Optimal Pricing of Biodiverse Natural Resources
for Sustainable Economic Growth

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Abstract
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Sustainability of renewable natural resources requires a pricing regime that can takes into account intrinsic rates of growth, heterogeneous harvesting rates, and the interdependence of species. Drawing on the theory of renewable natural resource pricing, we develop a discrete model of dynamic optimization of multiple species of renewable natural resources, along with an index of relative biodiversity. Using a sustainable yield framework, we then illustrate how interdependence can be factored within a dynamic optimization model to derive the corresponding shadow prices, and how the corresponding index of relative biodiversity will change over time. In turn, the corresponding shadow prices can be used to determine the value of harvesting permits consistent with species interdependence and sustainable growth.

Introduction

Biodiversity is a natural response to the management of risk. When not offset by new alternatives, exhaustion or extinction of resources increases the level of risk, thus reducing the prospect for sustainable choices in the future. Although natural forces can lead to extinctions, human intervention can lead either to a reduction in, preservation of, or expansion of diversity, thereby affecting the level of risk. While we can think of biodiversity as a public good, it leaves open the question of how it should be priced within a sustainable economic framework. The issue is complicated further by the potential of genetic banking and genetic intervention[1]. Yet until the frontiers of pure science acquire greater commercial application, biodiversity for sustainable growth merits some consideration.

Sustainable economic growth is any rate of change in real per capita output that can be maintained over a given time horizon. To do so requires that the rate of economic growth be matched by the rate of growth of resources, subject to adjustments in resource productivity and resource substitutability. In the case of exhaustible natural resources, sustainable growth is based on the underlying assumption of technological and resource substitution as the resource is depleted, an issue addressed elsewhere in the literature (Fisher, 1981; Dasgupta and Heal, 1979; Goeller, 1978; Hotelling, 1931)[2]. Where renewable natural resources are concerned, sustainable economic growth requires that the rate of consumption of a renewable natural resource be less than or equal to the rate of growth in the stock, again adjusted for changes in factor productivity. What complicates this arrangement is where the rate of growth in the stock of a renewable natural resource depends on the rate of growth of other natural resources, that is, on a given level of biodiversity. We examine here the choice of optimal pricing of biodiverse natural resources consistent with a target level of sustainable economic growth.

There is an extensive literature on sustainability and biodiversity. Outside of economics, it includes works in biology (Freese, 1997; Rhodes, et.al, 1996; Perrings, et.al., 1995; Murray, 1993; Wilson, 1992; Hilborn and Walters, 1992; Renshaw, 1991; Mangel and Clark, 1988; and Nitecki, 1984, among others). Many of these contributions draw on early work in biological systems (Thompson, 1992, 1942; Lotka, 1956, 1925; Verhulst, 1838). Subsequent work has focused on ecology and the environment, including the question of global warming (Jepma and Monasinghe, 1998; Singer, 1997; Moore, 1998; and Faber, 1996). In turn, natural resource economists have drawn on this literature to incorporate aspects of pricing for sustainable growth, and the impact of natural resource use on global climate change (Maestad, 2001; Brown, 2000; Conrad, 1999; Heal, 1998; Neher, 1990; Conrad and Clark, 1987; Hyde, 1980; Nordhaus, 1979; Daly, 1977; and Hammack and Brown, 1974).

Across this spectrum is a common concern for sustainability of economic activity where exhaustible and renewable natural resources are consumed. Where environmental and biological

work has pointed out the risk of non-sustainability and extinction, economists have looked to derive institutional mechanisms that determine whether sustainability and biodiversity can be achieved and at what cost relative to technological alternatives or the extinction of species (Rao, 2000; Levhari and Mirman, 1980; Goeller, 1978; Zellner, 1970).

There are three existing general approaches to the problem of biodiverse sustainable growth. The first and oldest is the creation and maintenance of a natural resource environment such as a national park or a biosphere. In principle, it addresses the property rights problem by restricting individual access and harvesting to levels consistent with the reproduction of renewable natural resources or with the optimal extraction of stocks of exhaustible natural resources. However, property rights may be incomplete, as in the case of an open access fishery, in which case extraordinary conventions may be needed to produce sustainable stocks (Southey, 1972; Scott, 1955). Yet even if this issue is resolved, access to the resource base does not always distinguish between natural dynamics among species from those posed by humans engaged in harvesting (Berck, 1979; Krutilla and Fisher, 1975). Moreover, it does not resolve the question of how to value the level of resource commitment necessary for the replication and sustainability of a given level of biodiversity.

The second approach is the use of contingent valuation models. Contingent valuation, or amenity valuation, refers to the willingness of individuals to pay for the right to have a resource preserved rather than consumed (Cummings, Brookshire, and Schultze, 1986; Portney, 1994). This approach seeks to address the question of sustainability by establishing a pricing regime essential to the conservation of a resource. While contingent valuation does help to address the valuation of access to a natural resource environment, most models seek to determine a solution to the demand for a natural resource amenity independent of the dynamics among individual species. Contingent valuation thus leaves open the question of whether selective harvesting may provide a more sustainable biodiversity path (Goldin and Winters, 1996, 1995; Pearce and Moran, 1995).

A third approach to the sustainable biodiversity problem is the use of buffer stocks for storable commodities to manage a given level of the stock of a species (Kremer and Morcom, 2000). While purchases and sales of buffer stocks can affect the demand for individual species, not all species contain storable byproducts such as ivory or hides, in which case the problem of optimal biodiverse stocks remains. Moreover, the Kremer and Morcom formulation does not consider biodiverse interdependency.

Previous work in this area covers the use of dynamic models for single species (Conrad, 1999; Clark, 1990, 1976; Conrad and Clark, 1991, 1987; Clark, Clarke, and Munro, 1979; Hoel, 1978; Beddington, and Leung and Wang, 1976, among others). Beddington, Watts, and Wright (1975)

consider the problem of a reproducible natural resource to determine under what conditions extinction will occur and suggest the use of a dynamic optimization model to determine the steady-state sustainable harvesting path. Leung and Wang (1976) draw on the Beverton and Holt model (1957) to derive steady-state conditions for the optimal operation of a fishery. Hoel (1978) provides a framework to examine the possibility of extinction of a reproducible natural resource under competitive conditions, which reflects the common resource pool problem and the assignment of property rights.

Brander and Taylor (1998) illustrate the problem of multi-species dynamics using a predator-prey model in the context of Easter Island[3]. Clark (1976, 1990) elaborates on the conditions for sustainable harvesting of renewable natural resources and the corresponding shadow price to achieve a dynamic equilibrium. Clark's model, and the extensions provided in Clark, Clarke and Munro (1976) and in Conrad (1999) illustrate an approach for deriving sustainable harvesting rates, with the latter utilizing a spreadsheet modeling approach.

In this paper, we propose a framework to address the optimal pricing of biodiverse renewable natural resources where more than two interdependent species are concerned and in which the harvested products may or may not be storable.[4] To do so, we first develop the framework of single species dynamics when we then extend to two interdependent species. We then consider the pricing problem where three interdependent species are concerned using a dynamic optimization model to derive optimal harvesting rates and efficient shadow prices. We then examine the question of property rights, poaching, and the choice of optimal prices under both certainty and stochastic conditions.

Single Species Dynamics

The framework of renewable natural resource pricing is well established[5]. For a single species, whether the stock of a renewable natural resource will be constant, increasing, or decreasing depends on several variables. These variables include the intrinsic rate of growth of the resource, the marginal cost of extraction, the prevailing rate of discount, the environmental carrying capacity of the resource, and the level of technology. Absent human intervention, the biological increase of a natural resource is considered to grow at a nonlinear rate until a carrying capacity limit is reached. Although there are several varieties of nonlinear growth models (Clark, 1990; Southey, 1972; Zellner, 1970; and Beverton and Holt, 1957), we will use a standard logistic curve model here to examine the biodiversity pricing problem[6].

Consider a renewable natural resource that grows at the following rate:

(1.) $\dfrac{dx}{dt} = rx\left(1 - \dfrac{x}{K}\right) = rx - \dfrac{rx^2}{K} = f(x)$, where:

   r = the intrinsic rate of growth

x = the stock of the renewable resource

K = the asymptotic limit, or carrying capacity, of the resource

Although x can be solved for explicitly, there are two equilibrium conditions in equation 1, namely, x = 0 and x = K. As long as $0 < x < K$, $dx > 0$.

We can use equation (1.) to derive two important conditions. The first is the stock level corresponding to the maximum sustainable yield, or msy. Taking the second derivative of (1.) and simplifying, we find that x(msy) will be:

(2.) $\dfrac{d^2 x}{dt}$ $x = \dfrac{K}{2}$, that is, the maximum sustainable yield will be where the stock of the

renewable resource is at one-half of the carrying-capacity level.

In turn, we can derive the time dimensionless maximum sustainable yield rate by substituting the value of x from equation (2.) in equation (1.), which reduces to:

(3.) $f(x)_{msy} = rK/4$.

The second condition is the logistic equation solution for x(t). To obtain the logistic equation solution to equation (1.), we first use the separation of variables and then integrate the resulting expression:

(4.) $\dfrac{dx}{x(K-x)} = \dfrac{r}{K} dt$, or $\left[\dfrac{1}{x} + \dfrac{1}{K-x}\right] dx = rdt$

Integrating (4.) yields:

(5.) $\ln \dfrac{x}{K-x} = rt + \ln \dfrac{x_0}{K-x_0}$, where $x_0 = x(0)$.

Equation (5.) can now be written as:

(6.) $x(t) = \dfrac{K}{1 + ce^{-rt}}$, where $c = \dfrac{K - x_0}{x_0}$, the basic logistic equation for the renewable resource.

Equation (5.) also can be used to solve for the time period when the yield is at a maximum, given that the stock will correspond to half of the carrying capacity level. Re-arranging the RHS of (5.), t(MSY) thus is:

(6.) $t = \dfrac{-\ln \dfrac{x_0}{K - x_0}}{r}$

Figure 1 maps the various rates of growth defined by equation (1.)[7]. As already noted, when integrated, we obtain the classic logistic equation defined in equation (9.), and which is mapped in Figure 2.
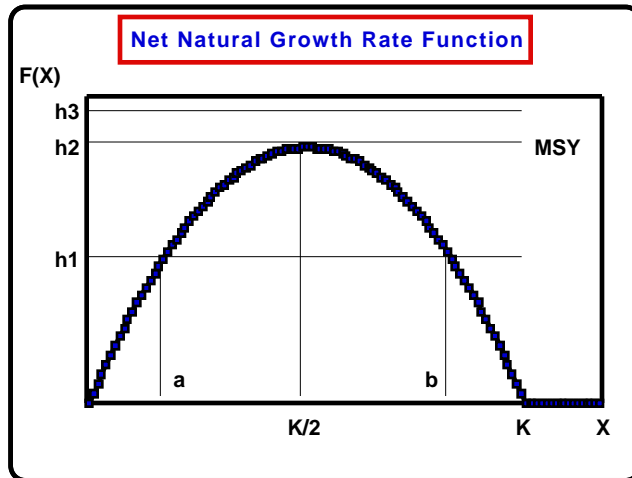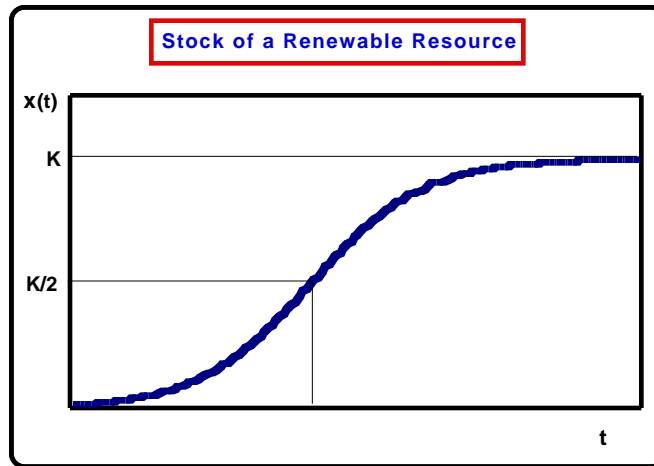
Figure 1

Figure 2

Biological sustainability requires that any harvest rate be no greater than the corresponding growth rate, or yield, of the renewable natural resource. In Figure 1, the maximum physical harvest rate consistent with sustainability of the resource is at the maximum sustainable yield (MSY) level shown above as h2. Any level above h2, such as h3, will eventually lead to the extinction of the renewable natural resource. In contrast, h1 illustrates two possible conditions. For the interval a-b, if the initial stock is greater than the harvesting rate, it will decline up to b and will be stable since it is below the maximum sustainable yield. On the other hand, if the initial stock is less than at the interval a, then h1 will eventually deplete the resource, though at a slower rate than at h3.

From equation 2, we note that the maximum sustainable yield rate is at half of the carrying capacity level of the resource, as is shown in Figure 2. In Figure 2, we map the logistic solution to the differential equation (1). Figure 2 shows that the slope of the logistic function is greatest at K/2, and it illustrates the corresponding time period when the maximum sustainable yield is achieved, based on application of equation (6.).

Technological change affects a renewable natural resource framework in two basic ways. Disembodied technical change that involves a better use of existing inputs, will result in increases in K, the carrying-capacity of the resource, thereby postponing the point of the maximum sustainable yield. On the other hand, embodied technical change results in a faster rate of intrinsic growth of the resource, which reflects qualitative changes in the inputs, including genetic engineering. In this case, the slope of the logistic curve rises upward to the carrying-capacity level at a faster rate, thus reducing the time period to reach the maximum sustainable yield level.

Technological change is but one of several variables that affect the choice of harvest rates of a renewable natural resource. While we could examine optimal harvesting policies from a static framework, it is more appropriate to do so within a dynamic setting. To do so, we can adapt the existing framework to the dynamic problem first through use of a continuous time-framework and then through use of difference equations to derive discrete-time solutions.

The value of a resource is a function of the rate of intrinsic growth relative to the harvest rate, the net benefit of the resource, the level of technology, the prevailing rate of discount, and the degree of sustainability and biodiversity. For a single species, the owner's problem can be stated as maximization of the net present value from extraction of the resource over time:

(8.) $\quad NPV = \int_0 e^{-\delta t} R(x, E) dt = \int_0 e^{-\delta t} \{ p - c[x(t)] \} h(t) dt$ subject to x(t) 0 and h(t) 0.

Since this integral has the form $\int \phi(t, x, \dot{x}) dt$, the necessary condition for a maximum will be:

(9.) $\quad \dfrac{\partial \phi}{\partial x} = \dfrac{d}{dt} \dfrac{\partial \phi}{\partial x} = \dfrac{\partial}{\partial x} \left\{ e^{-\partial t} [p - c(x)][F(x) - \dot{x}] \right\}$

(10.) $\quad = e^{-\partial t} \left\{ -c'(x)[F(x) - \dot{x}] + [p - c(x) F(x)] \right\}$

(11.) $\dfrac{\partial}{\partial t} \dfrac{\partial \phi}{\partial x} = \dfrac{d}{dt} \left\{ e^{-\partial t} [p - c(x)] \right\} = e^{-\partial t} \left\{ \delta [p - c(x)] + c'(x) \dot{x} \right\}$

When these expressions are set equal to each other, they reduce to

(12.) $-c'(x)F(x) + [p - c(x)]F'(x) = [p - c(x)] \quad F'(x) - \dfrac{\acute{c}(x)F(x)}{p - c(x)} = $. This expression states

that the optimal rate of growth of net benefits must be equal to the corresponding discount rate, analogous to the Hotelling rule for exhaustible resources that the user cost must rise at the prevailing rate of interest. Equation (12.) also stipulates is that as long as the rate of growth of a renewable natural resource exceeds the prevailing rate of discount, the optimal harvest policy will be set at a rate below the maximum sustainable yield. For a discount rate above the renewable natural resource rate, the optimal harvest policy will be greater than the maximum sustainable yield, and may even lead to eventual extinction of the resource. Since we are concerned with sustainability of a resource, we need to frame the problem in such a way that we can determine the optimal price necessary to achieve sustainability of the resource over a given time horizon.

From the single species framework, what is essential to retain is the derivation of the corresponding shadow price of the resource necessary to achieve sustainability at a given stock target level. Let us now reformulate the dynamic problem using a discrete-time formulation, where the dynamic constrained optimization takes the form:

(13). $L = \displaystyle\sum_{t=o}^{n} \rho^t \left\{ F(X) + \rho\lambda_{t+1}[R_t - q_t - R_{t+1}] \right\}$, where:

$F(X)$ = the harvesting rate of the resource, depends on the intrinsic rate of growth and the harvest rate, where:

      ^{t} = the discount factor at time t, and
      $_0$ = the shadow price of the resource.

The shadow price of the resource is the price necessary to conserve a rate of extraction and a stock in time t+1, or the sustainability price of the resource. If we stipulate that F(x) takes on a particular form such as $\ln(1+q_t)$, then we have the basic framework to solve the nonlinear dynamic optimization problem. Within this framework, if the initial rate of extraction exceeds the optimal rate, the solution may require that there be no harvesting of the resource up to the point where growth achieves the optimal rate of growth. In this case, the first-order condition must satisfy the Kuhn-Tucker condition and will take the form of $[1/(1+q_t) - {}_{t+1}]q_t = 0$.

Let us now reformulate equation (13.) in which we specify the form of F(X) as rX(1-X/K). Then F'(X)=r(1-2X/K)= , which defines the steady-state optimal stock. Solving for X yields:

(14.) $X* = \dfrac{K(r -)}{2r}$. From this expression, with $Y*=rX*(1-X*/K)$, we derive the optimal harvest rate as:

(15.) $Y* = \dfrac{K\left(r^2 - {}^2\right)}{4r}$. With $= {}'(Y)$, we get $= (1+)(a-bY*)$, or

(16). $* = \left(1 + \right)\left[a - bK\left(r^2 - {}^2\right)/(4r)\right].$

This last expression is the optimal shadow price of the natural resource sufficient to achieve a perpetual rate of harvest of the resource corresponding to the given rate of discount.

What happens when the market price of a resource exceeds or fails to achieve the optimal shadow price? Within a dynamic pricing framework, it depends on whether the initial rate of harvesting exceeds or is less than the minimum rate needed to achieve sustainability of harvests in the future. If the initial rate of harvesting is less than the critical rate (as in point b in Figure 1), then a market price below the optimal rate will lead to an increase in the rate of harvesting. As long as this rate of harvesting does not exceed the minimum level for survival of the species, sustainability will be achieved. Where the market price is less than the optimal shadow price, then there will be an over-harvesting non-sustainable solution (as in point a in Figure 1). What the optimal shadow price provides is a benchmark against which sustainable natural resource management can then evaluate the specific effects of regulation, including individual transferable quotas and other forms of resource pricing.

Multi-Species Dynamics

Sustainable biodiversity requires that the growth and harvesting rates of two or more renewable resources conform to some target range level of stocks over time[8]. Absent human intervention, harvesting can be construed in terms of predator-prey relationships, or a symbiotic relationship in which the health of each species alone depends on some critical mass of one or more other species. Since we are interested in the optimal pricing of bio-diverse renewable natural resources, let us first characterize the underlying growth functions for two individual species:

(17.) $\dfrac{dx}{dt} = rx\left(1 - \dfrac{X}{K}\right) - q_1 Ex$

(18.) $\dfrac{dy}{dt} = sy\left(1 - \dfrac{Y}{L}\right) - q_2 Ey$, where:

  r, s = intrinsic rates of growth of species X and Y, respectively,
  K,L = the environmental carrying capacity of species X and Y, respectively,
  q1, q2 = the catchability coefficients of the two populations, and
  E = the harvesting effort for each species

We can further stipulate that the basic market prices of each species are constant at p1 and p2, in which case, the net benefit function can be defined as:

(19.) $\quad (x, y, E) = p_1 q_1 x E + p_2 q_2 y E - cE$

In equilibrium dx/dt and dy/dt must be equal to zero. As long as 0 x K and 0 y L, then the bionomic equilibrium will be defined by:

(20.) $\quad \dfrac{r}{q_1}\left(1 - \dfrac{x}{K}\right) = \dfrac{s}{q_2}\left(1 - \dfrac{y}{L}\right)$. If $r/q_1 < s/q_2$, then the equilibrium will be given as

(21.) $\quad \tilde{y} =: \left(1 - \dfrac{rq_2}{sq_1}\right)$, where $\tilde{y}$ is the minimum stock of y consistent with a positive stock of x.

Where $r/q_1 < s/q_2$, and $c/(p_2 q_2) < y$, then there will be an ultimate extinction of x. The necessary and sufficient condition for sustainable harvesting of x thus is $c/(p_2 q_2) > \tilde{y}$. The task then is to specify a set of shadow prices that will preserve a given degree of biodiversity among the species.

To specify a set of bio-diverse sustainable prices we need first of all an index of biodiversity. While there is no single standard index in the literature, we propose the use of an index of relative biodiversity (IRB)[9]. This index takes into account the number of renewable resource species as well as the degree of inequality in the distribution of the corresponding biomass. We thus define relative biodiversity, IRB, as:

(22.) $\quad \text{IRB} = (1/(S/ha))(1 - \bar{g}/\bar{x})$, where:

$\quad\quad\quad$ S/ha = the number of species per hectare

$\quad\quad\quad$ $\bar{g}$ = the geometric mean of biomass across species

$\quad\quad\quad$ $\bar{x}$ = the arithmetic mean of biomass across species

The index ranges from 0 to 1, with values closer to 0 corresponding to greater biodiversity. While the index does not incorporate characteristics on the interdependence of species, it serves as a useful benchmark for the biodiversity tracking problem.

Table 1 provides an illustration of the relative biodiversity index for a hypothetical sample of 5 species. Each species has a different carrying capacity as well as a different natural rate of growth such that the corresponding differential maximum sustainable yields will be realized at different points in time. Figure 3 illustrates the growth of multi-species biomass from which Figure 4 provides the corresponding index of relative biodiversity over time. In this example, no harvesting takes place, and there is no specification of stochastic interactions among species. While there will be a corresponding variable level of relative biodiversity over time, the index reaches asymptotic stationarity as t approaches infinity.

Table 1
Multi-Species Logistic Growth Example

	S-1	S-2	S-3	S-4	S-5
r=	0.100	0.070	0.060	0.055	0.048
h=	200,000	230,000	100,000	140,000	120,000
a=	399.00	459.00	199.00	279.00	239.00
t(msy)=	11.56	16.90	17.02	19.76	22.02
P(t,msy))	1621.92	1668.30	1408.79	1508.87	1462.26
MSY(t)	160.88	115.93	83.34	82.09	69.33

Figure 3

Renewable Resource Biodiversity Growth Profile

Figure 4

Index of Relative Biodiversity
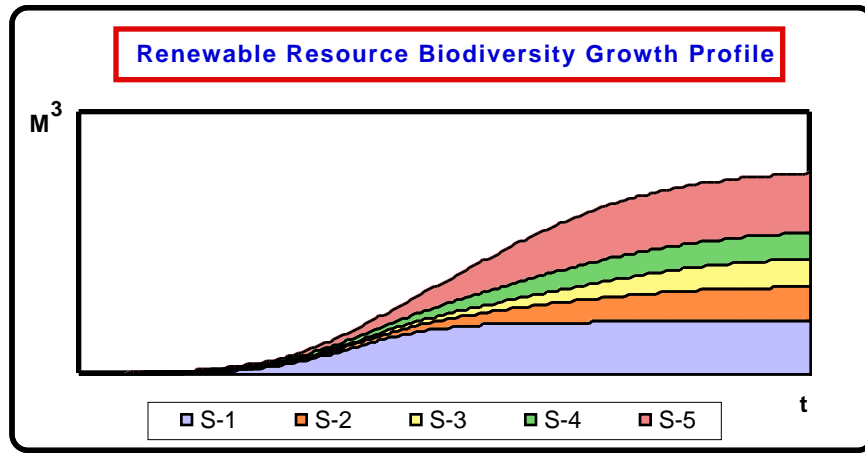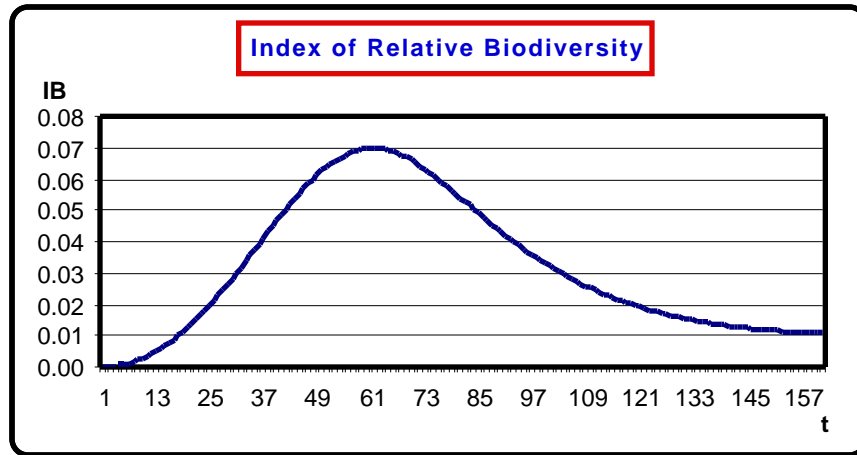
Let us now specify the pricing problem in terms of the relevant optimal control formulation. The objective is to maximize the present value integral:

$$(23.) \quad PV = \int_0 d^{-t}[p_1 q_1 x + p_2 q_2 y - c]E(t)dt , \text{ subject to the constraints defined in equation (17.)}$$

and, the control constraint $0 \leq E(t) \leq E_{max}$. The corresponding Hamiltonian thus is:

$$H = e^{-\delta t}\left[p_1 q_1 x + p_2 q_2 y - c\right]E + \lambda_1(t)\left[F(x) - q_1 Ex\right] + \lambda_2(t)\left[G(y) - q_2 Ey\right]$$

(24.)
$$= \sigma(t)E + \lambda_1 F(x) + \lambda_2 G(y)$$

where the respective lambda expressions are adjoint variables. The corresponding equations are:

(25.) $\dfrac{d\lambda_1}{dt} = -\dfrac{\partial H}{\partial x} = -e^{-\delta t} p_1 q_1 E - \lambda_1\left[F'(x) - q_1 E\right],$ and

(26.) $\dfrac{d\lambda_2}{dt} = -\dfrac{\partial H}{\partial y} = -e^{-\delta t} p_2 q_2 E - \lambda_2\left[G'(y) - q_2 E\right].$

Equilibrium requires that

(27.) $E = \dfrac{F(x)}{q_1 x} = \dfrac{G(y)}{q_2 y}.$ Under this condition, equations (24.) and (25.) reduce to:

(28.) $\dfrac{d\lambda_1}{dt} - \gamma_1 \lambda_1 = -p_1 q_1 E e^{-\delta t}$

(29.) $\dfrac{d\lambda_2}{dt} - \gamma_2 \lambda_2 = -p_2 q_2 E e^{-\delta t}.$

Since $-\gamma_1 = F'(x) - \dfrac{F(x)}{x} = \dfrac{rx}{K}$ and $-\gamma_2 = \dfrac{sy}{L}$, the solutions are:

(30.) $e^{\delta t}\lambda_1(t) = \dfrac{p_1 q_1 E}{\gamma_1 + \delta} = \text{constant},$ and

(31.) $e^{\delta t}\lambda_2(t) = \dfrac{p_2 q_2 E}{\gamma_2 + \delta} = \text{constant}.$

As long as shadow prices are bounded at t approaches infinity, and as long as optimal equilibrium satisfies 0 E Emax, then a closed form solution can be derived such that:

(32.) $\dfrac{\partial H}{\partial E} = e^{-\delta t}\left(p_1 q_1 x + p_2 q_2 y - c\right) - \lambda_1 q_1 x - \lambda_2 q_2 y = 0.$

From equation (26.), equations (28.) and (29.),

(33.) $p_1 q_1\left(x - \dfrac{F(x)}{\gamma_1 + \delta}\right) + p_2 q_2\left(y - \dfrac{G(y)}{\gamma_2 + \delta}\right) = c$

Application of the Discrete-Time Sequential Optimization Framework

It is not obvious that a closed form solution can be derived when the number of biodiverse species is greater than 2. However, it is possible to use the discrete-time framework specified above to derive sequentially the optimal shadow prices of individual species in a steady-state dynamic equilibrium. This discrete-time framework can then be used to examine biological sustainability constraints for the long-term sustainability of individual species under homogeneous and heterogeneous harvesting rates, and how these determinants affect the corresponding shadow prices of the resources.

To illustrate the discrete-time framework, let us consider an environment containing 3 species of renewable natural resources. X1 constitutes a floral biomass. X2 represents a faunal biomass dependent on the floral biomass. X3 is a faunal biomass dependent on the X2 faunal biomass. The floral biomass can be measured in terms of habitat density, as in cubic meters per hectare. Each faunal biomass can be measured likewise[10].

We can define the steady-state relations as follows:

(34.) $\qquad \alpha_2 X_2 = r_1 X_1 - \dfrac{r_1 X_1^2}{K}$,

where $_2$ represents the represents the herbivore rate of consumption of floral biomass. In turn the herbivore rate of growth is constrained by:

(35.) $\qquad r_2 - \dfrac{r_2 X_2}{\left(\beta X_1\right)} = \gamma X_3$

where reflects the rate of predation of the carnivore per herbivore. Finally, the carnivore population is determined by the intrinsic rate of growth but also by a carrying capacity factor, η:

(36.) $\qquad X_3 = \eta X_2$

While we could use these equations to derive the steady-state biological equilibrium for each species, we are interested in solving for the respective shadow prices today that correspond to the maximization of the present value of steady-state stocks and harvesting rates. Once this is determined, one can then measure the impact of imposing a constraint of an index of relative biodiversity over time, or some minimum rate of growth necessary to avoid extinction of a species. If the corresponding steady-state value of the index of relative biodiversity is not consistent with biological sustainability of a given species or a target level of biodiversity to achieve sustainability among several species, one can then introduce additional constraints on the dynamic harvesting of a species and then examine the corresponding effects on the respective

shadow prices. These new shadow prices can then be used to price harvesting rights to species consistent with the long-term sustainability of biodiverse resources.

To derive the optimal shadow price of a resource, we need to specify the corresponding net benefit function. While the net benefit function for each resource could be linear, to put the problem within the context of a single resource owner, we utilize a quadratic formulation:

(37.) $\quad \pi\left(Y_t\right) = aY_t - (b/2)Y_t^2$, where a > b > 0.

Net growth of the stock corresponds to   , as defined in equation 12. With this framework, we modify equation 24 to define the maximization problem as:

(38.) $\quad L = \sum_0^{t-1} \rho^t\left(aY_t - \dfrac{b}{2}\ Y_t^2\right) + \dfrac{\rho^{t-1}\left(a\left(rX_t\left(1 - \dfrac{X_t}{K}\right)\right) - \dfrac{b}{2}\left(rX_t\left(1 - \dfrac{X_t}{K}\right)\right)^2\right)}{\delta}$

The second term represents the terminal condition, which plays an important role in the sustainability condition. Basically, the last expression stipulates that whatever harvesting rate is chosen, at the terminal period, the stock level will be harvested sustainably for the foreseeable future.

We first examine the discrete-time maximization problem for each species for a 30-year time period. Parametric variations are introduced sequentially, beginning with a base case scenario in which biomass alone grows unharvested in the environment, followed by a biomass alone optimal harvest schedule. In turn, we then show the separate effects of the introduction of a herbivore species on the unconstrained biomass, then on the optimal biomass, then with both optimal herbivore and optimal biomass. We then repeat this sequence with the introduction of a carnivore species. In this base case scenario, the dynamics are deterministic, the net benefit function is homogeneous across species, with initial production and carrying capacity levels selected according to biological and environmental data. We have set arbitrarily the carrying capacity ratios as follows: herbivore to biomass at .33, and carnivore to herbivore at .10.

Table 2
Comparative Effects of Non-Harvesting and Sequential Optimal Harvesting
under Base Case Scenarios

	PVNB	IRB	λ1	λ2	λ3	λ∗1	λ∗2	λ∗3
Base Case								
biomass alone, no harvesting	$5,923.60	1.0000	60.89	73.58	43.00	65.66	69.49	71.21
biomass, herbivore no harvesting	$15,201.13	0.0670	64.48	73.58	43.00	65.66	69.49	71.21
all species no harvesting	$15,507.53	0.1817	64.26	73.58	43.00	65.66	69.49	71.21
biomass alone optimal harvest	$18,436.49	1.0000	65.66	50.47	43.00	65.66	69.49	71.21
biomass with herbivore optimal harvest	$17,530.98	0.1082	63.96	73.58	43.00	65.66	69.49	71.21
carnivore optimal harvest	$15,818.48	0.1859	64.35	73.58	43.00	65.66	69.49	71.21
biomass-herbivore optimal harvest	$19,705.43	0.0689	65.83	73.58	43.00	65.66	69.49	71.21
biomass-carnivore optimal harvest	$17,496.48	0.1715	65.93	73.58	43.00	65.66	69.49	71.21
herbivore-carnivore optimal harvest	$16,263.24	0.2024	63.67	73.58	43.00	65.66	69.49	71.21
all species optimal harvest	$18,026.86	0.1701	66.09	66.27	43.00	65.66	69.49	71.21

Base Case Parameters		X1	X2	X3
Net benefits 1st parameter	a =	70.00	70.00	70.00
Net benefits 2nd parameter	b =	1.00	1.00	1.00
Intrinsic growth rate	r =	0.0800	0.0800	0.0800
Carrying capacity	K=	300.00	99.99	10.00
Discount rate	=	0.0200	0.0200	0.0200
X* = K(r-)/2r		112.50	37.50	3.75
Y* = K(r²- ²)/4r		5.63	1.87	0.19

herbivore grass cons.rate, =	0.0200
predation rate per carnivore, =	0.0100
herbivore to grass ratio, =	0.3333
predator/herb.support ratio, h =	0.1000
n =	30.00

Table 2 provides a base reference case. Sequential inclusion of herbivore and carnivore species increases the present value of the natural resource environment. In turn, the initial shadow price of the biomass increases with the introduction of the herbivore and carnivore species. Because the carrying capacity ratios have been set arbitrarily, the index of relative biodiversity increases with the introduction of the herbivore species and then decreases with the introduction of the carnivore species. In these simulations, each species grows arbitrarily at a homogeneous uniform intrinsic rate, and we apply a homogeneous discount rate across all species. In turn, we can think of the zero harvest benefit function as an equivalent total revenue function were a resource harvested only in the terminal time period. This is similar to the contingent valuation procedure except that we now take into explicit account the relative valuation of individual species and a given level of biodiversity.

We now introduce dynamic optimization in a sequential pattern, with constraints imposed for carrying capacity ratios as defined above. In the absence of herbivore and carnivore species, optimal harvesting of biomass alone increases the present value of the natural resource, and the initial shadow price is equivalent to the sustainable solution value. When biomass and herbivore species are both present, optimal harvesting of the herbivore species with no optimization of the biomass in this case produces an initial shadow price above the sustainable solution value, with harvesting of the herbivore beginning in the base period 0. When carnivore species are introduced and no optimization is undertaken on either the biomass or herbivore species, the new

present value is reduced below the two preceding solutions, based on the rate of predation on herbivores by carnivores. In these latter two cases, the index of relative biodiversity is slightly greater than in the no harvesting cases.

Table 3
Comparative Effects of Initial Excess and Deficient Sustainable Stocks

	PVNB	IRB	λ1	λ2	λ3	λ*1	λ*2	λ*3
Excess Minimum Initial Stocks								
biomass alone, no harvesting	$11,009.66	1.0000	63.71	73.58	43.00	65.45	69.42	71.20
biomass, herbivore no harvesting	$12,516.33	0.0670	66.64	73.58	43.00	65.45	69.42	71.20
all species no harvesting	$12,838.25	0.1701	66.64	73.58	43.00	65.45	69.42	71.20
biomass alone optimal harvest	$17,021.86	1.0000	65.67	50.66	43.00	65.45	69.42	71.20
biomass with herbivore optimal harvest	$14,450.32	0.1518	61.96	73.58	43.00	65.45	69.42	71.20
carnivore optimal harvest	$13,155.52	0.2088	62.00	73.58	43.00	65.45	69.42	71.20
biomass-herbivore optimal harvest	$21,814.80	0.0690	65.78	61.54	43.00	65.45	69.42	71.20
biomass-carnivore optimal harvest	$20,218.29	0.1222	65.78	61.54	43.00	65.45	69.42	71.20
herbivore-carnivore optimal harvest	$13,483.51	0.1595	61.85	73.58	43.00	65.45	69.42	71.20
all species optimal harvest	$21,261.96	0.1811	65.63	61.14	43.00	65.45	69.42	71.20
Deficient Minimum Initial Stocks								
biomass alone, no harvesting	$5,923.60	1.0000	60.89	73.58	43.00	66.61	64.8	71.24
biomass, herbivore no harvesting	$10,879.03	0.0670	61.69	73.58	43.00	66.61	64.8	71.24
all species no harvesting	$11,678.15	0.2094	61.44	73.58	43.00	66.61	64.8	71.24
biomass alone optimal harvest	$18,243.26	1.0000	65.23	49.55	43.00	66.61	64.8	71.24
biomass with herbivore optimal harvest	$13,657.37	0.1440	61.46	73.58	43.00	66.61	64.8	71.24
carnivore optimal harvest	$11,985.14	0.2101	61.45	73.58	43.00	66.61	64.8	71.24
biomass-herbivore optimal harvest	$22,928.16	0.0670	65.23	52.86	43.00	66.61	64.8	71.24
biomass-carnivore optimal harvest	$21,328.78	0.0670	65.23	52.86	43.00	66.61	64.8	71.24
herbivore-carnivore optimal harvest	$11,803.45	0.2118	61.44	73.58	43.00	66.61	64.8	71.24
all species optimal harvest	$21,401.01	0.1707	65.19	52.86	43.00	66.61	64.8	71.24

Excess and Deficiency Stock Parameters		X1	X2	X3
Net benefits 1st parameter	a =	70.00	70.00	70.00
Net benefits 2nd parameter	b =	1.00	1.00	1.00
Intrinsic growth rate	r =	0.0800	0.0800	0.0800
Carrying capacity	K=	300.00	99.99	10.00
Discount rate	=	0.0200	0.0200	0.0200
Base Case X* = K(r-)/2r		112.50	37.50	3.75
Base Case Y* = K(r- ²)/4r		5.63	1.87	0.19
Excess Initial Stock, X* = K(r-)/2r		80.00	26.66	2.67
Excess Initial Harvest, Y* = K(r²- ²)/4r		5.63	1.87	0.19
Deficient Initial Stock, X* = K(r-)/2r		125.00	41.66	4.17
Deficient Initial Harvest, Y* = K(r²- ²)/4r		5.63	1.87	0.19
herbivore grass cons.rate, =	0.0200			
predation rate per carnivore, =	0.0100			
herbivore to grass ratio, =	0.3333			
predator/herb.support ratio, h =	0.1000			
n =	30.00			

What happens in the case where initial production levels are either in excess supply or insufficient to guarantee sustainability of a species? Based on biological knowledge of the degree of genetic diversity as a function of the population size, we can set some minimum level of stocks essential for sustainability, as is shown in Table 3. Under sequential introduction of species, we show the separate effects of no harvesting and optimal harvesting constrained by minimum stocks per species. In the absence of carnivore species, biomass and herbivore constrained optimization raises the level of the present value of benefits, while increasing the degree of biodiversity. Optimization of carnivore and biomass harvesting with no optimization on herbivores slightly reduces the level of net benefits, even as relative biodiversity declines. The initial shadow prices for the biomass and herbivore species are above the optimal level, but for carnivores, it is below the optimal level. Finally, optimal harvesting of all species subject to the

constraints of the carrying capacity ratios raises the level of benefits, and brings the index of relative biodiversity to a greater degree in comparison to the non-harvesting of all species.

Let us now examine the effects of technical change and changes in discount rates on the optimal pricing of the resources. Table 4 illustrates sequential optimization under embodied and disembodied technical change. Given the interdependence of species, whether net benefits are higher or lower depends on how many species are included in the environment and on whether single or multiple dynamic optimization is used. When all species are taken into account and when optimization is adopted for all three, embodied technical change produces a higher net present benefit outcome than under the disembodied alternative. At the same time, because disembodied technical change produces effects that occur in later time periods, the corresponding shadow prices are higher than under the embodied technical change version. There are no significant differences in the terminal value of the index of relative biodiversity when all species are included.

Table 4
Comparative Effects under Technical Change

	PVNB	IRB	$\lambda 1$	$\lambda 2$	$\lambda 3$	$\lambda*1$	$\lambda*2$	$\lambda*3$
Embodied Technical Change (r=.10 versus .08 base case)								
biomass alone, no harvesting	$4,588.87	1.0000	57.71	74.36	36.27	64.06	68.95	71.16
biomass, herbivore no harvesting	$10,084.86	0.0670	58.57	74.36	36.27	64.06	68.95	71.16
all species no harvesting	$11,936.57	0.2072	58.29	74.36	36.27	64.06	68.95	71.16
biomass alone optimal harvest	$23,051.76	1.0000	64.06	38.07	36.27	64.06	68.95	71.16
biomass with herbivore optimal harvest	$14,327.71	0.1611	58.22	74.36	36.27	64.06	68.95	71.16
carnivore optimal harvest	$12,315.00	0.2107	58.31	74.36	36.27	64.06	68.95	71.16
biomass-herbivore optimal harvest	$29,916.24	0.0678	64.12	50.37	36.27	64.06	68.95	71.16
biomass-carnivore optimal harvest	$27,396.27	0.1717	64.18	50.37	36.27	64.06	68.95	71.16
herbivore-carnivore optimal harvest	$13,075.24	0.2239	58.15	74.36	36.27	64.06	68.95	71.16
all species optimal harvest	$28,978.52	0.1748	64.09	50.37	36.27	64.06	68.95	71.16
Disembodied Technical Change (K=10% over base; r = .08)								
biomass alone, no harvesting	$5,321.02	1.0000	60.69	73.58	43.00	65.66	69.49	71.21
biomass, herbivore no harvesting	$10,255.64	0.0670	61.49	73.58	43.00	65.66	69.49	71.21
all species no harvesting	$11,148.01	0.2103	61.23	73.58	43.00	65.66	69.49	71.21
biomass alone optimal harvest	$19,109.58	1.0000	65.66	50.39	43.00	65.66	69.49	71.21
biomass with herbivore optimal harvest	$13,329.18	0.1613	61.17	73.58	43.00	65.66	69.49	71.21
carnivore optimal harvest	$11,530.39	0.2143	61.25	73.58	43.00	65.66	69.49	71.21
biomass-herbivore optimal harvest	$21,669.99	0.0670	65.78	61.31	43.00	65.66	69.49	71.21
biomass-carnivore optimal harvest	$14,081.04	0.2091	61.13	71.15	51.01	65.09	69.09	71.15
herbivore-carnivore optimal harvest	$14,381.01	0.2212	61.02	73.15	51.01	65.09	69.09	71.15
all species optimal harvest	$24,357.64	0.1744	65.14	61.86	51.01	65.09	69.09	71.15

Technical Change Parameters		X1	X2	X3
Net benefits 1st parameter	a =	70.00	70.00	70.00
Net benefits 2nd parameter	b =	1.00	1.00	1.00
Intrinsic growth rate	r =	0.0800	0.0800	0.0800
Embodied Technical Change growth rate	r'=	**0.1000**	**0.1000**	**0.1000**
Carrying capacity	K=	300.00	99.99	10.00
Disembodied Technical carrying capacity	K'=	**330.00**	**109.99**	**11.00**
Discount rate	=	0.0200	0.0200	0.0200
X* = K(r-)/2r		112.50	37.50	3.75
Y* = K(r̂- ²)/4r		5.63	1.87	0.19
Disembodied Optimal Stock, X* = K(r-)/2r		123.75	45.37	4.99
Disembodied Optimal Harvest, Y* = K(r̂- ²)/4r		6.19	2.27	0.25

herbivore grass cons.rate, =	0.0200
predation rate per carnivore, =	0.0100
herbivore to grass ratio, =	0.3333
predator/herb.support ratio, h =	0.1000
n =	30.00

Changes in the discount rate produce predictable effects. In Table 5, we illustrate change in a common discount rate, whose base case value is 2 percent. Increases in the discount rate reduce the net present benefits function, while reductions result in an increase. At the same time, reductions in the discount rate also lower the corresponding shadow price of a resource while increases tend to raise them as optimal harvesting shifts future production back toward the present.

Table 5
Comparative Effects under Alternative Discount Rates

	PVNB	IRB	λ1	λ2	λ3	λ*1	λ*2	λ*3
Increase in discount rate (δ = 5% vs. 2% base)								
biomass alone, no harvesting	$1,022.26	1.0000	62.75	74.28	43.04	69.66	72.22	73.37
biomass, herbivore no harvesting	$1,877.44	0.0670	63.58	74.28	43.04	69.66	72.22	73.37
all species no harvesting	$2,015.34	0.2094	63.32	74.28	43.04	69.66	72.22	73.37
biomass alone optimal harvest	$8,061.23	1.0000	69.65	67.41	43.04	69.66	72.22	73.37
biomass with herbivore optimal harvest	$3,946.90	0.2438	63.04	74.28	43.04	69.66	72.22	73.37
carnivore optimal harvest	$2,260.54	0.2322	63.46	74.28	43.04	69.66	72.22	73.37
biomass-herbivore optimal harvest	$8,902.54	0.0670	68.11	65.00	43.04	69.66	72.22	73.37
biomass-carnivore optimal harvest	$7,499.27	0.1701	68.11	65.84	43.04	69.66	72.22	73.37
herbivore-carnivore optimal harvest	$2,841.92	0.2257	63.22	74.28	43.04	69.66	72.22	73.37
all species optimal harvest	$6,927.46	0.2043	65.78	71.34	43.04	69.66	72.22	73.37
Decrease in discount rate (δ = 0% vs. 2% base)								
biomass alone, no harvesting	$204,377.12	1.0000	60.54	73.44	42.99	64.06	68.07	69.87
biomass, herbivore no harvesting	$375,350.45	0.0670	61.33	73.44	42.99	64.06	68.07	69.87
all species no harvesting	$402,921.75	0.2094	61.08	73.44	42.99	64.06	68.07	69.87
biomass alone optimal harvest	$399,004.89	1.0000	64.06	3.13	42.99	64.06	68.07	69.87
biomass with herbivore optimal harvest	$401,733.37	0.1217	61.16	73.44	42.99	64.06	68.07	69.87
carnivore optimal harvest	$407,601.18	0.2050	61.11	73.44	42.99	64.06	68.07	69.87
biomass-herbivore optimal harvest	$534,197.95	0.0678	64.11	52.31	42.99	64.06	68.07	69.87
biomass-carnivore optimal harvest	$544,730.73	0.1734	64.11	52.31	42.99	64.06	68.07	69.87
herbivore-carnivore optimal harvest	$410,327.70	0.2018	61.13	73.44	42.99	64.06	68.07	69.87
all species optimal harvest	$410,342.71	0.2018	61.13	73.44	42.99	64.06	68.07	69.87

Alternative Discount Parameters		X1	X2	X3
Net benefits 1st parameter	a =	70.00	70.00	70.00
Net benefits 2nd parameter	b =	1.00	1.00	1.00
Intrinsic growth rate	r =	0.0800	0.0800	0.0800
Carrying capacity	K=	300.00	99.99	10.00
Base case discount rate	=	0.0200	0.0200	0.0200
Alternative discount rate 1	' =	**0.0500**	**0.0500**	**0.0500**
Alternative discount rate 2	' =	**0.0000**	**0.0000**	**0.0000**
X* = K(r-)/2r		112.50	37.50	3.75
Y* = K(r̂- ²)/4r		5.63	1.87	0.19
Optimal Stock, X* = K(r-)/2r		123.75	45.37	4.99
Optimal Harvest, Y* = K(r²- ²)/4r		6.19	2.27	0.25
herbivore grass cons.rate, =	0.0200			
predation rate per carnivore, =	0.0100			
herbivore to grass ratio, =	0.3333			
predator/herb.support ratio, h =	0.1000			
n =	30.00			

Uncertainty and the Role of Property Rights in the Optimal Pricing of Renewable Natural Resources

We now consider the impact of uncertainty and property rights in the optimal pricing of renewable natural resources. For the first condition, one can introduce a random number generator that affects the dynamic stock function. We do so here and leave the optimal harvesting as a residual from the constrained optimal growth scenario. As to the magnitude of random effects, we also can set a control over the degree of randomness. As long as the degree

of randomness is less than the intrinsic rate of growth of a species, under both non-harvesting and constrained optimization harvesting scenarios there may be net growth or not as stocks asymptotically approach the underlying carrying capacity level of a species. Setting the degree of randomness in the model can be done through the following control process:

(38.) $Random = \left(1 - RAND()^{1/c}\right)$, where:

   c = a control number ranging from 1 to infinity.

The greater the value of c, the smaller will be the degree of randomness in the growth of a stock. Selection of a given value of c should be based on the actual degree of randomness in the growth of a stock from observed field behavior. How much harvesting should be constrained to zero in the presence of uncertainty is a function of the degree of randomness in the growth rates of species. While the cause of uncertainty may be due to purely random effects such as climatic variation, where property rights are variable, the problem will also be affected.

Property rights are central to the selection of the optimal pricing of biodiverse renewable natural resources. While we have developed a framework for the efficient pricing of biodiverse resources, they can not be achieved where property rights are imperfectly defined. Brown (2000) notes several characteristics of renewable natural resources where market prices fail to achieve economically efficient outcomes.[11] They are: 1. jurisdictional externalities; 2. public goods and non-use values; 3. public goods for which the location of beneficiaries and cost bearers are spatially separated; 4. ecological complexity that obscures cause (benefits) and effect (costs); 5. dramatic time lags between individual actions and subsequent social consequences; and 6. the unintended, unforeseen, environmental consequences of public policies pursuing non-environmental objectives. In each of these instances, property rights are poorly defined with the result that there is a classical externality divergence between value and opportunity cost of a resource.

The most promising approach to the property rights assignment problem is individual transferable quotas, or ITQ's. The necessary condition for an efficient use of ITQ's, like the analogous Tradable Pollution Permits in environmental management, is that the price capture both the externality effect as well as the dynamic efficiency conditions for sustainable use of the resource. We have not extended the present model to the selection of optimal ITQ's, nor have we addressed the corollary question of the stochastic behavior of species[12]. What the optimal biodiverse pricing model does suggest is a way to price ITQ's in ways that achieve both sustainability as well as a given target level of biodiversity.

Conclusion

The efficient pricing of biodiverse renewable natural resources depends on several considerations. Intrinsic rates of biological growth, the carrying capacity of a habitat, interdependence of species, existing market prices, property rights, and the underlying time frame each are important to the derivation of an optimal solution. Existing approaches to sustainable biodiversity rely largely on the use of quantity-based restrictions for the harvesting of species. In this paper we have developed an alternative approach that derives the corresponding shadow prices necessary to achieve an equivalent allocation of resources over time. By incorporating environmental and biological information into the economic framework of constrained dynamic optimization, one can achieve a more efficient allocation of renewable natural resources than through the use of single-species quota models.

Notes

[1] Michael Crichton's *Jurassic Park*, and Richard Stone's *Mammoth*, illustrate both the fascination and challenge of using genetics to sustain the diversity of species.

[2] See, for example, Geoffrey Heal, *Valuing the Future: Economic Theory and Sustainability*. (New York: Columbia University Press, 1998); Anthony C. Fisher, *Resource and Environmental Economics*. (New York: Cambridge University Press, 1981); Partha S. Dasgupta and Geoffrey M. Heal, *Economic Theory and Exhaustible Resources.* (New York: Cambridge University Press, 1979); and H.E. Goeller, "The Age of Substitutability", *Science*, vol. 191 (February 20, 1976), pp. 683-689, and Phillip G. LeBel, *Energy Economics and Technology*. (Baltimore: The Johns Hopkins University Press, 1982).

[3] Predator-prey models were first developed independently by Alfred Lotka and V. Volterra. See, *Mathematical Biology* (New York: Dover Publications, 1956 reprint of 1925 edition), and V. Volterra, "Variazionie fluttuazioni del numero d'individui in specie animali conviventi", *Mem. Acad. Lincei*. 2, 31-113 (1926).

[4] Jon Conrad provides a simulation for three species in a concluding chapter of his book on resource economics but does not point to an optimal sustainable pricing solution. Jon Conrad, *Resource Economics* (New York: Cambridge University Press, 1999).

[5] See, for example, Jon M. Conrad and Colin W. Clark, *Natural Resource Economics* (New York: Cambridge University Press, 1991, 1987); Colin W. Clark *Mathematical Bioeconomics: The Optimal Management of Renewable Resources,* second edition. (New York: John Wiley and Sons, 1990); and Lee G. Anderson, *The Economics of Fisheries Management,* second edition. (Baltimore, Md.: The Johns Hopkins University Press, 1986, 1977).

[6] Verhulst, P.F. "Notice sur la loi que la population suit dans son accroissement," *Correspondance Mathématique et Physique* 10, 113-121. Verhulst's formulation was popularized in Lotka, Alfred J. *Elements of Mathematical Biology*, 1924. Reprint. (New York: Dover Publications, 1956). For a treatment of some alternative formulations, see Jon Conrad, *Resource Economics* (New York: Cambridge University Press, 1999).

[7] If we mapped the natural growth rate as a function of time alone, it would resemble a classic bell curve, analogous to the relationship between a probability density function and a cumulative probability density function.

[8] A target range level must take into consideration the stochastic behavior of species as well as a limit value essential to the level of sustainable reproduction.

[9] Most treatments on biodiversity track the status of species as well as their numbers. Coverage includes measures of healthy species, endangered species, and extinct species. See, for example, David Pearce and Dominic Moran, *The Economic Value of Biodiversity* (London: Earthscan Publications for IUCN, 1994); Charles Perrings, et.al., *Biodiversity Loss* (Cambridge, U.K.: Cambridge University Press, 1995); and E.O. Wilson, *The Diversity of Life* (New York: W.W. Norton, 1992). One effort to derive an index of biodiversity is in Jon Conrad, *Resource Economics* (New York: Cambridge University Press, 1999), pp. 176-177. Conrad's first index takes extinction into account to produce an extreme value of zero, and then offers a second that takes into account species interdependence. Our index is in the spirit of his second approach, though it explicit considers the distribution of biomass as well as the number of species.

[10] Determination of the population size can be done either in terms of the population stock or in terms of the ratio of total biomass to the mean biomass per species to derive the corresponding total population level of each species.

[11] Gardner M. Brown(2000), "Renewable Natural Resource Management and Use without Markets", *Journal of Economic Literature* XXXVIII, December, pp.875-914.

[12] Stochasticity can be achieved through a random number process that operates on the growth rate of each individual species. The corresponding shadow prices would then represent a range of values over the corresponding range of random behavior.

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