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THE UTILITY ANALYSIS OF CHOICES INVOLVING RISK[1]

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I. THE PROBLEM AND ITS BACKGROUND

The purpose of this paper is to suggest that an important class of reactions of individuals to risk can be rationalized by a rather simple extension of orthodox utility analysis.

Individuals frequently must, or can, choose among alternatives that differ, among other things, in the degree of risk to which the individual will be subject. The clearest examples are provided by insurance and gambling. An individual who buys fire insurance on a house he owns is accepting the certain loss of a small sum (the insurance premium) in preference to the combination of a small chance of a much larger loss (the value of the house) and a large chance of no loss. That is, he is choosing certainty in preference to uncertainty. An individual who buys a lottery ticket is subjecting himself to a large chance of losing a small amount (the price of the lottery ticket) plus a small chance of winning a large amount (a prize) in preference to avoiding both risks. He is choosing uncertainty in preference to certainty.

This choice among different degrees of risk so prominent in insurance and gambling, is clearly present and important in a much broader range of economic choices. Occupations differ greatly in the variability of the income they promise: in some, for example, civil service employment, the prospective income is rather clearly defined and is almost certain to be within rather narrow limits; in others, for example, salaried employment as an accountant, there is somewhat more variability yet almost no chance of either an extremely high or an extremely low income; in still others, for example, motion-picture acting, there is extreme variability, with a small chance of an extremely high income and a larger chance of an extremely low income. Securities vary similarly, from government bonds and industrial "blue chips" to "blue-sky" common stocks; and so do business enterprises or lines of business activity. Whether or not they realize it and whether or not they take explicit account of the varying degree of risk involved, individuals choosing among occupations, securities, or lines of business activity are making choices analogous to those that they make when they decide whether to buy insurance or to gamble. Is there

[1] The fundamental ideas of this paper were worked out jointly by the two authors. The paper was written primarily by the senior author.

any consistency among the choices of this kind that individuals make? Do they neglect the element of risk? Or does it play a central role? If so, what is that role?

These problems have, of course, been considered by economic theorists, particularly in their discussions of earnings in different occupations and of profits in different lines of business.[2] Their treatment of these problems has, however, never been integrated with their explanation of choices among riskless alternatives. Choices among riskless alternatives are explained in terms of maximization of utility: individuals are supposed to choose as they would if they attributed some common quantitative characteristic—designated utility—to various goods and then selected the combination of goods that yielded the largest total amount of this common characteristic. Choices among alternatives involving different degrees of risk, for example, among different occupations, are explained in utterly different terms—by ignorance of the odds or by the fact that "young men of an adventurous disposition are more attracted by the prospects of a great success than they are deterred by the fear of failure," by "the overweening conceit which the greater part of men have of their own abilities," by "their absurd presumption in their own good fortune," or by some similar *deus ex machina*.[3]

The rejection of utility maximization as an explanation of choices among different degrees of risk was a direct consequence of the belief in diminishing marginal utility. If the marginal utility of money diminishes, an individual seeking to maximize utility will never participate in a "fair" game of chance, for example, a game in which he has an equal chance of winning or losing a dollar. The gain in utility from winning a dollar will be less than the loss in utility from losing a dollar, so that the expected utility from participation in the game is negative. Diminishing marginal utility plus maximization of expected utility would thus imply that individuals would always have to be paid to induce them to bear risk.[4] But this implication is clearly contradicted by actual behavior. People not only engage in fair games of chance, they engage freely and often eagerly in such unfair games as lotteries. Not only do risky occupations and risky investments not always yield a higher average return than relatively safe occupations or investments, they frequently yield a much lower average return.

Marshall resolved this contradiction by rejecting utility maximization as an explanation of choices involving risk. He need not have done so, since he did not need diminishing marginal utility—or, indeed, any quantitative concept of utility—for the analysis of riskless choices.

[2] E.g., see Adam Smith, *The Wealth of Nations*, Book I, chap. x (Modern Library reprint of Cannan ed.), pp. 106–11; Alfred Marshall, *Principles of Economics* (8th ed.; London: Macmillan & Co., Ltd., 1920), pp. 398–400, 554–55, 613.

[3] Marshall, *op. cit.*, p. 554 (first quotation); Smith, *op. cit.*, p. 107 (last two quotations).

[4] See Marshall, *op. cit.*, p. 135 n.; Mathematical Appendix, n. ix (p. 843). "Gambling involves an economic loss, even when conducted on perfectly fair and even terms. . . . A theoretically fair insurance against risks is always an economic gain" (p. 135). "The argument that fair gambling is an economic blunder . . . requires no further assumption than that, firstly the pleasures of gambling may be neglected; and, secondly $\phi''(x)$ is negative for all values of x, where $\phi(x)$ is the pleasure derived from wealth equal to x. . . . It is true that this loss of probable happiness need not be greater than the pleasure derived from the excitement of gambling, and we are then thrown back upon the induction that pleasures of gambling are in Bentham's phrase 'impure'; since experience shows that they are likely to engender a restless, feverish character, unsuited for steady work as well as for the higher and more solid pleasures of life" (p. 843).

The shift from the kind of utility analysis employed by Marshall to the indifference-curve analysis of F. Y. Edgeworth, Irving Fisher, and Vilfredo Pareto revealed that to rationalize riskless choices, it is sufficient to suppose that individuals can rank baskets of goods by total utility. It is unnecessary to suppose that they can compare differences between utilities. But diminishing, or increasing, marginal utility implies a comparison of differences between utilities and hence is an entirely gratuitous assumption in interpreting riskless choices.

The idea that choices among alternatives involving risk can be explained by the maximization of expected utility is ancient, dating back at least to D. Bernoulli's celebrated analysis of the St. Petersburg paradox.[5] It has been repeatedly referred to since then but almost invariably rejected as the correct explanation—commonly because the prevailing belief in diminishing marginal utility made it appear that the existence of gambling could not be so explained. Even since the widespread recognition that the assumption of diminishing marginal utility is unnecessary to explain riskless choices, writers have continued to reject maximization of expected utility as "unrealistic."[6] This rejection of maximization of expected utility has been challenged by John von Neumann and Oskar Morgenstern in their recent book, *Theory of Games and Economic Behavior*.[7] They argue that "under the conditions on which the indifference curve analysis is based very little extra effort is needed to reach a numerical utility," the expected value of which is maximized in choosing among alternatives involving risk.[8] The present paper is based on their treatment but has been made self-contained by the paraphrasing of essential parts of their argument.

If an individual shows by his market

[5] See Daniel Bernoulli, *Versuch einer neuen Theorie der Wertbestimmung von Glücksfällen* (Leipzig, 1896), translated by A. Pringsheim from "Specimen theoriae novae de mensura sortis," *Commentarii academiae scientiarum imperialis Petropolitanae*, Vol. V, for the years 1730 and 1731, published in 1738.

In an interesting note appended to his paper Bernoulli points out that Cramer (presumably Gabriel Cramer [1704–52]), a famous mathematician of the time, had anticipated some of his own views by a few years. The passages that he quotes from a letter in French by Cramer contain what, to us, is the truly essential point in Bernoulli's paper, namely, the idea of using the mathematical expectation of utility (the "moral expectation") instead of the mathematical expection of income to compare alternatives involving risk. Cramer has not in general been attributed this much credit, apparently because the essential point in Bernoulli's paper has been taken to be the suggestion that the logarithm of income is an appropriate utility function.

[6] "It has been the assumption in the classical literature on this subject that the individual in question will always try to maximize the mathematical expectation of his gain or utility. . . . This may appear plausible, but it is certainly not an assumption which must hold true in all cases. It has been pointed out that the individual may also be interested in, and influenced by, the range or the standard deviation of the different possible utilities derived or some other measure of dispersion. It appears pretty evident from the behavior of people in lotteries or football pools that they are not a little influenced by the skewness of the probability distribution" (Gerhard Tintner, "A Contribution to the Non-Static Theory of Choice," *Quarterly Journal of Economics*, LVI [February, 1942], 278).

"It would be definitely unrealistic . . . to confine ourselves to the mathematical expectation only, which is the usual but not justifiable practice of the traditional calculus of 'moral probabilities' " (J. Marschak, "Money and the Theory of Assets," *Econometrica*, VI [1938], 320).

Tintner's inference, apparently also shared by Marschak, that the facts he cites are necessarily inconsistent with maximization of expected utility is erroneous (see secs. 3 and 4 below). He is led to consider a formally more general solution because of his failure to appreciate the real generality of the kinds of behavior explicable by the maximization of expected utility.

[7] Princeton University Press, 1st ed., 1944; 2d ed., 1947; pp. 15–31 (both eds.), pp. 617–32 (2d ed. only); succeeding references are to 2d ed.

[8] *Ibid.*, p. 17.

behavior that he prefers A to B and B to C, it is traditional to rationalize this behavior by supposing that he attaches more utility to A than to B and more utility to B than to C. All utility functions that give the same ranking to possible alternatives will provide equally good rationalizations of such choices, and it will make no difference which particular one is used. If, in addition, the individual should show by his market behavior that he prefers a 50-50 chance of A or C to the certainty of B, it seems natural to rationalize this behavior by supposing that the *difference* between the utilities he attaches to A and B is greater than the *difference* between the utilities he attaches to B and C, so that the *expected* utility of the preferred combination is greater than the utility of B. The class of utility functions, if there be any, that can provide the same ranking of alternatives that involve risk is much more restricted than the class that can provide the same ranking of alternatives that are certain. It consists of utility functions that differ only in origin and unit of measure (i.e., the utility functions in the class are linear functions of one another).[9] Thus, in effect, the ordinal properties of utility functions can be used to rationalize riskless choices, the numerical properties to rationalize choices involving risk.

It does not, of course, follow that there will exist a utility function that will rationalize in this way the reactions of individuals to risk. It may be that individuals behave inconsistently—sometimes choosing a 50-50 chance of A or C instead of B and sometimes the reverse; or sometimes choosing A instead of B, B instead of C, and C instead of A—or that in some other way their behavior is different from what it would be if they were seeking rationally to maximize expected utility in accordance with a given utility function. Or it may be that some types of reactions to risk can be rationalized in this way while others cannot. Whether a numerical utility function will in fact serve to rationalize any particular class of reactions to risk is an empirical question to be tested; there is no obvious contradiction such as was once thought to exist.

This paper attempts to provide a crude empirical test by bringing together a few broad observations about the behavior of individuals in choosing among alternatives involving risk (sec. 2) and investigating whether these observations are consistent with the hypothesis revived by von Neumann and Morgenstern (secs. 3 and 4). It turns out that these empirical observations are entirely consistent with the hypothesis if a rather special shape is given to the total utility curve of money (sec. 4). This special shape, which can be given a tolerably satisfactory interpretation (sec. 5), not only brings under the aegis of rational utility maximization much behavior that is ordinarily explained in other terms but also has implications about observable behavior not used in deriving it (sec. 6). Further empirical work should make it possible to determine whether or not these implications conform to reality.

It is a testimony to the strength of the belief in diminishing marginal utility that it has taken so long for the possibility of interpreting gambling and similar phenomena as a contradiction of universal diminishing marginal utility, rather than of utility maximization, to be recognized. The initial mistake must have been at least partly a product of a strong introspective belief in diminishing marginal utility: a dollar must mean less to a rich man than to a poor man; see how

[9] *Ibid.*, pp. 15–31, esp. p. 25.

much more a man will spend when he is rich than when he is poor to avoid any given amount of pain or discomfort.[10] Some of the comments that have been published by competent economists on the utility analysis of von Neumann and Morgenstern are even more remarkable testimony to the hold that diminishing marginal utility has on economists. Vickrey remarks: "There is abundant evidence that individual decisions in situations involving risk are not always made in ways that are compatible with the assumption that the decisions are made rationally with a view to maximizing the mathematical expectation of a utility function. The purchase of tickets in lotteries, sweepstakes, and 'numbers' pools would imply, on such a basis, that the marginal utility of money is an increasing rather than a decreasing function of income. Such a conclusion is obviously unacceptable as a guide to social policy."[11] Kaysen remarks, "Unfortunately, these postulates [underlying the von Neumann and Morgenstern discussion of utility measurement] involve an assumption about economic behavior which is contrary to experience. . . . That this assumption is contradicted by experience can easily be shown by hundreds of examples [including] the participation of individuals in lotteries in which their mathematical expectation of gain (utility) is negative."[12]

[10] This elemental argument seems so clearly to justify diminishing marginal utility that it may be desirable even now to state explicitly how this phenomenon can be rationalized equally well on the assumption of increasing marginal utility of money. It is only necessary to suppose that the avoidance of pain and the other goods that can be bought with money are related goods and that, while the marginal utility of money increases as the amount of money increases, the marginal utility of avoiding pain increases even faster.

[11] William Vickrey, "Measuring Marginal Utility by Reactions to Risk," *Econometrica*, XIII (1945), 319–33. The quotation is from pp. 327 and 328. "The purchase of tickets in lotteries, sweepstakes, and 'numbers' pools" does not imply that marginal utility of money increases with income everywhere (see sec. 4 below). Moreover, it is entirely unnecessary to identify the quantity that individuals are to be interpreted as maximizing with a quantity that should be given special importance in public policy.

2. OBSERVABLE BEHAVIOR TO BE RATIONALIZED

The economic phenomena to which the hypothesis revived by von Neumann and Morgenstern is relevant can be divided into, first, the phenomena ordinarily regarded as gambling and insurance; second, other economic phenomena involving risk. The latter are clearly the more important, and the ultimate significance of the hypothesis will depend primarily on the contribution it makes to an understanding of them. At the same time, the influence of risk is revealed most markedly in gambling and insurance, so that these phenomena have a significance for testing and elaborating the hypothesis out of proportion to their importance in actual economic behavior.

At the outset it should be confessed that we have conducted no extensive empirical investigation of either class of phenomena. For the present, we are content to use what is already available in the literature, or obvious from casual observation, to provide a first test of the hypothesis and to impose significant substantive restrictions on it.

The major economic decisions of an individual in which risk plays an important role concern the employment of the resources he controls: what occupation to follow, what entrepreneurial activity to engage in, how to invest (nonhuman) capital. Alternative possible uses of resources can be classified into three broad

[12] C. Kaysen, "A Revolution in Economic Theory?" *Review of Economic Studies*, XIV, No. 35 (1946–47), 1–15; quotation is from p. 13.

groups according to the degree of risk involved: (*a*) those involving little or no risk about the money return to be received—occupations like schoolteaching, other civil service employment, clerical work; business undertakings of a standard, predictable type like many public utilities; securities like government bonds, high-grade industrial bonds; some real property, particularly owner-occupied housing; (*b*) those involving a moderate degree of risk but unlikely to lead to either extreme gains or extreme losses—occupations like dentistry, accountancy, some kinds of managerial work; business undertakings of fairly standard kinds in which, however, there is sufficient competition to make the outcome fairly uncertain; securities like lower-grade bonds, preferred stocks, higher-grade common stocks; (*c*) those involving much risk, with some possibility of extremely large gains and some of extremely large losses—occupations involving physical risks, like piloting aircraft, automobile racing, or professions like medicine and law; business undertakings in untried fields; securities like highly speculative stocks; some types of real property.

The most significant generalization in the literature about choices among these three uses of resources is that, other things the same, uses *a* or *c* tend in general to be preferred to use *b;* that is, people must in general be paid a premium to induce them to undertake moderate risks instead of subjecting themselves to either small or large risks. Thus Marshall says: "There are many people of a sober steady-going temper, who like to know what is before them, and who would far rather have an appointment which offered a certain income of say £400 a year than one which was not unlikely to yield £600, but had an equal chance of affording only £200. Uncertainty, therefore, which does not appeal to great ambitions and lofty aspirations, has special attractions for very few; while it acts as a deterrent to many of those who are making their choice of a career. And as a rule the certainty of moderate success attracts more than an expectation of an uncertain success that has an equal actuarial value.

"But on the other hand, if an occupation offers a few extremely high prizes, its attractiveness is increased out of all proportion to their aggregate value."[13]

Adam Smith comments similarly about occupational choices and, in addition, says of entrepreneurial undertakings: "The ordinary rate of profits always rises more or less with the risk. It does not, however, seem to rise in proportion to it, or so as to compensate it completely. . . . The presumptuous hope of success seems to act here as upon all other occasions, and to entice so many adventurers into those hazardous trades, that their competition reduces the profit below what is sufficient to compensate the risk."[14]

Edwin Cannan, in discussing the rate of return on investments, concludes that "the probability is that the classes of investments which on the average return most to the investor are neither the very safest of all nor the very riskiest, but the intermediate classes which do not appeal either to timidity or to the gambling instinct."[15]

This asserted preference for extremely safe or extremely risky investments over

[13] *Op. cit.*, pp. 554–55.

[14] *Op. cit.*, p. 111.

[15] Article on "Profit," in *Dictionary of Political Economy*, ed. R. H. Inglis Palgrave (new edition, ed. Henry Higgs; London, 1926); see also the summary of the views of different writers on risk-taking in F. H. Knight, *Risk, Uncertainty, and Profit* (New York, 1921; reprint London School of Economics and Political Science, 1933), pp. 362–67.

investments with an intermediate degree of risk has its direct counterpart in the willingness of persons to buy insurance and also to buy lottery tickets or engage in other forms of gambling involving a small chance of a large gain. The extensive market for highly speculative stocks —the kind of stocks that "blue-sky" laws are intended to control—is a border-line case that could equally well be designated as investment or gambling.

The empirical evidence for the willingness of persons of all income classes to buy insurance is extensive.[16] Since insurance companies have costs of operation that are covered by their premium receipts, the purchaser is obviously paying a larger premium than the average compensation he can expect to receive for the

[16] E.g., see U.S. Bureau of Labor Statistics, *Bulletin 648: Family Expenditures in Selected Cities, 1935–36*, Vol. I: *Family Expenditures for Housing, 1935–36;* Vol. VI: *Family Expenditures for Transportation, 1935–36;* and Vol. VIII: *Changes in Assets and Liabilities, 1935–36.*

Table 6 of the Tabular Summary of Vol. I gives the percentage of home-owning families reporting the payment of premiums for insurance on the house. These percentages are given separately for each income class in each of a number of cities or groups of cities. Since premiums are often paid less frequently than once a year, the percentages given definitely understate the percentage of families carrying insurance. Yet the bulk of the percentages are well over 40.

Table 5 of the Tabular Summary of Vol. VI gives the percentage of families (again by income classes and cities or groups of cities) reporting expenditures for automobile insurance. These figures show a very rapid increase in the percentage of automobile operators that had insurance (this figure is derived by dividing the percentage of families reporting automobile insurance by the percentage of families operating cars) as income increases. In the bottom income classes, where operation of a car is infrequent, only a minority of those who operate cars carry insurance. In the upper income classes, where most families operate cars, the majority of operators carry insurance. A convenient summary of these percentages for selected income classes in six large cities, given in text Table 10 (p. 26), has forty-two entries. These vary from 4 per cent to 98 per cent and twenty-three are over 50 per cent.

Table 3 of the Tabular Summary of Vol. VIII gives the percentage of families in each income class in various cities or groups of cities reporting the payment of life, endowment, or annuity insurance premiums. The percentages are uniformly high. For example, for New York City the percentage of white families reporting the payment of insurance premiums is 75 per cent or higher for every income class listed and varies from 75 per cent in the income class \$500–\$749 to over 95 per cent in the upper-income classes; the percentage of Negro families purchasing insurance was 38 per cent for the \$1,000–\$1,249 class but 60 per cent or higher for every other class. This story is repeated for city after city, the bulk of the entries in the table for the percentage of families purchasing insurance being above 80 per cent.

These figures cannot be regarded as direct estimates of the percentage of families willing to pay something—that is, to accept a smaller actuarial value—in order to escape risk, the technical meaning of the purchase of insurance that is relevant for our purpose. (1) The purchase of automobile and housing insurance may not be a matter of choice. Most owned homes have mortgages (see I, 361, Table L) and the mortgage may require that insurance be carried. The relevant figure for mortgaged homes would be the fraction of owners carrying a larger amount of insurance than is required by the mortgage. Similarly, finance companies generally require that insurance be carried on automobiles purchased on the instalment plan and not fully paid for, and the purchase of automobile insurance is compulsory in some states. (2) For automobile property damage and liability insurance (but not collision insurance) the risks to the operator and to the insurance company may not be the same, particularly to persons in the lower-income classes. The loss to the uninsured operator is limited by his wealth and borrowing power, and the maximum amount that he can lose may be well below the face value of the policy that he would purchase. The excess of the premium over the expected loss is thus greater for him than for a person with more wealth or borrowing power. The rise in the percentage of persons carrying automobile insurance as income rises may therefore reflect not an increased willingness to carry insurance but a reduction in the effective price that must be paid for insurance. (3) This tendency may be reversed for the relatively high-income classes for both automobile and housing insurance by the operation of the income tax. Uninsured losses are in many instances deductible from income before computation of income tax under the United States federal income tax, while insurance premiums are not. This tends to make the net expected loss less for the individual than for the insurance company. This effect is almost certainly negligible for the figures cited above, both because they do not effectively cover very high incomes and because the federal income tax was relatively low in 1935–36. (4) Life insurance at times comes closer to gambling (the losses against which he carries insurance. That is, he is paying something to escape risk.

The empirical evidence for the willingness of individuals to purchase lottery tickets, or engage in similar forms of gambling, is also extensive. Many governments find, and more governments have found, lotteries an effective means of raising revenue.[17] Though illegal, the "numbers" game and similar forms of gambling are reported to flourish in the United States,[18] particularly among the lower income classes.

It seems highly unlikely that there is a sharp dichotomy between the individuals who purchase insurance and those who gamble. It seems much more likely that many do both or, at any rate, would be willing to. We can cite no direct evidence for this asserted fact, though indirect evidence and casual observation give us considerable confidence that it is correct. Its validity is suggested by the extensiveness of both gambling and the purchase of insurance. It is also suggested by some of the available evidence on how people invest their funds. The widespread legislation against "bucket shops" suggests that relatively poor people must have been willing to buy extremely speculative stocks of a "blue-sky" variety. Yet the bulk of the property income of the lower-income classes consists of interest and rents and relatively little of dividends, whereas the reverse is true for the upper-income classes.[19] Rents and interest are types of receipts that tend to be derived from investments with relatively little risk, and so correspond to the purchase of insurance, whereas investment in speculative stocks corresponds to the purchase of lottery tickets.

Offhand it appears inconsistent for the same person both to buy insurance and to gamble: he is willing to pay a premium, in the one case, to avoid risk, in the other, to bear risk. And indeed it would be inconsistent for a person to be

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choice of an uncertain alternative in preference to a certain alternative with a higher expected value) than to the payment of a premium to escape risk. For example, special life-insurance policies purchased to cover a single railroad or airplane trip are probably more nearly comparable to a lottery ticket than a means of achieving certainty. (5) Even aside from these qualifications, actual purchase of insurance would give at best a lower limit to the number willing to buy insurance, since there will always be some who will regard the price asked as too high.

These qualifications offset one another to some extent. It seems highly unlikely that their net effect could be sufficient to reverse the conclusion suggested by the evidence cited that a large fraction of people in all income classes are willing to buy insurance.

[17] France, Spain, and Mexico, to name but three examples, currently conduct lotteries for revenue. Russia attaches a lottery feature to bonds sold to the public. Great Britain conducted lotteries from 1694 to 1826. In the United States lotteries were used extensively before the Revolution and for some time thereafter, both directly by state governments and under state charters granted to further specific projects deemed to have a state interest. For the history of lotteries in Great Britain see C. L'Estrange Ewen, *Lotteries and Sweepstakes* (London, 1932); in New York State, A. F. Ross, "History of Lotteries in New York," *Magazine of History*, Vol. V (New York, 1907). There seem to be no direct estimates of the fraction of the people who purchase tickets in state or other legal lotteries, and it is clear that such figures would be difficult to get from data obtained in connection with running the lotteries. The receipts from legal lotteries, and casual impressions of observers, suggest that a substantial fraction of the relevant units (families or, alternatively, individual income recipients) purchase tickets.

[18] Evidence from wagering on horse races, where this has been legalized, is too ambiguous to be of much value. Since most legal wagering is at the track, gambling is available only to those who go to watch the races and is combined with participation in the mechanics of the game of chance.

[19] *Delaware Income Statistics*, Vol. I (Bureau of Economic and Business Research, University of Delaware, 1941, Table 1; *Minnesota Incomes, 1938–39*, Vol. II (Minnesota Resources Commission, 1942), Table 27; F. A. Hanna, J. A. Pechman, S. M. Lerner, *Analysis of Wisconsin Income* ("Studies in Income and Wealth," Vol. IX [National Bureau of Economic Research, 1948]), Part II, Table 1.

willing to pay something (no matter how little) in excess of actuarial value to avoid every possible risk and also something in excess of actuarial value to assume every possible risk. One must distinguish among different kinds of insurance and different kinds of gambling, since a willingness to pay something for only some kinds of insurance would not necessarily be inconsistent with a willingness to engage in only some kinds of gambling. Unfortunately, very little empirical evidence is readily available on the kinds of insurance that people are willing to buy and the kinds of gambling that they are willing to engage in. About the only clear indication is that people are willing to enter into gambles that offer a small chance of a large gain—as in lotteries and "blue-sky" securities.

Lotteries seem to be an extremely fruitful, and much neglected, source of information about reactions of individuals to risk. They present risk in relatively pure form, with little admixture of other factors; they have been conducted in many countries and for many centuries, so that a great deal of evidence is available about them; there has been extensive experimentation with the terms and conditions that would make them attractive, and much competition in conducting them, so that any regularities they may show would have to be interpreted as reflecting corresponding regularities in human behavior.[20] It is, of course, not certain that inferences from lotteries would carry over to other choices involving risk. There would, however, seem to be some presumption that they would do so, though of course the validity of this presumption would have to be tested.[21]

The one general feature of lotteries that is worth noting in this preliminary survey, in addition to the general willingness of people to participate in them, is the structure of prizes that seems to have developed. Lotteries rarely have just a single prize equal to the total sum to be paid out as prizes. Instead, they tend to have several or many prizes. The largest prize is ordinarily not very much larger than the next largest, and often there is not one largest prize but several of the same size.[22] This tendency is so general that one would expect it to reflect some consistent feature of individual reactions, and any hypothesis designed to explain reactions to uncertainty should explain it.

3. THE FORMAL HYPOTHESIS

The hypothesis that is proposed for rationalizing the behavior just summarized can be stated compactly as follows: In choosing among alternatives open to it, whether or not these alternatives involve risk, a consumer unit (generally a family, sometimes an individual) behaves as if (*a*) it had a consistent set of preferences; (*b*) these preferences could be completely described by a function attaching a numerical value—to be designated "utility"—to alternatives each of which is regarded as certain; (*c*) its objective were to make its expected util-

[20] Aside from their value in providing information about reactions to risk, data from lotteries may be of broader interest in providing evidence about the stability of tastes and preferences over time and their similarity in different parts of the world. Here is a "commodity" which has remained unchanged over centuries, which is the same all over the globe, and which has been dealt in widely for the entire period and over much of the globe. It is hard to conceive of any other commodity for which this is true.

[21] See Smith, *op. cit.*, p. 108, for a precedent.

[22] See Ewen, *op. cit.*, *passim*, but esp. descriptions of state lotteries in chap. vii, pp. 199–244; see also the large numbers of bills advertising lotteries in John Ashton, *A History of English Lotteries* (London: Leadenhall Press, 1893).

ity as large as possible. It is the contribution of von Neumann and Morgenstern to have shown that an alternative statement of the same hypothesis is: An individual chooses in accordance with a system of preferences which has the following properties:

1. The system is complete and consistent; that is, an individual can tell which of two objects he prefers or whether he is indifferent between them, and if he does not prefer C to B and does not prefer B to A, then he does not prefer C to A.[23] (In this context, the word "object" includes combinations of objects with stated probabilities; for example, if A and B are objects, a 40-60 chance of A or B is also an object.)
2. Any object which is a combination of other objects with stated probabilities is never preferred to every one of these other objects, nor is every one of them ever preferred to the combination.
3. If the object A is preferred to the object B and B to the object C, there will be some probability combination of A and C such that the individual is indifferent between it and B.[24]

This form of statement is designed to show that there is little difference between the plausibility of this hypothesis and the usual indifference-curve explanation of riskless choices.

These statements of the hypothesis conceal by their very compactness most of its implications. It will pay us, therefore, to elaborate them. It simplifies matters, and involves no loss in generality, to regard the alternatives open to the consumer unit as capable of being expressed entirely in terms of money or money income. Actual alternatives are not, of course, capable of being so expressed: the same money income may be valued very differently according to the terms under which it is to be received, the nonpecuniary advantages or disadvantages associated with it, and so on. We can abstract from these factors, which play no role in the present problem, by supposing either that they are the same for different incomes compared or that they can be converted into equivalent sums of money income.[25] This permits us to consider total utility a function of money income alone.

Let I represent the income of a consumer unit per unit time, and $U(I)$ the utility attached to that income if it is regarded as certain. Measure I along the horizontal axis of a graph and U along the vertical. In general, $U(I)$ will not be defined for all values of I, since there will be a lower limit to the income a consumer unit can receive, namely, a negative in-

[23] The transitivity of the relation of indifference assumed in this postulate is, of course, an idealization. It is clearly possible that the difference between successive pairs of alternatives in a series might be imperceptible to an individual, yet the first of the series definitely preferable to the last. This idealization, which is but a special case of the idealization involved in the geometric concept of a dimensionless point, seems to us unobjectionable. However, the use of this idealization in indifference-curve analysis is the major criticism offered by W. E. Armstrong in an attack on indifference-curve analysis in his article "The Determinateness of the Utility Function," *Economic Journal*, XLIX (September, 1939), 453–67. In a more recent article ("Uncertainty and the Utility Function," *Economic Journal*, LVIII [March, 1948], 1–10) Armstrong repeats this criticism and adds to it the criticism that choices involving risk cannot be rationalized by the ordinal properties of utility functions.

[24] For a rigorous presentation of the second statement and a rigorous proof that the statements are equivalent see von Neumann and Morgenstern, *op. cit.*, pp. 26–27, 617–32.

[25] The other factors abstracted from must not, of course, include any that cannot in fact be held constant while money income varies. For example, a higher income is desired because it enables a consumer unit to purchase a wider variety of commodities. The consumption pattern of the consumer unit must not therefore be supposed to be the same at different incomes. As another example, a higher income may mean that a consumer unit must pay a higher price for a particular commodity (e.g., medical service). Such variation in price should not be impounded in *ceteris paribus*, though price changes not necessarily associated with changes in the consumer unit's income should be.

come equal (in absolute value) to the maximum amount that the consumer unit can lose per unit time for the period to which the utility curve refers.

Alternatives open to the consumer unit that involve no risk consist of possible incomes, say I', I'', ... The hypothesis then implies simply that the consumer unit will choose the income to which it attaches the most utility. Other things the same, we know from even casual observation that the consumer unit will in general choose the largest income: put differently, we consider it pathological for an individual literally to throw money away, yet this means of choosing a smaller income is always available. It follows that the hypothesis can rationalize riskless choices of the limited kind considered here if, and only if, the utility of money income is larger, the higher the income. Consideration of riskless choices imposes no further requirements on the utility function.

Alternatives involving risk consist of probability distributions of possible incomes. Fortunately, it will suffice for our purpose to consider only a particularly simple kind of alternative involving risk, namely (A) a chance $a(0 < a < 1)$ of an income I_1, and a chance $(1 - a)$ of an income I_2, where for simplicity I_2 is supposed always greater than I_1. This simplification is possible because, as we shall see later, the original hypothesis implies that choices of consumer units among more complicated alternatives can be predicted from complete knowledge of their preferences among alternatives like A and a riskless alternative (B) consisting of a certain income I_0.

Since "other things" are supposed the same for alternatives A and B, the utility of the two alternatives may be taken to be functions solely of the incomes and probabilities involved and not also of attendant circumstances. The utility of alternative B is $U(I_0)$. The expected utility of A is given by

$$\bar{U}(A) = aU(I_1) + (1 - a)U(I_2).$$

According to the hypothesis, a consumer unit will choose A if $\bar{U} > U(I_0)$, will choose B if $\bar{U} < U(I_0)$, and will be indifferent between A and B if $\bar{U} = U(I_0)$.

Let $\bar{I}(A)$ be the actuarial value of A, i.e., $\bar{I}(A) = aI_1 + (1 - a)I_2$. If I_0 is equal to $\bar{I}$, the "gamble" or "insurance" is said to be "fair" since the consumer unit gets the same actuarial value whichever alternative it chooses. If, under these circumstances, the consumer unit chooses A, it shows a preference for this risk. This is to be interpreted as meaning that $\bar{U} > U(\bar{I})$ and indeed $\bar{U} - U(\bar{I})$ may be taken to measure the utility it attaches to this particular risk.[26] If the consumer unit chooses B, it shows a preference for certainty. This is to be interpreted as meaning that $\bar{U} < U(\bar{I})$. Indifference between A and B is to be interpreted as meaning that $\bar{U} = U(\bar{I})$.

Let I^* be the certain income that has the same utility as A, that is, $U(I^*) = \bar{U}$.[27] Call I^* the income equivalent to A. The requirement, derived from consider-

[26] This interpretation of $\bar{U} - U(\bar{I})$ as the utility attached to a particular risk is directly relevant to a point to which von Neumann and Morgenstern and commentators on their work have given a good deal of attention, namely, whether there may "not exist in an individual a (positive or negative) utility of the mere act of 'taking a chance,' of gambling, which the use of the mathematical expectation obliterates" (von Neumann and Morgenstern, *op. cit.*, p. 28). In our view the hypothesis is better interpreted as a rather special explanation why gambling has utility or disutility to a consumer unit, and as providing a particular measure of the utility or disutility, than as a denial that gambling has utility (see *ibid.*, pp. 28, 629–32).

[27] Since U has been assumed strictly monotonic to rationalize riskless choices, there will be only one income, if any, that has the same utility as A. There will be one if U is continuous which, for simplicity, we assume to be the case throughout this paper.

ation of riskless choices, that utility increase with income means that

$$\bar{U} \gtreqless U(\bar{I})$$

implies

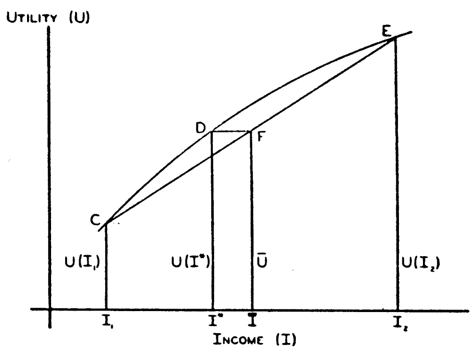
$$I^* \gtreqless \bar{I} \,.$$

If I^* is greater than $\bar{I}$, the consumer unit prefers this particular risk to a certain income of the same actuarial value and would be willing to pay a maximum of $I^* - \bar{I}$ for the privilege of "gambling." If I^* is less than $\bar{I}$, the consumer unit prefers certainty and is willing to pay a maximum of $\bar{I} - I^*$ for "insurance" against this risk.

These concepts are illustrated for a consumer unit who is willing to pay for insurance ($\bar{I} > I^*$) in Figure 1, *a*, and for a consumer unit who is willing to pay for the privilege of gambling ($\bar{I} < I^*$) in Figure 1, *b*. In both figures, money income is measured along the horizontal axis, and utility along the vertical. On the horizontal axis, designate I_1 and I_2. $\bar{I}$, the actuarial value of I_1 and I_2, is then represented by a point that divides the interval I_1 to I_2 in the proportion

$$\frac{1-a}{a}\left(\text{i.e., } \frac{\bar{I}-I_1}{I_2-\bar{I}}=\frac{1-a}{a}\right).$$

Draw the utility curve (CDE in both figures). Connect the points $(I_1, U[I_1])$, $(I_2, U[I_2])$ by a straight line (CFE). The vertical distance of this line from the horizontal axis at $\bar{I}$ is then equal to $\bar{U}$. (Since $\bar{I}$ divides the distance between I_1 and I_2 in the proportion $[1-a]/a$, F divides the vertical distance between C and E in the same proportion, so the vertical distance from F to the horizontal axis is the expected value of $U[I_1]$ and $U[I_2]$). Draw a horizontal line through F and find the income corresponding to its intersection with the utility curve (point D). This is the income the utility of which is the same as the expected utility of A, hence by definition is I^*.



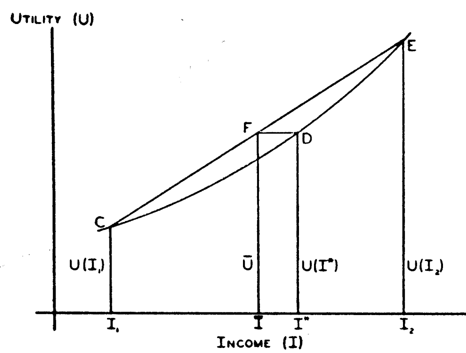


FIG. 1.—Illustration of utility analysis of choices involving risk: *a*, preference for certainty; *b*, preference for risk.

In Figure 1, *a*, the utility curve is so drawn as to make I^* less than $\bar{I}$. If the consumer unit is offered a choice between A and a certain income I_0 greater than I^*, it will choose the certain income. If this certain income I_0 were less than $\bar{I}$, the consumer unit would be paying $\bar{I} - I_0$ for certainty—in ordinary parlance it would be "buying insurance"; if the certain income were greater than $\bar{I}$, it would be being paid $I_0 - \bar{I}$ for accepting certainty, even though it is willing to pay for certainty—we might say that it is "selling a gamble" rather than "buying insurance." If the consumer unit were offered a choice between A and a certain income I_0 less than I^*, it would choose A because, while it is willing to pay a price for certainty, it is being asked to pay more than the maximum amount ($\bar{I} - I^*$) that it is willing to pay. The price of insurance has become so high that it has, as it were, been converted into a seller rather than a buyer of insurance.

In Figure 1, b, the utility curve is so drawn as to make I^* greater than $\bar{I}$. If the consumer unit is offered a choice between A and a certain income I_0 less than I^*, it will choose A. If this certain income I_0 were greater than $\bar{I}$, the consumer unit would be paying $I_0 - \bar{I}$ for this risk—in ordinary parlance, it would be choosing to gamble or, one might say, "to buy a gamble"; if the certain income were less than $\bar{I}$, it would be being paid $\bar{I} - I_0$ for accepting this risk even though it is willing to pay for the risk—we might say that it is "selling insurance" rather than "buying a gamble." If the consumer unit is offered a choice between A and a certain income I_0 greater than I^*, it will choose the certain income because, while it is willing to pay something for a gamble, it is not willing to pay more than $I^* - \bar{I}$. The price of the gamble has become so high that it is converted into a seller, rather than a buyer, of gambles.

It is clear that the graphical condition for a consumer unit to be willing to pay something for certainty is that the utility function be above its chord at $\bar{I}$. This is simply a direct translation of the condition that $U(\bar{I}) > \bar{U}$. Similarly, a consumer unit will be willing to pay something for a risk if the utility function is below its chord at $\bar{I}$.

The relationship between these formalized "insurance" and "gambling" situations and what are ordinarily called insurance and gambling is fairly straightforward. A consumer unit contemplating buying insurance is to be regarded as having a current income of I_2 and as being subject to a chance of losing a sum equal to $I_2 - I_1$, so that if this loss should occur its income would be reduced to I_1. It can insure against this loss by paying a premium equal to $I_2 - I_0$. The premium, in general, will be larger than $I_2 - \bar{I}$, the "loading" being equal to $\bar{I} - I_0$. Purchase of insurance therefore means accepting the certainty of an income equal to I_0 instead of a pair of alternative incomes having a higher expected value. Similarly, a consumer unit deciding whether to gamble (e.g., to purchase a lottery ticket) can be interpreted as having a current income equal to I_0. It can have a chance $(1 - a)$ of a gain equal to $I_2 - I_0$ by subjecting itself to a chance a of losing a sum equal to $I_0 - I_1$. If it gambles, the actuarial value of its income is $\bar{I}$, which in general is less than I_0. $I_0 - \bar{I}$ is the premium it is paying for the chance to gamble (the "take" of the house, or the "banker's cut").

It should be emphasized that this analysis is all an elaboration of a particular hypothesis about the way consumer units choose among alternatives involving risk. This hypothesis describes the reactions of consumer units in terms of a utility function, unique except for origin and unit of measure, which gives the utility assigned to certain incomes and which has so far been taken for granted. Yet for choices among certain incomes only a trivial characteristic of this function is relevant, namely, that it rises with income. The remaining characteristics of the function are relevant only to choices among alternatives involving risk and can therefore be inferred only from observation of such choices. The precise manner in which these characteristics are implicit in the consumer unit's preferences among alternatives involving risk can be indicated most easily by describing a conceptual experiment for determining the utility function.

Select any two incomes, say \$500 and \$1,000. Assign any arbitrary utilities to these incomes, say 0 utiles and 1 utile, respectively. This corresponds to an arbitrary choice of origin and unit of measure. Select any intermediate income, say \$600. Offer the consumer unit the choice between (A) a chance a of \$500 and $(1 - a)$ of \$1,000 or (B) a certainty of \$600, varying a until the consumer unit is indifferent between the two (i.e., until $I^* =$ \$600). Suppose this indifference value of a is $\frac{2}{5}$. If the hypothesis is correct, it follows that

$$U(600) = \tfrac{2}{5}U(500) + \tfrac{3}{5}U(1000)$$
$$= \tfrac{2}{5}\ 0 + \tfrac{3}{5}\cdot 1 = \tfrac{3}{5} = .60\ .$$

In this way the utility attached to every income between \$500 and \$1,000 can be determined. To get the utility attached to any income outside the interval \$500 to \$1,000, say \$10,000, offer the consumer unit a choice between (A) a chance a of \$500 and $(1 - a)$ of \$10,000 or (B) a certainty of \$1,000, varying a until the consumer unit is indifferent between the two (i.e., until $I^* =$ \$1,000). Suppose this indifference value of a is $\frac{4}{5}$. If the hypothesis is correct, it follows that

$$\tfrac{4}{5}U(500) + \tfrac{1}{5}U(10{,}000) = U(1000)\ ,$$

or

$$\tfrac{4}{5}\cdot 0 + \tfrac{1}{5}U(10{,}000) = 1\ ,$$

or

$$U(10{,}000) = 5\ .$$

In principle, the possibility of carrying out this experiment, and the reproducibility of the results, would provide a test of the hypothesis. For example, the consistency of behavior assumed by the hypothesis would be contradicted if a repetition of the experiment using two initial incomes other than \$500 and \$1,000 yielded a utility function differing in more than origin and unit of measure from the one initially obtained.

Given a utility function obtained in this way, it is possible, if the hypothesis is correct, to compute the utility attached to (that is, the expected utility of) any set or sets of possible incomes and associated probabilities and thereby to predict which of a number of such sets will be chosen. This is the precise meaning of the statement made toward the beginning of this section that, if the hypothesis were correct, complete knowledge of the preferences of consumer units among alternatives like A and B would make it possible to predict their reactions to any other choices involving risk.

The choices a consumer unit makes that involve risk are typically far more complicated than the simple choice between A and B that we have used to elaborate the hypothesis. There are two chief sources of complication: Any particular alternative typically offers an indefinitely large number of possible incomes, and "other things" are generally not the same.

The multiplicity of possible incomes is very general: losses insured against ordinarily have more than one possible value; lotteries ordinarily have more than one prize; the possible income from a particular occupation, investment, or business enterprise may be equal to any of an indefinitely large number of values. A hypothesis that the essence of choices among the degrees of risk involved in such complex alternatives is contained in such simple choices as the choice between A and B is by no means tautological.

The hypothesis does not, of course, pretend to say anything about how consumer choices will be affected by differences in things other than degree of risk. The significance for our purposes of such differences is rather that they greatly increase the difficulty of getting evidence about reactions to differences in risk alone. Much casual experience, particularly experience bearing on what is ordinarily regarded as gambling, is likely to be misinterpreted, and erroneously regarded as contradictory to the hypothesis, if this difficulty is not explicitly recognized. In much so-called gambling the individual chooses not only to bear risk but also to participate in the mechanics of a game of chance; he buys, that is, a gamble, in our technical sense, and entertainment. We can conceive of separating these two commodities: he could buy entertainment alone by paying admission to participate in a game using valueless chips; he could buy the gamble alone by having an agent play the game of chance for him according to detailed instructions.[28] Further, insurance and gambles are often purchased in almost pure form. This is notably true of insurance. It is true also of gambling by the purchase of lottery tickets when the purchaser is not a spectator to the drawing of the winners (e.g., Irish sweepstakes tickets bought in this country or the "numbers" game), and of much stock-market speculation.

An example of behavior that would definitely contradict the assertion, contained in the hypothesis, that the same utility function can be used to explain choices that do and do not involve risk would be willingness by an indiviual to pay more for a gamble than the maximum amount he could win. In order to explain riskless choices it is necessary to suppose that utility increases with income. It follows that the average utility of two incomes can never exceed the utility of the larger income and hence that an individual will never be willing to pay, for example, a dollar for a chance of winning, at most, 99 cents.

More subtle observation would be required to contradict the assertion that the reactions of persons to complicated gambles can be inferred from their reactions to simple gambles. For example, suppose an individual refuses an opportunity to toss a coin for a dollar and also to toss a coin for two dollars but then accepts an opportunity to toss two coins in succession, the first to determine whether the second toss is to be for one dollar or for two dollars. This behavior would definitely contradict the hypothesis. On the hypothesis, the utility of the third gamble is an average of the utility of the first two. His refusal of the first two indicates that each of them has a lower utility than the alternative of not gambling; hence, if the hypothesis were correct, the third should have a lower utility than the same alternative, and he should refuse it.

4. RESTRICTIONS ON UTILITY FUNCTION REQUIRED TO RATIONALIZE OBSERVABLE BEHAVIOR

The one restriction imposed on the utility function in the preceding section is that total utility increase with the size of money income. This restriction was imposed to rationalize the first of the facts listed below. We are now ready to see whether the behavior described in section 2 can be nartioalized by the hy-

[28] It does not, of course, follow that the price an individual is willing to pay for the joint commodity is simply the sum of the prices he is willing to pay for them separately. Indeed, it may well be the possible existence of such a difference that people have in mind when they speak of a "specific utility of gambling."

pothesis, and, if so, what additional restrictions this behavior imposes on the utility function. To simplify the task, we shall take as a summary of the essential features of the behavior described in section 2 the following five statements, alleged to be facts: (1) consumer units prefer larger to smaller certain incomes; (2) low-income consumer units buy, or are willing to buy, insurance; (3) low-income consumer units buy, or are willing to buy, lottery tickets; (4) many low-income consumer units buy, or are willing to buy, both insurance and lottery tickets; (5) lotteries typically have more than one prize.

These particular statements are selected not because they are the most important in and of themselves but because they are convenient to handle and the restrictions imposed to rationalize them turn out to be sufficient to rationalize all the behavior described in section 2.

It is obvious from Figure 1 and our discussion of it that if the utility function were everywhere convex from above (for utility functions with a continuous derivative, if the marginal utility of money does not increase for any income), the consumer unit, on our hypothesis, would be willing to enter into any fair insurance plan but would be unwilling to pay anything in excess of the actuarial value for any gamble. If the utility function were everywhere concave from above (for functions with a continuous derivative, if the marginal utility of money does not diminish for any income), the consumer unit would be willing to enter into any fair gamble but would be unwilling to pay anything in excess of the actuarial value for insurance against any risk.

It follows that our hypothesis can rationalize statement 2, the purchase of insurance by low-income consumer units, only if the utility functions of the corresponding units are not everywhere concave from above; that it can rationalize statement 3, the purchase of lottery tickets by low-income consumer units, only if the utility functions of the corresponding units are not everywhere convex from above; and that it can rationalize statement 4, the purchase of both insurance and lottery tickets by low-income consumer units, only if the utility functions of the corresponding units are neither everywhere concave from above nor everywhere convex from above.

The simplest utility function (with a continuous derivative) that can rationalize all three statements simultaneously is one that has a segment convex from above followed by a segment concave from above and no other segments.[29] The convex segment must precede the concave segment because of the kind of insurance and of gambling the low-income consumer units are said to engage in: a chord from the existing income to a lower income must be below the utility function to rationalize the purchase of insurance against the risk of loss; a chord from the immediate neighborhood of the existing income to a higher income must be above the utility function at the existing income to rationalize the purchase for a small sum of a small chance of a large gain.[30]

Figure 2 illustrates a utility function satisfying these requirements. Let this utility function be for a low-income con-

[29] A kink or a jump in the utility function could rationalize either the gambling or the insurance. For example, the utility function could be composed of two convex or two concave segments joined in a kink. There is no essential loss in generality in neglecting such cases, as we shall do from here on, since one can always think of rounding the kink ever so slightly.

[30] If there are more than two segments and a continuous derivative, a convex segment necessarily precedes a concave segment.

sumer unit whose current income is in the initial convex segment, say at the point designated I^*. If some risk should arise of incurring a loss, the consumer unit would clearly (on our hypothesis) be willing to insure against the loss (if it did not have to pay too much "loading") since a chord from the utility curve at I^* to the utility curve at the lower income that would be the consequence of the actual occurrence of the loss would everywhere be below the utility function. The consumer unit would not be willing to engage in small gambling. But suppose it is offered a fair gamble of the kind represented by a lottery involving a small chance of winning a relatively large sum equal to $I_2 - I^*$ and a large chance of losing a relatively small sum equal to $I^* - I_1$. The consumer unit would clearly prefer the gamble, since the expected utility (I^*G) is greater than the utility of I^*. Indeed it would be willing to pay any premium up to $I^* - \bar{I}$ for the privilege of gambling; that is, even if the expected value of the gamble were almost as low as $\bar{I}$, it would accept the gamble in preference to a certainty of receiving I^*. The utility curve in Figure 2 is therefore clearly consistent with statements 2, 3, and 4.

These statements refer solely to the behavior of relatively low-income consumer units. It is tempting to seek to restrict further the shape of the utility function, and to test the restrictions so far imposed, by appealing to casual observation of the behavior of relatively high-income consumer units.[31] It does not seem desirable to do so, however, for two major reasons: (1) it is far more difficult to accumulate reliable information about the behavior of relatively high-income consumer units than about the behavior of the more numerous low-income units; (2) perhaps even more important, the progressive income tax so affects the terms under which the relatively high-income consumer units purchase insurance or gamble as to make

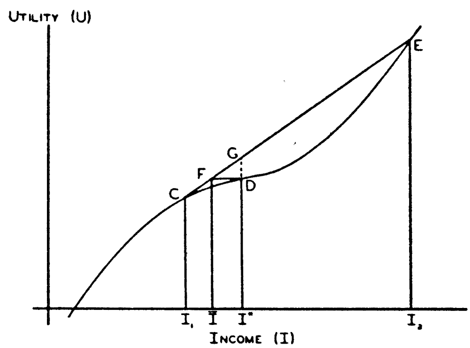


FIG. 2.—Illustration of utility function consistent with willingness of a low-income consumer unit both to purchase insurance and to gamble.

evidence on their behavior hard to interpret for our purposes.[32] Therefore, in-

[31] For example, a high-income consumer unit that had a utility function like that in Fig. 2 and a current income of I_2 would be willing to participate in a wide variety of gambling, including the purchase of lottery tickets; it would be unwilling to insure against losses that had a small expected value (i.e., involved payment of a small premium) though it might be willing to insure against losses that had a large expected value. Consequently, unwillingness of relatively high-income consumer units to purchase lottery tickets, or willingness to purchase low-premium insurance, would contradict the utility function of Fig. 2 and require the imposition of further restrictions.

[32] The effect of the income tax, already referred to in n. 16 above, depends greatly on the specific provisions of the tax law and of the insurance or gambling plan. For example, if an uninsured loss is deductible in computing taxable income (as is loss of an owned home by fire under the federal income tax) while the premium for insuring against the loss is not (as a fire-insurance premium on an owned home is not), the expected value of the loss is less to the consumer unit than to the firm selling insurance. A premium equal to the actuarial value of the loss to the insurance company then exceeds the actuarial value of the loss to the consumer unit. That is, the government in effect pays part of the loss but none of the premium. On the other hand, if the premium is deductible (as a health-insurance premium may be), while an uninsured loss is not (as the excess of medical bills over $2,500 for a family is not), the net

stead of using observations about the behavior of relatively high-income consumer units, we shall seek to learn more about the upper end of the curve by using statement 5, the tendency for lotteries to have more than one prize.

In order to determine the implications of this statement for the utility function, we must investigate briefly the economics of lotteries. Consider an entrepreneur conducting a lottery and seeking to maximize his income from it. For simplicity, suppose that he conducts the lottery by deciding in advance the number of tickets to offer and then auctioning them off at the highest price he can get.[33] Aside from advertising and the like, the variables at his disposal are the terms of the lottery: the number of tickets to sell, the total amount to offer as prizes (which together, of course, determine the actuarial value of a ticket), and the structure of prizes to offer. For any given values of the first two, the optimum structure of prizes is clearly that which maximizes the price he can get per ticket or, what is the same thing, the excess of the price of a ticket over its actuarial value—the "loading" per ticket.

In the discussion of Figure 2, it was noted that $I^* - \bar{I}$ was the maximum amount in excess of the actuarial value that the corresponding consumer unit would pay for a gamble involving a chance $(1 - a)$ of winning $I_2 - I^*$ and a chance a of losing $I^* - I_1$. This gamble is equivalent to a lottery offering a chance $(1 - a)$ of a prize $I_2 - I_1$ in return for the purchase of a ticket at a price of $I^* - I_1$, the chance of winning the prize being such that $\bar{I} - I_1$ is the actuarial worth of a ticket (i.e., is equal to $[1 - a] \times [I_2 - I_1]$). If the consumer unit won the prize, its net winnings would be $I_2 - I^*$, since it would have to subtract the cost of the ticket from the gross prize. The problem of the entrepreneur, then, is to choose the structure of prizes that will maximize $I^* - \bar{I}$ for a given actuarial value of a ticket, that is, for a given value of $\bar{I} - I_1$. Changes in the structure of prizes involve changes in $I_2 - I_1$. If there is a single prize, $I_2 - I_1$ is equal to the total amount to be distributed ($[1 - a]$ is equal to the reciprocal of the number of tickets). If there are two equal prizes, $I_2 - I_1$ is cut in half ($[1 - a]$ is then equal to twice the reciprocal of the number of tickets). Suppose Figure 2 referred to this latter situation in which there were two equal prizes, I^* on the diagram designating both the current income of the consumer unit and the income equivalent to the lottery. If the price and actuarial worth of the ticket were kept unchanged, but a single prize was substituted for the two prizes (and $[1 - a]$ correspondingly reduced), the gamble would clearly become more at-

premium to the consumer unit is less than the premium received by the insurance company. Similarly, gambling gains in excess of gambling losses are taxable under the federal income tax, while gambling losses in excess of gambling gains are not deductible. The special treatment of capital gains and losses under the existing United States federal income tax adds still further complications.

Even if both the premium and the uninsured loss are deductible, or a gain taxable and the corresponding loss deductible, the income tax may change the terms because of the progressive rates. The tax-saving from a large loss may be a smaller fraction of the loss than the tax payable on the gain is of the gain.

These comments clearly apply not only to insurance and gambling proper but also to other economic decisions involving risk—the purchase of securities, choice of occupation or business, etc. The neglect of these considerations has frequently led to the erroneous belief that a progressive income tax does not affect the allocation of resources and is in this way fundamentally different from excise taxes.

[33] This was, in fact, the way in which the British government conducted many of its official lotteries. It frequently auctioned off the tickets to lottery dealers, who served as the means of distributing the tickets to the public (see Ewen, *op. cit.*, pp. 234–40).

tractive to the consumer unit. I_2 would move to the right, the chord connecting $U(I_1)$ and $U(I_2)$ would rotate upward, $\bar{U}$ would increase, and the consumer unit would be paying less than the maximum amount it was willing to pay. The price of the ticket could accordingly be increased; that is, I_2, $\bar{I}$, and I_1 could be moved to the left until the I^* for the new gamble were equal to the consumer unit's current income (the I^* for the old gamble). The optimum structure of prizes clearly consists therefore of a single prize, since this makes $I_2 - I_1$ as large as possible.

Statement 5, that lotteries typically have more than one prize, is therefore inconsistent with the utility function of Figure 2. This additional fact can be rationalized by terminating the utility curve with a suitable convex segment. This yields a utility curve like that drawn in Figure 3. With such a utility curve, $I^* - \bar{I}$ would be a maximum at the point at which a chord from $U(I_1)$ was tangent to the utility curve, and a larger prize would yield a smaller value of $I^* - \bar{I}$.[34]

A utility curve like that drawn in Figure 3 is the simplest one consistent with the five statements listed at the outset of this section.

5. A DIGRESSION

It seems well to digress at this point to consider two questions that, while not strictly relevant to our main theme, are likely to occur to many readers: first, is not the hypothesis patently unrealistic; second, can any plausible interpretation be given to the rather peculiar utility function of Figure 3?

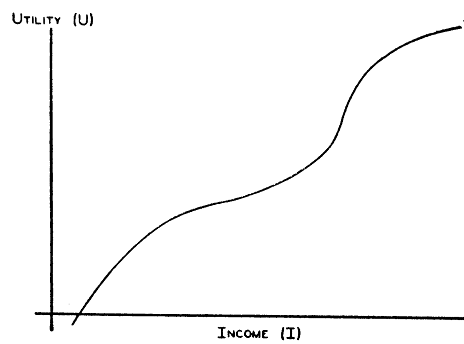


FIG. 3.—Illustration of typical shape of utility curve.

a) THE DESCRIPTIVE "REALISM" OF THE HYPOTHESIS

An objection to the hypothesis just presented that is likely to be raised by many, if not most, readers is that it conflicts with the way human beings actually behave and choose. Is it not patently unrealistic to suppose that individuals consult a wiggly utility curve before gambling or buying insurance, that they know the odds involved in the gambles or insurance plans open to them, that they can compute the expected utility of a gamble or insurance plan, and that they base their decision on the size of the expected utility?

While entirely natural and understandable, this objection is not strictly

[34] An additional convex segment guarantees that there will always exist current incomes of the consumer unit for which (*a*) attractive gambles exist and (*b*) the optimum prize for attractive gambles has a maximum. It does not guarantee that *b* will be true for every income for which attractive gambles exist. The condition on the current income that attractive gambles exist is that the tangent to the utility curve at the current income be below the utility curve for some income (this argument, like many in later technical footnotes, holds not only for the utility function of Fig. 3 but for any differentiable utility function). A single prize will be the optimum, no matter what the amount distributed in prizes or the fixed actuarial worth of the prize if, and only if, every chord from the utility curve at the current income to the utility of a higher income is everywhere above the utility curve. A particular, and somewhat interesting, class of utility functions for which *b* will be true for every income for which *a* is true is the class for which utility approaches a finite limit as income increases.

relevant. The hypothesis does not assert that individuals explicitly or consciously calculate and compare expected utilities. Indeed, it is not at all clear what such an assertion would mean or how it could be tested. The hypothesis asserts rather that, in making a particular class of decisions, individuals behave *as if* they calculated and compared expected utility and *as if* they knew the odds. The validity of this assertion does not depend on whether individuals know the precise odds, much less on whether they say that they calculate and compare expected utilities or think that they do, or whether it appears to others that they do, or whether psychologists can uncover any evidence that they do, but solely on whether it yields sufficiently accurate predictions about the class of decisions with which the hypothesis deals. Stated differently, the test by results is the only possible method of determining whether the *as if* statement is or is not a sufficiently good approximation to reality for the purpose at hand.

A simple example may help to clarify the point at issue. Consider the problem of predicting, before each shot, the direction of travel of a billiard ball hit by an expert billiard player. It would be possible to construct one or more mathematical formulas that would give the directions of travel that would score points and, among these, would indicate the one (or more) that would leave the balls in the best positions. The formulas might, of course, be extremely complicated, since they would necessarily take account of the location of the balls in relation to one another and to the cushions and of the complicated phenomena introduced by "english." Nonetheless, it seems not at all unreasonable that excellent predictions would be yielded by the hypothesis that the billiard player made his shots *as if* he knew the formulas, could estimate accurately by eye the angles, etc., describing the location of the balls, could make lightning calculations from the formulas, and could then make the ball travel in the direction indicated by the formulas. It would in no way disprove or contradict the hypothesis, or weaken our confidence in it, if it should turn out that the billiard player had never studied any branch of mathematics and was utterly incapable of making the necessary calculations: unless he was capable in some way of reaching approximately the same result as that obtained from the formulas, he would not in fact be likely to be an expert billiard player.

The same considerations are relevant to our utility hypothesis. Whatever the psychological mechanism whereby individuals make choices, these choices appear to display some consistency, which can apparently be described by our utility hypothesis. This hypothesis enables predictions to be made about phenomena on which there is not yet reliable evidence. The hypothesis cannot be declared invalid for a particular class of behavior until a prediction about that class proves false. No other test of its validity is decisive.

b) A POSSIBLE INTERPRETATION OF THE UTILITY FUNCTION

A possible interpretation of the utility function of Figure 3 is to regard the two convex segments as corresponding to qualitatively different socioeconomic levels, and the concave segment to the transition between the two levels. On this interpretation, increases in income that raise the relative position of the consumer unit in its own class but do not shift the unit out of its class yield diminishing marginal utility, while increases that shift it into a new class, that give it a new social and economic status, yield increasing marginal utility. An unskilled worker may prefer the certainty of an income about the same as that of the majority of unskilled workers to an actuarially fair gamble that at best would make him one of the most prosperous unskilled workers and at worst one of the least prosperous. Yet he may jump at an actuarially fair gamble that offers a small chance of lifting him out of the class of unskilled workers and into the "middle" or "upper" class, even though it is far more likely than the preceding gamble to make him one of the least prosperous unskilled workers. Men will and do take great risks to distinguish themselves, even when they know what the risks are. May not the concave segment of the utility curve of Figure 3 translate the economic counterpart of this phenomenon appropriately?

A number of additions to the hypothesis are suggested by this interpretation. In the first place, may there not be more than two qualitatively distinguishable socioeconomic classes? If so, might not each be reflected by a convex segment in the utility function? At the moment, there seems to be no observed behavior that requires the introduction of additional convex segments, so it seems undesirable and unnecessary to complicate the hypothesis further. It may well be, however, that it will be necessary to add such segments to account for behavior revealed by further empirical evidence. In the second place, if different segments of the curve correspond to different socioeconomic classes, should not the dividing points between the segments occur at roughly the same income for different consumer units in the same community? If they did, the fruitfulness of the hypothesis would be greatly extended. Not only could the general shape of the utility function be supposed typical; so also could the actual income separating the various segments. The initial convex segment could be described as applicable to "relatively low-income consumer units" and the terminal convex segment as applicable to "relatively high-income consumer units"; and the groups so designated could be identified by the actual income or wealth of different consumer units.

Interpreting the different segments of the curve as corresponding to different socioeconomic classes would, of course, still permit wide variation among consumer units in the exact shape and height of the curve. In addition, it would not be necessary to suppose anything more than rough similarity in the location of the incomes separating the various segments. Different socioeconomic classes are not sharply demarcated from one another; each merges into the next by imperceptible gradations (which, of course, accounts for the income range encompassed by the concave segment); and the generally accepted dividing line between classes will vary from time to time, place to place, and consumer unit to consumer unit. Finally, it is not necessary that every consumer unit have a utility curve like that in Figure 3. Some may be inveterate gamblers; others, inveterately cautious. It is enough that many consumer units have such a utility curve.

6. FURTHER IMPLICATIONS OF HYPOTHESIS

To return to our main theme, we have two tasks yet to perform: first, to show that the utility function of Figure 3 is consistent with those features of the behavior described in section 2 not used in deriving it; second, to suggest additional implications of the hypothesis capable of providing a test of it.

The chief generalization of section 2

not so far used is that people must in general be paid a premium to induce them to bear moderate risks instead of either small or large risks. Is this generalization consistent with the utility function of Figure 3?

It clearly is for a consumer unit whose income places it in the initial convex segment. Such a relatively low-income consumer unit will be willing to pay something more than the actuarial value for insurance against any kind of risk that may arise; it will be averse to small fair gambles; it may be averse to all fair gambles; if not, it will be attracted by fair gambles that offer a small chance of a large gain; the attractiveness of such gambles, with a given possible loss and actuarial value, will initially increase as the size of the possible gain increases and will eventually decrease.[35] Such consumer units therefore prefer either certainty or a risk that offers a small chance of a large gain to a risk that offers the possibility of moderate gains or losses. They will therefore have to be paid a premium to induce them to undertake such moderate risks.

The generalization is clearly false for a consumer unit whose income places it in the concave segment. Such an "intermediate-income" consumer unit will be attracted by every small fair gamble; it may be attracted by every fair gamble; it may be averse to all fair insurance; if not, it will be attracted by insurance against relatively large losses.[36] Such consumer units will therefore be willing to pay a premium in order to assume moderate risks.

[35] The willingness of a consumer unit in the initial convex segment to pay something more than the actuarial value for insurance against any kind of risk follows from the fact that a chord connecting the utility of its current income with the utility of any lower income to which it might be reduced by the risk in question will everywhere be below the utility curve. The expected utility is therefore less than the utility of the expected income.

To analyze the reaction of such a consumer unit to different gambles, consider the limiting case in which the gamble is fair, i.e., $\bar{I} = I_0$. $\bar{I}$ then is both the expected income of the consumer unit if it takes the gamble and its actual income if it does not (i.e., its current income). The possible gains (and associated probabilities) that will be attractive to the unit for a given value of I_1 (i.e., a given possible loss) can be determined by drawing a straight line through $U(I_1)$ and $U(\bar{I})$. All values of $I_2 > \bar{I}$ for which $U(I_2)$ is greater than the ordinate of the extended straight line will be attractive; no others will be.

Since $\bar{I}$ is assumed to be in the first convex segment, there will always exist some values of $I_2 > \bar{I}$ for which $U(I_2)$ is less than the ordinate of the extended straight line. This is the basis for the statement that the consumer unit will be averse to small gambles.

Consider the line that touches the curve at only two points and is nowhere below the utility curve. Call the income at the first of the points at which it touches the curve, which may be the lowest possible income, I', and the income at the second point, I''. The consumer unit will be averse to all gambles if its income ($I_0 = \bar{I}$) is equal to or less than I'. This follows from the fact that a tangent to the curve at $\bar{I}$ will then be steeper than the "double tangent" and will intersect the latter prior to I'; a chord from $\bar{I}$ to a lower income will be even steeper. This is the basis for the statement that the consumer unit may be averse to all gambles.

If the income is above I', there will always be some attractive gambles. These will offer a small chance of a large gain. The statement about the changing attractiveness of the gamble as the size of the possible gain changes follows from the analysis in sec. 4 of the conditions under which it would be advantageous to have a single prize in a lottery.

[36] Consider the tangent to the utility curve at the income the consumer unit would have if it did not take the gamble ($\bar{I} = I_0$). If this income is in the concave section, the tangent will be below the utility curve at least for an interval of incomes surrounding $\bar{I}$. A chord connecting any two points of the utility curve on opposite sides of $\bar{I}$ and within this interval will always be above the utility curve at $\bar{I}$ (i.e., the expected utility will be above the utility of the expected income), so these gambles will be attractive. The tangent may lie below the utility curve for all incomes. In this case, every fair gamble will be attractive. The unit will be averse to insuring against a loss, whatever the chance of its occurring, if a chord from the current income to the lower income to which it would be reduced by the loss is everywhere above the utility curve. This will surely be true for small losses and may be true for all possible losses.

The generalization is partly true, partly false, for a consumer unit whose income places it in the terminal convex segment. Such a relatively high-income consumer unit will be willing to insure against any small possible loss and may be attracted to every fair insurance plan; the only insurance plans it may be averse to are plans involving rather large losses; it may be averse to all fair gambles; if not, it will be attracted by gambles that involve a reasonably sure, though fairly small, gain, with a small possibility of a sizable loss; it will be averse to gambles of the lottery variety.[37] These consumer units therefore prefer certainty to moderate risks; in this respect they conform to the generalization. However, they may prefer moderate risks to extreme risks, though these adjectives hardly suffice to characterize the rather complex pattern of risk preferences implied for high-income consumer units by a utility curve like that of Figure 3. Nonetheless, in this respect the implied behavior of the high-income consumer units is either neutral or contrary to the generalization.

Our hypothesis does not therefore lead inevitably to a rate of return higher to uses of resources involving moderate risk than to uses involving little or much risk. It leads to a rate of return higher for uses involving moderate risk than for uses involving little risk only if consumer units in the two convex segments outweigh in importance, for the resource use in question, consumer units in the concave segment.[38] Similarly, it leads to a rate of return higher for uses involving moderate risk than for uses involving much risk only if consumer units in the initial convex segment outweigh in importance consumer units in both the concave and the terminal convex segments—though this may be a more stringent condition than is necessary in view of the uncertainty about the exact role of consumer units in the terminal convex segment.

This relative distribution of consumer units among the various segments could be considered an additional restriction that would have to be imposed to rationalize the alleged higher rate of return to moderately risky uses of resources. It is not clear, however, that it need be so considered, since there are two independent lines of reasoning that, taken together, establish something of a presumption that relatively few consumer units are in the concave segment.

One line of reasoning is based on the interpretation of the utility function suggested in section 5*b* above. If the concave segment is a border line between two qualitatively different social classes, one would expect relatively few consumer units to be between the two classes.

The other line of reasoning is based on the implications of the hypothesis for the relative stability of the economic status of consumer units in the different segments. Units in the intermediate segment are tempted by every small gamble and at least some large ones. If opportunities are available, they will be continually subjecting themselves to risk. In consequence, they are likely to move out of the segment; upwards, if they are lucky; downwards, if they are not. Consumer units in the two convex segments, on the other hand, are less likely to move into the intermediate segment. The gambles that units in the initial segment

[37] These statements follow directly from considerations like those in the two preceding footnotes.

[38] This statement is deliberately vague. The actual relative rates of return will depend not only on the conditions of demand for risks of different kinds but also on the conditions of supply, and both would have to be taken into account in a comprehensive statement.

accept will rarely pay off and, when they do, are likely to shift them all the way into the terminal convex segment. The gambles that units in the terminal segment accept will rarely involve losses and, when they do, may shift them all the way into the lower segment. Under these conditions, maintenance of a stable distribution of the population among the three segments would require that the two convex segments contain many more individuals than the concave segment. These considerations, while persuasive, are not, of course, conclusive. Opportunities to assume risks may not exist. More important, the status of consumer units is determined not alone by the outcome of risks deliberately assumed but also by random events over which they cannot choose and have no control; and it is conceivable that these random events might be distributed in such a way that their main effect was to multiply the number in the concave segment.

The absolute number of persons in the various segments will count most for choices among the uses of human resources; wealth will count most for choices among uses of nonhuman resources.[39] In consequence, one might expect that the premium for bearing moderate risks instead of large risks would be greater for occupations than for investments. Indeed, for investments, the differential might in some cases be reversed, since the relatively high-income consumer units (those in the terminal segment) count for more in wealth than in numbers and they may prefer moderate to extreme risks.

[39] This distinction requires qualification because of the need for capital to enter some types of occupations and the consequent existence of "noncompeting groups"; see Milton Friedman and Simon Kuznets, *Income from Independent Professional Practice* (New York: National Bureau of Economic Research, 1945), chap. iii, sec. 3; chap. iv, sec. 2.

In judging the implications of our hypothesis for the market as a whole, we have found it necessary to consider separately its implications for different income groups. These offer additional possibilities of empirical test. Perhaps the most fruitful source of data would be the investment policies of different income groups.

It was noted in section 2 that, although many persons with low incomes are apparently willing to buy extremely speculative stocks, the low-income group receives the bulk of its property income in the form of interest and rents. These observations are clearly consistent with our hypothesis. Relatively high-income groups might be expected, on our hypothesis, to prefer bonds and relatively safe stocks. They might be expected to avoid the more speculative common stocks but to be attracted to higher-grade preferred stocks, which pay a higher nominal rate of return than high-grade bonds to compensate for a small risk of capital loss. Intermediate income groups might be expected to hold relatively large shares of their assets in moderately speculative common stocks and to furnish a disproportionate fraction of entrepreneurs.

Of course, any empirical study along these lines will have to take into account, as noted above, the effect of the progressive income tax in modifying the terms of investment. The current United States federal income tax has conflicting effects: the progressive rates discourage risky investments; the favored treatment of capital gains encourages them. In addition, such a study will have to consider the risk of investments as a group, rather than of individual investments, since the rich may be in a position to "average" risks.

Another implication referred to above

that may be susceptible of empirical test, and the last one we shall cite, is the implied difference in the stability of the relative income status of various economic groups. The unattractiveness of small risks to both high- and low-income consumer units would tend to give them a relatively stable status. By contrast, suppose the utility curve had no terminal convex segment but was like the curve of Figure 2. Low-income consumer units would still have a relatively stable status: their willingness to take gambles at long odds would pay off too seldom to shift many from one class to another. High-income consumer units would not. They would then take almost any gamble, and those who had high incomes today almost certainly would not have high incomes tomorrow. The average period from "shirt sleeves to shirt sleeves" would be far shorter than "three generations."[40] Unlike the other two groups, the middle-income class might be expected to display considerable instability of relative income status.[41]

7. CONCLUSION

A plausible generalization of the available empirical evidence on the behavior of consumer units in choosing among alternatives open to them is provided by the hypothesis that a consumer unit (generally a family, sometimes an individual) behaves as if

1. It had a consistent set of preferences;
2. These preferences could be completely described by attaching a numerical value—to be designated "utility"—to alternatives each of which is regarded as certain;
3. The consumer unit chose among alternatives not involving risk that one which has the largest utility;
4. It chose among alternatives involving risk that one for which the expected utility (as contrasted with the utility of the expected income) is largest;
5. The function describing the utility of money income had in general the following properties:
   a) Utility rises with income, i.e., marginal utility of money income everywhere positive;
   b) It is convex from above below some income, concave between that income and some larger income, and convex for all higher incomes, i.e., diminishing marginal utility of money income for incomes below some income, increasing marginal utility of money income for incomes between that income and some larger income, and diminishing marginal utility of money income for all higher incomes;
6. Most consumer units tend to have incomes that place them in the segments of the utility function for which marginal utility of money income diminishes.

Points 1, 2, 3, and 5*a* of this hypothesis are implicit in the orthodox theory of choice; point 4 is an ancient idea recently revived and given new content by von Neumann and Morgenstern; and points 5*b* and 6 are the consequence of the attempt in this paper to use this idea to rationalize existing knowledge about the choices people make among alternatives involving risk.

Point 5*b* is inferred from the following phenomena: (*a*) low-income consumer units buy, or are willing to buy, insurance; (*b*) low-income consumer units

[40] We did not use the absence of such instability to derive the upper convex segment because of the difficulty of allowing for the effect of the income tax.

[41] The existing data on stability of relative income status are too meager to contradict or to confirm this implication. In their study of professional incomes Friedman and Kuznets found that relative income status was about equally stable at all income levels. However, this study is hardly relevant, since it was for homogeneous occupational groups that would tend to fall in a single one of the classes considered here. Mendershausen's analysis along similar lines for family incomes in 1929 and 1933 is inconclusive. See Friedman and Kuznets, *op. cit.*, chap vii; Horst Mendershausen, *Changes in Income Distribution during the Great Depression* (New York: National Bureau of Economic Research, 1946), chap. iii.

buy, or are willing to buy, lottery tickets; (*c*) many consumer units buy, or are willing to buy, both insurance and lottery tickets; (*d*) lotteries typically have more than one prize. These statements are taken as a summary of the essential features of observed behavior not because they are the most important features in and of themselves but because they are convenient to handle and the restrictions imposed to rationalize them turn out to be sufficient to rationalize all the behavior described in section 2 of this paper.

A possible interpretation of the various segments of the utility curve specified in 5*b* is that the segments of diminishing marginal utility correspond to socioeconomic classes, the segment of increasing marginal utility to a transitional stage between a lower and a higher socioeconomic class. On this interpretation the boundaries of the segments should be roughly similar for different people in the same community; and this is one of several independent lines of reasoning leading to point 6.

This hypothesis has implications for behavior, in addition to those used in deriving it, that are capable of being contradicted by observable data. In particular, the fundamental supposition that a single utility curve can generalize both riskless choices and choices involving risk would be contradicted if (*a*) individuals were observed to choose the larger of two certain incomes offered to them but (*b*) individuals were willing to pay more than the largest possible gain for the privilege of bearing risk. The supposition that individuals seek to maximize expected utility would be contradicted if individuals' reactions to complicated gambles could not be inferred from their reactions to simple ones. The particular shape of the utility curve specified in 5*b* would be contradicted by any of a large number of observations, for example, (*a*) general willingness of individuals, whatever their income, who buy insurance against small risks to enter into small fair gambles under circumstances under which they are not also buying "entertainment," (*b*) the converse of *a*, namely an unwillingness to engage in small fair gambles by individuals who are not willing to buy fair insurance against small risks, (*c*) a higher average rate of return to uses of resources involving little risk than to uses involving a moderate amount of risk when other things are the same, (*d*) a concentration of investment portfolios of relatively low-income groups on speculative (but not highly speculative) investments or of relatively high-income groups on either moderately or highly speculative investments, (*e*) great instability in the relative income status of high-income groups or of low-income groups as a consequence of a propensity to engage in speculative activities.

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