Horne, James C. (1989). *Fundamentals of Financial Management*, 7th, ed. (Englewood Cliffs, N.J.: Prentice-Hall).