

## Measuring Inequality

### Modern Approaches to Inequality and Social Welfare

While philosophy may provide but an opaque guide to distributive justice, economists and statisticians have forged ahead with measures of inequality. We put forth here various ways in which inequality can be measured, how it relates to standard measures of social welfare, and what it may imply for public policy choices.

Whether one accepts Adam Smith's notion that mercantilism is the root cause of inequality, or whether one embraces Marx's notion that under the labor theory of value, even a pure capitalist economy is by nature exploitative and doomed to its own destruction, when such notions were first advanced, they relied on exceptionally weak empirical foundations. Since then, economists have worked to develop more refined measures of income, wealth, growth, and distribution, largely starting with the work of the League of Nations in the 1920's and 1930's, but more recently with the adoption of more universal systems of national accounts by various countries under what is known as the UN system. It is this system of national accounts that serves as a first order basis on which to approach the question of income and wealth distribution, and from which specific indices have been developed. In the process, what economists have tried to do is to eschew subjective measures of well-being in favor of more explicit measures.

Key to this was the work of Vilfredo Pareto (1840-1923), an Italian economist and sociologist, who developed one of the first welfare standards. A Pareto optimum can be said to exist where any effort to improve the welfare of one individual results in a reduction in the welfare of someone else. This would seem to expunge from economists the subjective notions found in nineteenth century writings, notably those of Karl Marx, though he was far from alone in his preferences.

A Pareto standard is difficult to achieve in practice. Most economists agree that it becomes relevant once an economy has achieved a level of full employment in which redistribution is then invoked in favor of some presumably superior outcome, but for which the Pareto standard would rule out by definition.

During the Great Depression of the 1930s, economists were concerned not only with full employment, but also with its distribution. John Maynard Keynes (1883-1946) put macroeconomics front and center with his *General Theory of Employment, Interest and Money* in 1936, a treatise that provided a rationale for discretionary fiscal deficits as a means to achieve full employment. The *General Theory* offers practically no insight on how achieving full employment would affect the distribution of income, and Keynes basically relegated this question to a secondary status as long as unemployment remained the chief preoccupation of economists.

Despite the understandable acceptance of Keynesian economics during the Great Depression, some economists were still drawn to the question of welfare economics. Arthur Cecil Pigou (1877-1959), a contemporary of Keynes, devoted much of his work on applied microeconomics, invoking a Pareto standard to questions of individual

resource allocation, with an emphasis on how market imperfections could produce inaccurate policy choices. But Pigou did not advance much of the debate over welfare criteria. That would be done by others, notably, Nicholas Kaldor (1908-1986), Abram Bergson (1914-2003), Paul Samuelson (1915-2009), the philosophers John Rawls (1921-2002) and Robert Nozick (1938-2002).

With a Pareto standard difficult to achieve, Nicholas Kaldor pointed to a Benthamite standard whereby if the gains of the gainers exceed the losses of the losers, then one can claim that a given redistribution represents an increase in social welfare. Determining whether the gains of the gainers exceed the losses of the losers, however, is a difficult proposition requiring that one make inevitable interpersonal value judgments, thus placing the Pareto standard back into a subjective box.

Abram Bergson and Paul Samuelson wrote simultaneously and independently in 1938 that insofar as a social welfare standard is likely to involve interpersonal value judgments, how much equality should be achieved is best left to a democratic political process, leaving to economists the most efficient way to bring it about. This seemed to have put an end for a while to the question of distributive justice.

What renewed the debate over distributive justice was the publication of *A Theory of Justice* (1971) by the Harvard philosopher John Rawls. Reviewing previous economic writings on inequality and economic efficiency, Rawls proposed a veil of ignorance that removed in theory pre-existing biases to make a straightforward egalitarian standard: Any redistribution that improves the welfare of the least advantaged member of society can be said to be an improvement.

Needless to say, this was greeted with skepticism by many writers, in part because a monotonic pattern of moving to greater equality would inevitably involve some measure of coercion by the state since individuals in a purely private economy failed to do so through purely voluntary acts of charity. One key writer in this strain of thought was fellow Harvard Philosophy Professor Robert Nozick. In his 1974 treatise, *Anarchy, State, and Utopia*, Nozick took a libertarian position that unless voluntary exchange would produce a Rawlsian outcome, then any redistribution would be inconsistent with individual freedom.

A further restatement on inequality was put forth by Amartya Sen (1933- ), in *Inequality Reexamined* (1992). Sen argued that capabilities should define the optimal degree of inequality. These capabilities included the elementary functions of good health, adequate nourishment, and fundamental shelter. In addition, capabilities also include the social functions of self-respect, taking part in the life of the community and other activities that from the time of Aristotle would be considered as the capacity of civic engagement. Sen overlaps with Nozick in setting an anchor of inequality in the pursuit of freedom, but does not rule out intervention by the state in its achievement, in particular in reducing various forms of discrimination that limit human fulfillment.

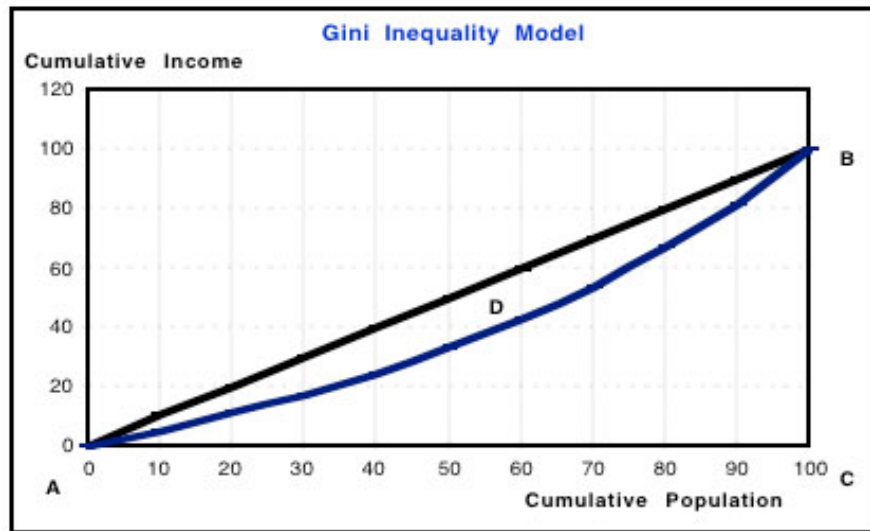
### Measuring Inequality

One of the first to portray an index of inequality was Corrado Gini (1884-1965), an Italian statistician who derived an index to examine inequality across countries and time and for which he looked to some natural law as justifying persistent inequality. As we will see, through use of integral calculus, Gini arrives at a visual method of characterizing the degree of inequality across the income spectrum. Fortunately, we have a simpler method that we can use, which we will derive once we have defined the original Gini-Lorenz coefficient.

### The Basic Geometry of the Gini Inequality Index

The Gini index is defined as a ratio of the areas on the Lorenz curve diagram. If the area between the line of perfect equality and the Lorenz curve is D, the Gini coefficient is defined as the area D divided by the triangle ABC. Perfect inequality implies that  $D=0$  such that the ratio  $D/ABC=0$ . Perfect inequality implies that  $D=ABC$ , in which case the ratio  $D/ABC=1$ .

Figure 2.1



As you might imagine, estimating the associated area for the Lorenz curve can involve some complex mathematics, notably the use of integral calculus.

### Estimating the Gini Inequality Index

If the Lorenz curve is represented by the function  $Y = L(X)$ , the value of B can be found with integration and:

$$(1.) B = \int_0^1 L(X) dX.$$

In some cases, this equation can be applied to calculate the Gini coefficient without direct reference to the Lorenz curve. For example (taking  $y$  to mean the income or wealth of a person or household):

For a population uniform on the values  $y_i$ ,  $i = 1$  to  $n$ , indexed in non-decreasing order ( $y_i \leq y_{i+1}$ ):

$$(2.) \quad G = \frac{1}{n} \left( n + 1 - 2 \left( \frac{\sum_{i=1}^n (n+1-i)y_i}{\sum_{i=1}^n y_i} \right) \right)$$

This may be simplified to:

$$(3.) \quad G = \frac{2 \sum_{i=1}^n i y_i}{n \sum_{i=1}^n y_i} - \frac{n+1}{n}$$

This formula actually applies to any real population, since each person can be assigned his or her own  $y_i$ .

For a discrete probability function  $f(y)$ , where  $y_i$ ,  $i = 1$  to  $n$ , are the points with nonzero probabilities and which are indexed in increasing order ( $y_i < y_{i+1}$ ):

$$(4.) \quad G = 1 - \frac{\sum_{i=1}^n f(y_i)(S_{i-1} + S_i)}{S_n}$$

where:

$$(5.) \quad S_i = \sum_{j=1}^i f(y_j) y_j \text{ and } S_0 = 0$$

For a cumulative distribution function  $F(y)$  that has a mean  $\mu$  and is zero for all negative values of  $y$ :

$$(6.) \quad G = 1 - \frac{1}{\mu} \int_0^{\infty} (1 - F(y))^2 dy = \frac{1}{\mu} \int_0^{\infty} F(y)(1 - F(y)) dy$$

(This formula can be applied when there are negative values if the integration is taken from minus infinity to plus infinity.)

Since the Gini coefficient is half the relative mean difference, it can also be calculated using formulas for the relative mean difference. For a random sample  $S$  consisting of values  $y_i$ ,  $i = 1$  to  $n$ , that are indexed in non-decreasing order ( $y_i \leq y_{i+1}$ ), the statistic:

$$(7.) \quad G(S) = \frac{1}{n-1} \left( n + 1 - 2 \left( \frac{\sum_{i=1}^n (n+1-i)y_i}{\sum_{i=1}^n y_i} \right) \right)$$

is a consistent estimator of the population Gini coefficient, but is not, in general, unbiased. Like  $G$ ,  $G(S)$  has a simpler form:

$$(8.) G(S) = 1 - \frac{2}{n-1} \left( n - \frac{\sum_{i=1}^n i y_i}{\sum_{i=1}^n y_i} \right)$$

### The Champernowne Gini Inequality Proxy Index

Fortunately, there is a much more accessible way to derive a comparable Gini inequality index. It was developed by David Champernowne (1912-2000) and for a series of observations, can be expressed as:

$$(9.) G = 1 - g/x,$$

Where:  $g$  = the geometric mean of a series, and  
 $x$  = the arithmetic mean of a series.

Consider, for example the following comparative distributions of income for a population of three individuals:

**Table 2.1**

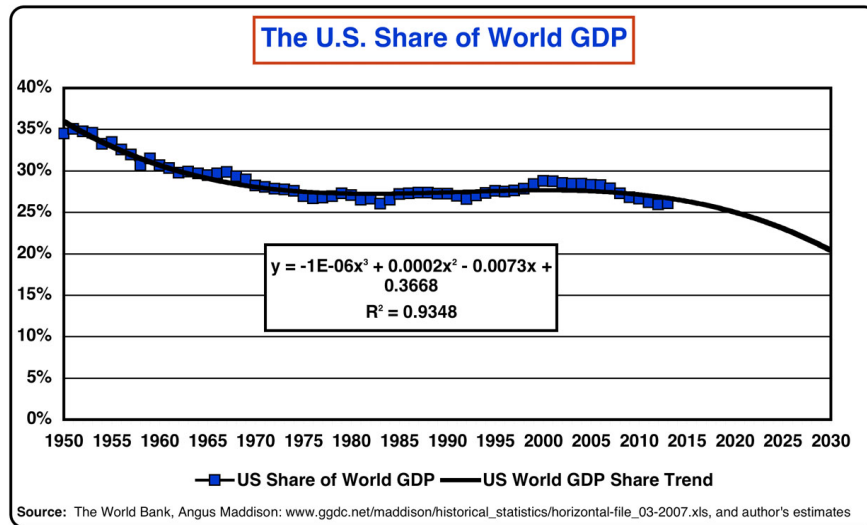
<b>Alternative Income Distributions</b>			
	<b>A</b>	<b>B</b>	<b>C</b>
<b>Tom</b>	\$20,000.00	\$36,666.67	\$8,000.00
<b>Harry</b>	\$29,000.00	\$36,666.67	\$12,000.00
<b>Sally</b>	\$61,000.00	\$36,666.67	\$90,000.00
<b>Total</b>	\$110,000.00	\$110,000.00	\$110,000.00
<b>g =</b>	\$32,828.62	\$36,666.67	\$20,519.71
<b>x =</b>	\$36,666.67	\$36,666.67	\$36,666.67
<b>I = 1-g/x</b>	0.1047	0.0000	0.4404

All three distributions, A, B, and C, have the same level of total income and the same average level, but differ significantly in the distribution. Distribution A has a Champernowne Inequality Index of .1047, with .000 for B, and .4404 for C. B represents perfect equality, while C has a higher degree of inequality than A. The ultimate question is what rank ordering is preferable. That brings us back to the original discussion regarding the question regarding process fairness versus outcomes fairness. Before we further take up this question so, let us look at some data on the distribution of income.

### Comparative Patterns of Inequality

While the Gini Inequality Index provides a summary measure of income inequality, there are several complementary ways to portray data within and across time. We could, for example, look at the U.S. Share of Global Gross Domestic Product. For some time it has been declining, even though the U.S. economy has been growing and, for now, remains as the largest single economy in the world. This has implications for distributive justice, along with standards for global governance. While we will concentrate on inequality within the U.S., we note simply that the U.S. accounts for five percent of global population while accounting for a quarter of global GDP. On a purchasing power parity (PPP) basis, the U.S. also has one of the highest levels of per capita GDP.

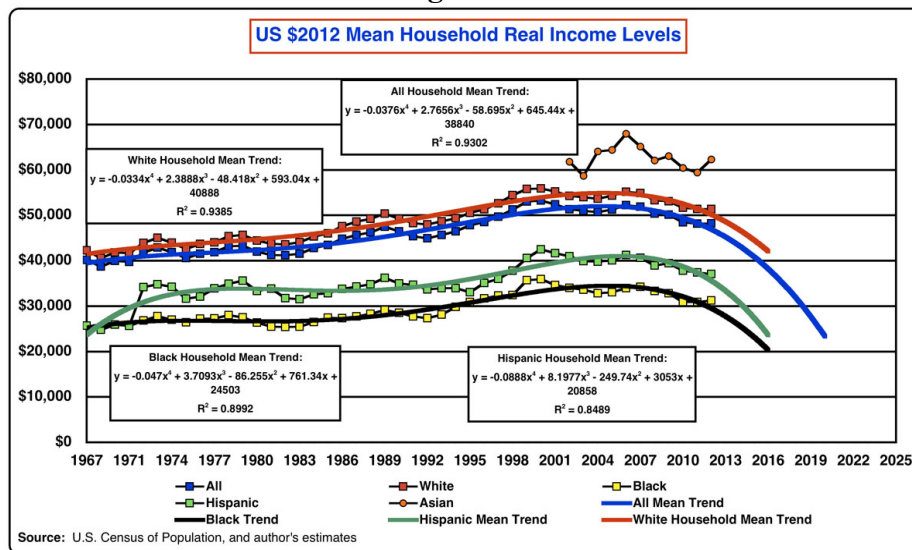
Figure 2.2



When it comes to policies on income distribution, it usually is at the national level that comparisons and choices are made within time and across countries over time. For the U.S., even as its economy grows, its declining share of the global economy implies less leverage in shaping the course of world affairs. Countries can sometimes offset the lack of economic size through the building of military strength, which the U.S. has done for some time, as well as through the appeal of the form of governance, in this case democratic institutions. At any rate, when it comes to distributive justice, it usually is addressed within states, even if international distributive justice may also be of interest and merit.

Turning to the U.S., we gain some perspective on distributive justice through data compiled by the U.S. Census Bureau. One building block is to track inflation-adjusted mean or median household or family income over time.

Figure 2.3

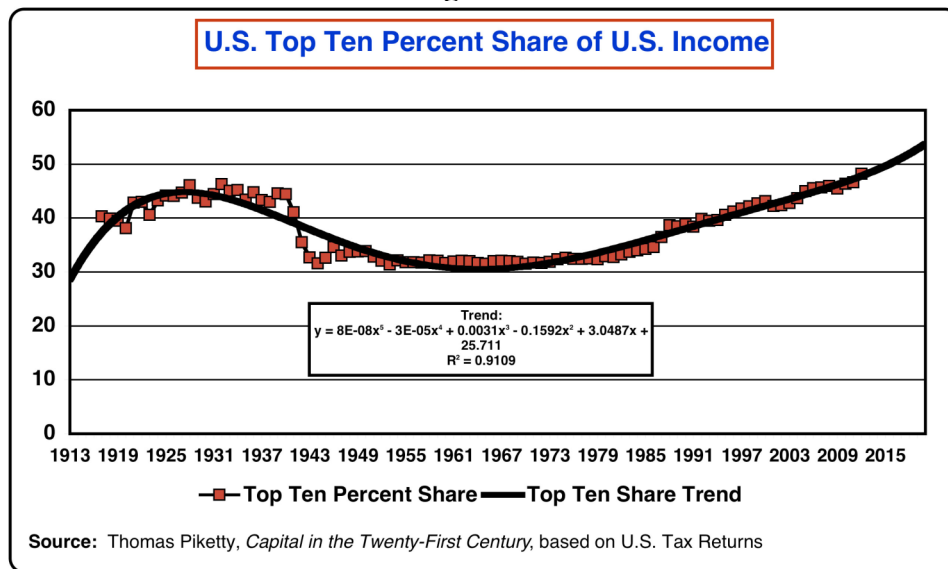


Data from Figure 2.3 show that real per capita household income for all census groups has been either stagnant or in decline for the past ten years. Income is a flow, and does not capture or reflect household wealth. Given that the most important single source of household wealth has been in residential housing, and given that residential housing values went through a steep decline during the Great Recession of 2008, real wealth may also have stagnated across income groups in recent years.

As to differences in real household income, trends across these broad groupings also may reflect continuing disparities in education, along with other determinants of income, including employment discrimination. What the data in Figure 2.3 show is that mean household real incomes for Blacks have remained at the bottom, followed by Hispanic households, then Whites, with Asian households at the top of the spectrum. In a world of perfect competition, one would expect no significant differences in income by these groupings, with any index of inequality to be similar for any population sample. This clearly is not the case.

We now look at overall inequality in the distribution of income. Based on tax return data, if we look at the top ten percent of incomes in the U.S. inequality is higher today than it was in the 1920's, and may reach even higher levels if current trends continue. While it stood at 45 percent during the 1920's, and declined to some 30 percent during the Great Depression, World War II, and in the first two decades thereafter, it has since risen to the 50 percent level, the highest since data have been gathered in the U.S.

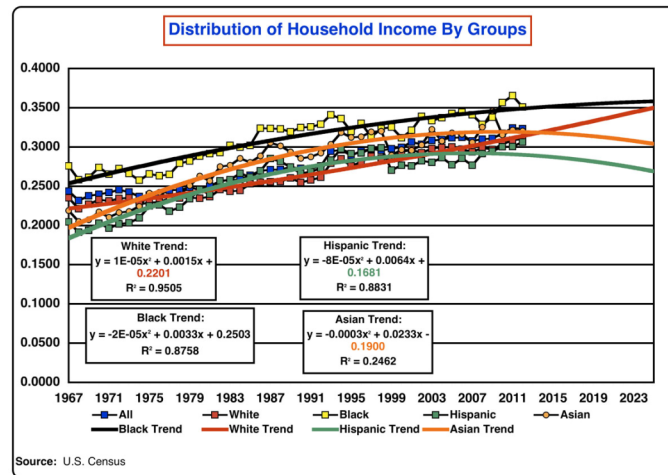
Figure 2.4



What about inequality within household income groups? Using our Champernowne inequality index, we find that while Black households have the lowest level of mean real income, they have the highest degree of income inequality. But this difference does not provide a consistent explanation for differences in mean real incomes across household groupings. For example, Hispanic households, the second lowest income grouping, have

the lowest degree of income inequality, while Asian households, with the highest level of income, have a lower degree of inequality than among White households. Explaining these differences, and formulating policies that pursue some standard of distributive justice requires first and foremost an understanding of the determinants of income levels as well as their distribution, a matter that has been pursued by social scientists for some time with no clear consensus up to this point.

**Figure 2.4**  
**Income Distribution in the U.S.**



What about geography or other factors? Internationally it would seem that temperate zone countries display a higher level of income than tropical ones, but data are both incomplete and inconsistent with this proposition. Eastern European countries and the Russian Federation have far lower per capita incomes than those in Western Europe, North America, and now China. Thus, geography in this sense provides little useful insight in terms of our concern with distributive justice.

Within the United States, we do have a richer data set that provides some insight as to the relationship between inequality and per capita income. Based on information gathered by the U.S. Census, if we compare 2010 per capita income and inequality with presidential voter outcomes in 2012, we find a pattern that is counter-intuitive to popular conceptions.

**Table 2.2**  
**Per Capita Income and Inequality within the 2012 Political Spectrum**

	Per Capita Income 2010	Gini 2010	Romney /Obama Ratio 2012
GOP Mean	\$36,005.78	0.4503	1.5567
Dem Mean	\$42,225.45	0.4570	0.7330
GOP Median	\$34,604.00	0.4550	1.4500
Dem Median	\$40,952.00	0.4520	0.7500
National Mean	\$39,474.44	0.4541	1.0973
National Median	\$38,283.50	0.4545	0.9412



From the 2012 presidential election, in states where Mitt Romney had a higher popular vote share than Barack Obama, these were parts of the country that had lower mean and median per capita income in comparison to those won by Obama. At the same time, using mean values alone, inequality was lower in those states won by Mitt Romney than in those states where Barack Obama won. The only counterfactual to this is if we calculate median inequality by state, keeping in mind that those still are states where per capita income is lower than in those won by Obama. If we now juxtapose mean values, we find that GOP states in 2012 had average incomes that were almost 9 percent lower than the national average while Democratic states had average incomes that were just under 7 percent higher than the national average. Within this simple framework, one could conclude that the price of higher per capita income is a higher degree of inequality, and voting Democratic doesn't hurt. So much for political clichés.

Suppose we now test for the relationship between inequality and per capita income, using state-level data for the U.S.

**Table 2.3**

Correlation Table				
	PC Income	Gini	Gini^2	R/O Ratio
Per Capita Income	1.0000			
Gini	0.4345	1.0000		
Gini^2	0.4573	0.9992	1.0000	
2012 R/O Ratio	-0.4666	-0.3915	-0.3910	1.0000

Our ratio the Romney to Obama voting percentage does generate the paradoxical relationship we have already noted. Turning to inequality, if we use the Gini coefficient and the squared value of the Gini coefficient, we find a quadratic relationship, as the results in the following table indicate, and which is strengthened by factoring in the Romney/Obama 2012 ratio.

**Table 2.4**

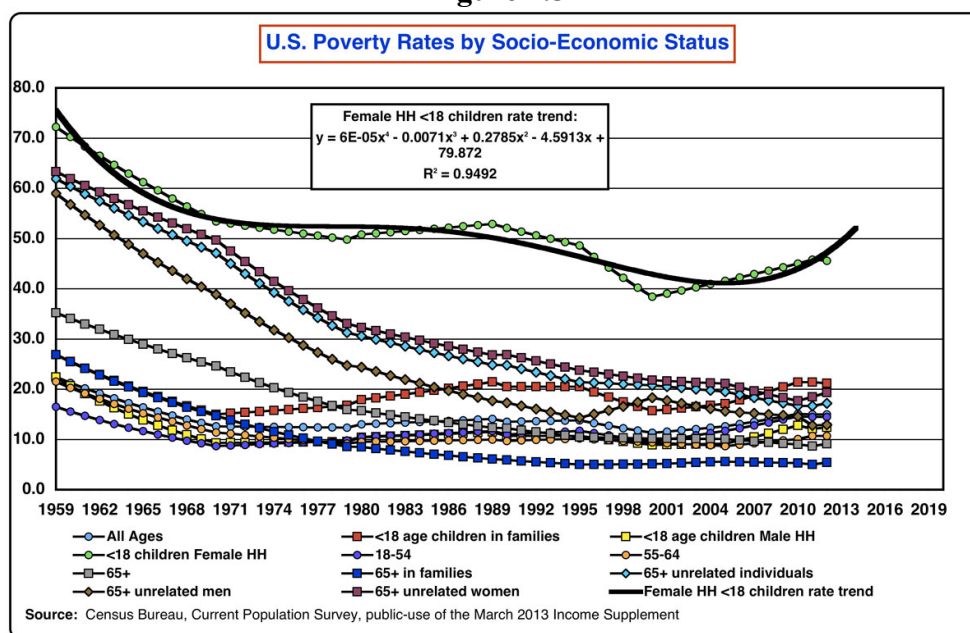
2010 State-Level Per Capita Income						
Regression Statistics						
Multiple R		0.7842				
R Square		0.6149				
Adjusted R Square		0.5909				
Standard Error	4,557.49					
Observations	52					
ANOVA						
	df	SS	MS	F	Sig. F	
Regression	3	1,592,202,910.64	530,734,303.55	25.55	0.0000	
Residual	48	996,995,666.19	20,770,743.05			
Total	51	2,589,198,576.83				
	Coefficients		Standard Error	t Stat		
Intercept	1,098,035.42		174,192.22	6.30		
Gini	-4,651,433.76		750,636.32	-6.20		
Gini-Sqd	5,124,697.72		808,478.30	6.34		
2012 R/O Ratio	-4,798.34		1,324.74	-3.62		

Within the sample values for inequality in 2010, the optimal Gini coefficient turns out to be .4492, which results in a predicted mean per capita value of \$37,184. Factoring in the political voting ratio reduces the optimal Gini to .4487 for marginally higher predicted level of per capita income at \$37,185.92. What should be kept in mind here is that our "optimal" inequality index is derived simply from the state-level sample and may not correspond to a normative standard that might be produced through the political process.

## Inequality and Poverty

In framing public policy, public discourse often confounds income levels, the distribution of income, and poverty. Poverty is a standard by which many public policies have been framed to achieve some measure of distributive justice. Yet poverty can be seen in either relative or absolute terms, leading to different policy implications. In absolute terms, poverty might be defined as that level of income that maximizes life expectancy, for example. In an international context, there is a direct relationship between income and life expectancy, but it is not linear, suggesting that differences in health and lifestyles may explain much, if not all, of the variation. First, let us take a look at data compiled by the U.S. Census Bureau.

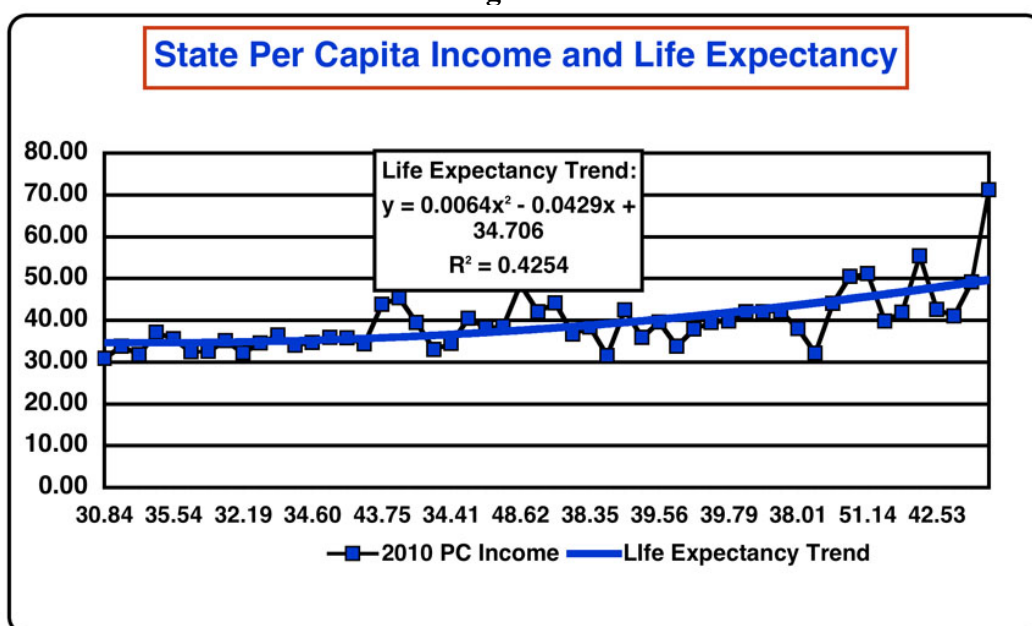
Figure 2.5



The first thing to note is that regardless of grouping, overall poverty rates have declined significantly since systematic data collection began in the late 1950s. When economic growth slows down or goes into recession, poverty rates tend to go up. During the recent Great Recession, poverty rates for children under the age of 18 in a Female-headed household were the highest and saw the sharpest increase from 2001 to 2012. The lowest poverty rate is for those aged 65 and above living in family household units. Clearly, family structure is an important determinant in affecting poverty rates.

Poverty thresholds in the U.S. have been held relatively constant in real constant dollar terms for decades. For one, two, three, four, five, six, seven, and eight the respective poverty thresholds in \$2013 dollars are: \$11,658, \$15,737, \$19,816, \$23,895, \$27,974, \$32,053, \$36,132, and \$40,210, which on a per capita basis are, respectively, \$11,658, \$7,868.59, \$6,605.36, \$5,973.74, \$5,594.77, \$5,342.12, \$5,161.66, and \$5,026.31.

Figure 2.6



If we look at life expectancy, we also can take stock of the impact of income, and from which we can derive an optimal life expectancy maximizing level of income. Using state-level data, a household income level of \$51,060 will generate a life expectancy of 80.38 years, which is higher than the current actual level of 78.66 years and household income of \$39,474. This also is substantially more income than current poverty threshold levels.

At this point, we should note the work on Inequality undertaken by Thomas Piketty and Immanuel Saez, respectively, of the Paris School of Economics and the University of California at Berkeley. In *Capital in the Twenty-First Century*, Piketty puts forth a striking thesis, namely, that outside of wars, including destructive ones and those to reduce poverty, along with economic recessions and depressions, there is an inherent tendency in market capitalist economies for income inequality to increase.

Drawing on an extensive compilation of data, Piketty's thesis can be summarized by his observation that the rate of return on capital tends to exceed the rate of economic growth, thus increasing the degree of income and inequality. In his view, the only reasonable way to bring about a golden rule of accumulation whereby the rate of return on capital is reduced to the rate of economic growth is by imposition of steeply progressive income taxes as well as substantial taxation of inherited wealth. We will return to this proposition once we have examined what kinds of measures now exist to affect the distribution of income.