Measuring Investment Risk

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Because investments involve an expected stream of returns over a future period of time, by definition they carry inherent risk. While interest rates are used to discount the stream of benefits and costs of an investment, beyond the pure rate of time preference between the present and future, perceived risk above and beyond the pure rate of time preference translates into a risk premium that adds the corresponding level of interest used in making investment choices.

Risk exists in a variety of forms ranging from the purely economic to the purely political. What all indicators of risk share in common is that one attaches specific probabilities to various possible events. *Value at Risk* models provide specific probability estimates of various outcomes and which in turn help to shape interest rate selection choices by financial institutions. Where bonds and collateral securities are concerned, measures of risk are compiled and reported by independent agencies such as the Economist Intelligence Unit and International Business Communications, among others. As broader measures of risk are compiled, individual agencies such as Moody's Bond Services and Standard and Poor's issue specific bond ratings that provide a guide to prospective investors, as well as to traders in the secondary market. In an ideal world of quasi-perfect information, decisions built around these information flows produce rational expectations and rational decisions. The challenge is that despite the increasing volume and flow of information, there still are imperfections in the level of information, some of which are symmetric, but often are asymmetric, thus leading to skewed measures of default risk, as events in East Asia have recently shown. Finally, it should be noted that no measure of risk can capture all probable events, in which case one is confronting the presence of uncertainty. As Frank Knight (*Risk, Uncertainty, and Profit* , Boston: Houghton-Mifflin, 1921) long ago pointed out there is a discernible difference between risk and uncertainty. Risk incorporates information from existing events, while uncertainty requires the supposition of subjective estimates of probable events. Entrepreneurial behavior is associated typically with uncertain events, for which standard measures of prediction are difficult to assess. Thus, while the challenge of investment is to seek the highest rate of expected return for the lowest level of risk, it also is true that there is a positive relationship between risk and return, and for which additional risk must be underwritten by some economic agent. This is especially so in the case of bringing new innovations to fruition.

Table 1

Global Economic Output and Composite Regional Risk

	Total GDP	Median C.Risk,1996	Median C.Risk,1991	Median PPP PCGNP
Western Europe	$8,828,092	85.5	81.5	$19,950
Japan	$5,108,540	89.5	84.5	$22,110
Eastern and Central Europe	$619,087	64.8	56.5	$4,480
North Africa and Middle East	$1,061,724	68.5	55.5	$5,320
South Asia	$624,835	65.0	44.0	$2,230
East Asia	$1,733,147	77.8	66.0	$5,230
North America	$7,770,986	85.0	83.0	$21,130
Central and Latin America	$1,365,961	67.0	58.0	$3,870
Sub-Saharan Africa	$302,164	58.3	51.3	$1,175
Australia and New Zealand	$405,852	85.3	79.3	$17,650
Total	$27,820,388	68.5	56.5	$4,360

Source: *The Wall Street Journal*

Figure 1

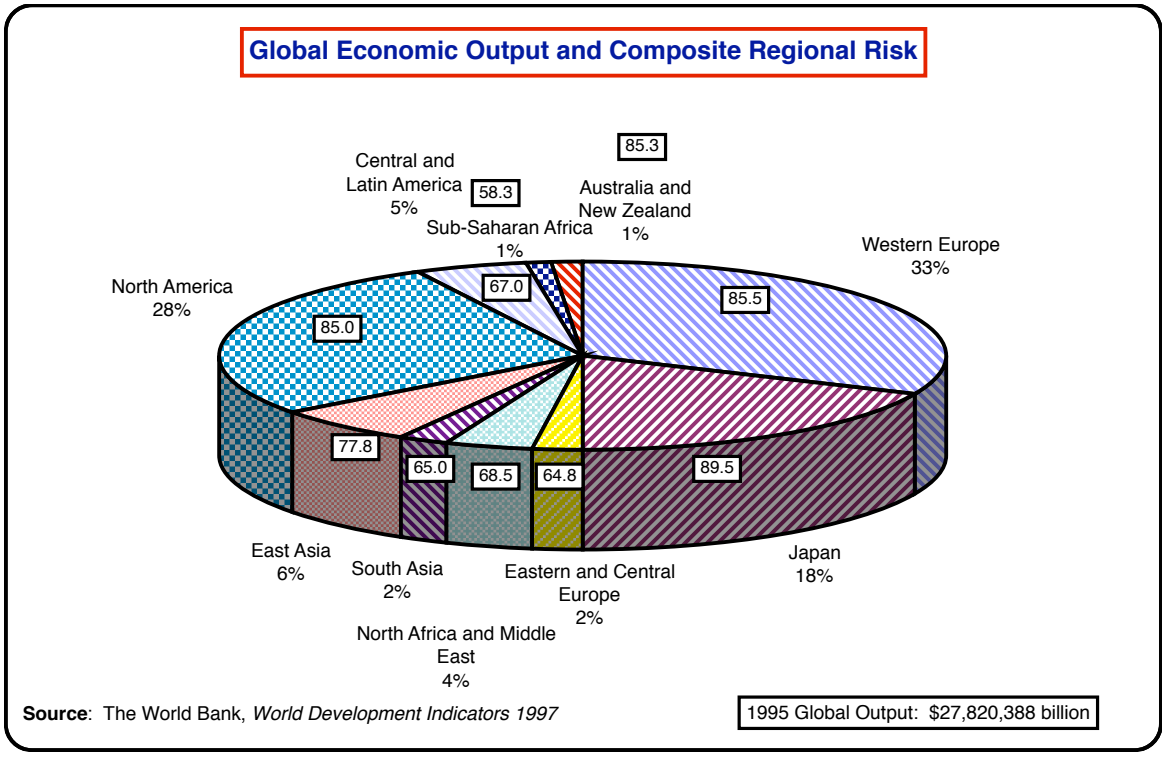


Figure 2

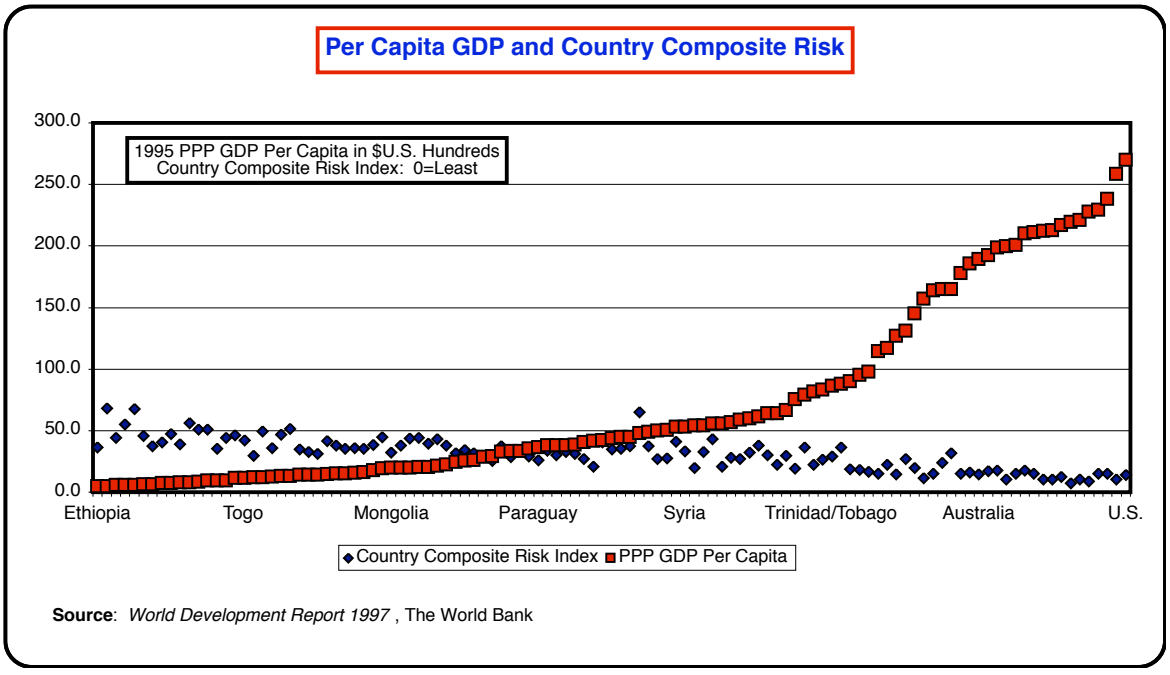


Table 2

Global Ranking of Country Risk

(**Source**: *International Country Risk Guide* , August 1991. International Business Communications Ltd., London)
(as reported in the *Wall Street Journal* , September 20, 1991, p. R4, and *World Development Indicators 1997*)

	Composite 1996	Composite 1991	1991 Political	1991 Financial	1991 Economic
Norway	92.5	88.0	87.0	57.0	42.0
Singapore	91.0	83.5	79.0	48.0	39.5
Switzerland	89.5	91.5	93.0	50.0	39.5
Netherlands	89.5	86.0	85.0	46.0	40.5
Japan	89.5	84.5	80.0	50.0	39.0
Denmark	89.5	82.0	86.0	41.0	37.0
Austria	89.5	87.5	88.0	57.0	39.5
Ireland	88.5	80.0	80.0	42.0	37.5
Belgium	87.5	82.0	82.0	45.0	36.5
U.S.	86.0	83.5	78.0	49.0	39.5
Portugal	85.5	75.0	69.0	42.0	38.5
Australia	85.5	79.0	76.0	45.0	37.0
New Zealand	85.0	79.5	78.0	46.0	35.0
Kuwait	85.0	46.0	38.0	24.0	29.5
Korea, Republic of	85.0	73.5	63.0	47.0	36.5
Hong Kong	85.0	67.5	58.0	42.0	35.0
Germany	85.0	86.0	83.0	50.0	38.5
Finland	85.0	80.5	85.0	44.0	32.0
Canada	85.0	83.0	81.0	48.0	37.0
Sweden	84.0	81.5	81.0	47.0	35.0
Czechoslovakia	83.5	69.5	73.0	36.0	30.0
United Kingdom	83.0	81.0	76.0	50.0	36.0
Italy	82.5	77.0	72.0	47.0	35.0
France	82.5	80.0	79.0	46.0	34.5
Chile	82.0	70.0	67.0	42.0	30.5
Malaysia	81.5	77.5	71.0	45.0	38.5
Thailand	81.0	68.0	57.0	42.0	37.0
Spain	80.5	71.0	65.0	42.0	35.0
Poland	80.0	61.0	62.0	29.0	31.0
Namibia	79.0	54.5	47.0	24.0	38.0
Botswana	79.0	73.0	70.0	34.0	42.0
Oman	77.5	70.5	65.0	34.0	42.0
Hungary	77.5	62.0	68.0	32.0	24.0
Greece	77.5	64.0	65.0	33.0	29.5
U.A.E.	76.0	63.0	53.0	33.0	39.5
China	74.5	60.0	58.0	24.0	38.0
Paraguay	74.0	66.5	59.0	39.0	34.5
Argentina	73.5	58.0	63.0	30.0	23.0
Tunisia	73.0	54.5	54.0	23.0	32.0
Saudi Arabia	73.0	63.5	60.0	31.0	35.5
Jordan	73.0	48.0	45.0	20.0	30.5
Costa Rica	73.0	69.0	71.0	35.0	32.0
South Africa	72.5	59.5	56.0	30.0	32.5
Iran	72.0	55.5	56.0	28.0	26.5
Philippines	71.5	46.5	41.0	22.0	29.5
Morocco	71.5	55.5	52.0	28.0	30.5
Trinidad/Tobago	71.0	63.0	59.0	35.0	31.5
Jamaica	71.0	63.5	66.0	37.0	24.0
Vietnam	70.5	44.0	50.0	18.0	20.0
Uruguay	70.5	68.5	66.0	39.0	32.0
Mexico	70.0	70.5	71.0	41.0	28.5
Indonesia	70.0	68.5	57.0	44.0	35.5
India	69.0	43.0	34.0	25.0	27.0
Dominican Republic	69.0	53.5	53.0	23.0	30.5
Papua New Guinea	68.5	54.0	54.0	26.0	28.0
Israel	68.5	63.0	58.0	33.0	34.5
El Salvador	68.5	43.5	37.0	18.0	32.0

Panama	68.0	54.5	47.0	28.0	34.0
Mongolia	68.0	64.5	65.0	36.0	28.0
Kenya	67.5	50.0	48.0	26.0	26.0
Egypt	67.5	56.5	54.0	30.0	29.0
Brazil	67.0	62.0	67.0	34.0	23.0
Syria	66.5	56.0	53.0	23.0	36.0
Peru	66.0	47.5	45.0	28.0	21.5
Guatemala	65.5	48.0	41.0	24.0	30.5
Bolivia	65.5	59.0	52.0	34.0	32.0
Romania	65.0	49.5	55.0	29.0	15.0
Bangladesh	65.0	40.0	33.0	18.0	29.0
Gambia	64.5	61.0	53.0	33.0	35.5
Gabon	64.5	64.5	57.0	33.0	39.0
Bulgaria	64.5	57.5	61.0	28.0	25.5
Albania	64.5	52.0	55.0	33.0	16.0
Yemen	64.0	42.0	49.0	23.0	12.0
Côte d'Ivoire	64.0	59.0	66.0	29.0	23.0
Venezuela	63.5	75.5	75.0	40.0	36.0
Libya	63.5	56.5	52.0	27.0	34.0
Ethiopia	63.5	31.5	22.0	16.0	25.0
Tanzania	62.5	53.0	56.0	27.0	23.0
Sri Lanka	62.5	47.5	36.0	26.0	32.5
Russian Fed. (U.S.S.R.)	62.5	55.5	53.0	36.0	21.5
Lebanon	62.5	39.0	32.0	11.0	35.0
Pakistan	62.0	44.0	34.0	22.0	32.0
Ghana	62.0	55.0	53.0	30.0	27.0
Cuba	62.0	41.0	54.0	16.0	12.0
Colombia	62.0	67.5	60.0	41.0	34.0
Senegal	61.5	58.0	53.0	29.0	33.5
Burkina Faso	61.0	50.0	41.0	23.0	36.0
Congo	60.5	53.0	52.0	20.0	33.5
Malawi	59.5	52.0	51.0	28.0	25.0
Ecuador	59.5	56.5	58.0	29.0	26.0
Algeria	59.0	58.5	54.0	30.0	32.5
Uganda	58.5	31.0	36.0	21.0	5.0
Togo	58.0	51.0	41.0	26.0	35.0
Turkey	57.0	49.5	52.0	19.0	27.5
Cameroon	57.0	54.0	47.0	27.0	34.0
Nicaragua	56.5	44.0	44.0	27.0	17.0
Zimbabwe	56.0	50.0	51.0	25.0	23.5
Zambia	56.0	45.0	45.0	19.0	25.5
Mali	56.0	45.0	40.0	19.0	30.5
Honduras	55.5	52.5	49.0	28.0	28.0
Madagascar	54.5	51.5	57.0	20.0	26.0
Yugoslavia	53.5	46.5	45.0	24.0	23.5
Guinea	53.0	48.5	48.0	21.0	28.0
Niger	52.5	49.0	45.0	24.0	29.0
Nigeria	50.5	56.5	49.0	29.0	35.0
Mozambique	49.0	43.0	44.0	26.0	15.5
Haiti	49.0	33.5	28.0	12.0	26.5
Angola	48.5	52.5	45.0	19.0	41.0
Korea, North	45.5	45.0	59.0	15.0	16.0
Sierra Leone	45.0	39.0	37.0	20.0	20.5
Guinea-Bissau	44.0	38.5	46.0	19.0	12.0
Iraq	35.0	24.5	19.0	4.0	25.5
Sudan	32.5	24.0	15.0	10.0	22.5
Zaire	32.0	34.0	30.0	18.0	20.0
Unweighted Mean:	**68.74**	**59.11**			
Unweighted st.dev.	**13.14**	**15.02**			
Unweighted c.var.	**0.19**	**0.25**			

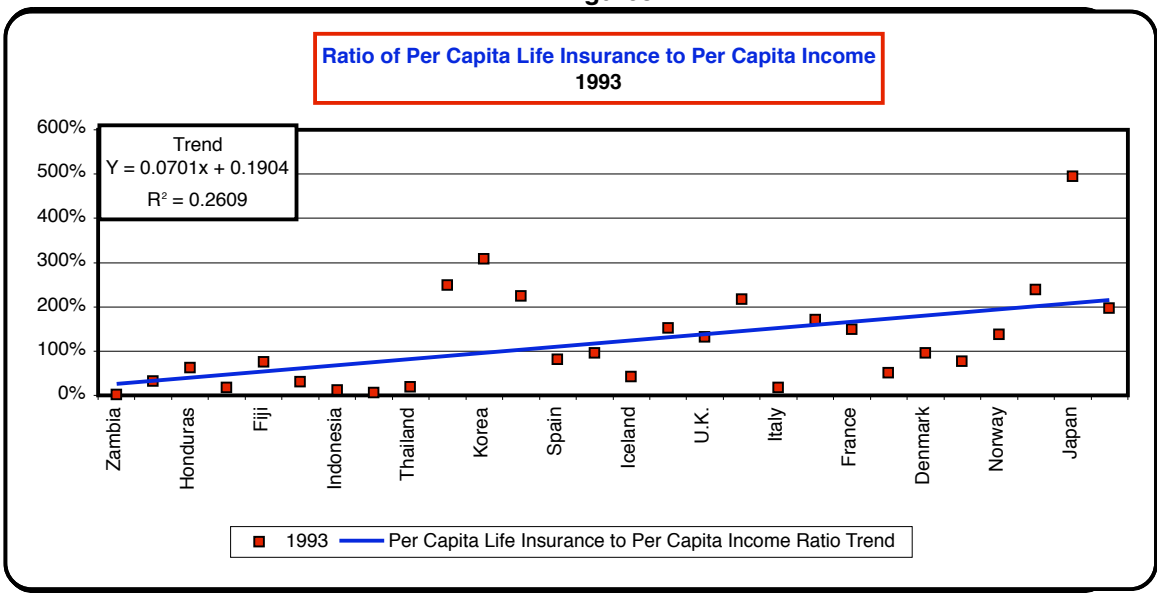
Table 3

Ratio of Life Insurance in Force to National Income

	1988	1993	PPP PC GDP, 1993	1993 PC Life Insur.
Zambia	7%	1%	$1,039	$10.39
India	22%	32%	$1,212	$387.92
Honduras	42%	62%	$1,905	$1,181.09
Pakistan	18%	17%	$2,177	$370.11
Fiji	56%	75%	$2,193	$1,644.75
Philippines	23%	31%	$2,672	$828.30
Indonesia	6%	11%	$3,142	$345.62
Tunisia	5%	6%	$4,775	$286.49
Thailand	15%	19%	$6,259	$1,189.25
South Africa	194%	249%	$8,659	$21,560.91
Korea	156%	308%	$9,624	$29,641.49
Ireland	210%	224%	$13,483	$30,202.59
Spain	50%	81%	$13,508	$10,941.51
Germany	77%	96%	$16,848	$16,174.02
Iceland	30%	42%	$16,849	$7,076.58
Sweden	125%	152%	$17,194	$26,135.34
U.K.	117%	131%	$17,219	$22,556.94
Netherlands	182%	217%	$17,318	$37,580.06
Italy	13%	17%	$17,838	$3,032.38
Australia	124%	170%	$17,912	$30,449.99
France	126%	149%	$19,000	$28,310.48
Austria	43%	50%	$19,421	$9,710.45
Denmark	77%	96%	$19,569	$18,786.57
Belgium	68%	76%	$19,644	$14,929.11
Norway	97%	137%	$19,792	$27,115.04
Canada	192%	239%	$20,237	$48,367.19
Japan	381%	494%	$20,856	$103,027.75
U.S.	183%	196%	$24,740	$48,490.40

Source: American Council on Life Insurance, *Life Insurance Fact Book*

Figure3



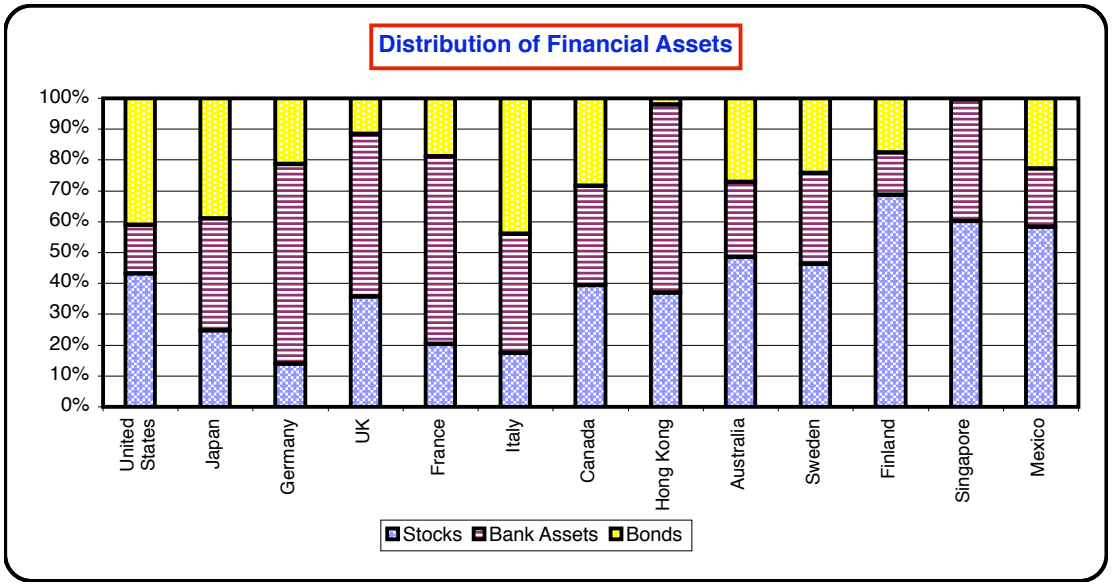
As countries achieve higher levels of income, they may become more risk averse, as measured by the level of per capita life insurance relative to the level of per capita income.

Another way of tracking attitudes toward risk is in terms of the distribution of financial assets by basic risk categories. Stocks, bank assets, and bonds constitute the principal form of financial assets that are used to generate investment. The volatility of these assets tends to be highest among stocks, second among bank assets, and third among bonds. The higher the proportion of a country's financial assets in riskier assets, the greater the tolerance for risk. In turn, assets with higher risk generally carry higher average rates of return. We report here a profile of the distribution of assets by major classes at the end of 1999 as compiled by Zurich Financial Services and published in the January 25, 2000 issue of the *Financial Times* (p. 16).

Distribution of Major Financial Assets by Country

(in $U.S. billions, end 1999)

	Stocks	Bank Assets	Bonds	Total	GDP	Ratio
United States	$15,300.00	$5,600.00	$14,607.00	$35,507.00		
Japan	$3,300.00	$4,857.20	$5,228.10	$13,385.30		
Germany	$1,200.00	$5,547.20	$1,842.00	$8,589.20		
UK	$2,600.00	$3,820.50	$849.50	$7,270.00		
France	$1,200.00	$3,564.60	$1,110.50	$5,875.10		
Italy	$570.00	$1,255.80	$1,435.50	$3,261.30		
Canada	$700.00	$568.00	$505.00	$1,773.00		
Hong Kong	$530.00	$871.00	$30.50	$1,431.50		
Australia	$580.00	$290.00	$323.70	$1,193.70		
Sweden	$450.00	$285.00	$236.70	$971.70		
Finland	$300.00	$60.00	$76.80	$436.80		
Singapore	$260.00	$170.00	$2.00	$432.00		
Mexico	$120.00	$39.00	$46.90	$205.90		
Total	$27,110.00	$26,928.30	$26,294.20	$80,332.50		



Risk and Uncertainty

Risk is a possible outcome that can be measured by the assignment of specific probabilities. Uncertainty involves the assignment of subjective probabilities based on beliefs regarding prospective outcomes.

I. Types of Risk.

Listed below is a grouping of types of risk as reported in Anand Shetty, Francis J. McGrath, Irene M. Hammerbacher, *Finance: An Integrated Global Approach* , (Burr Ridge, Illinois: R.D. Irwin, 1995)

A. Domestic

1. Default
2. Illiquidity
3. Purchasing Power
4. Interest Rate
5. Market
6. Call
7. Expiration

B. International

1. Exchange Rate
2. Interest Rate
3. Country

II. Basic Measurement of Risk

A. Standard Deviation

Consider the following 5 possible outcomes from an investment:

		(Xi-Xm)	(Xi-Xm)^2	fi(Xi-Xm)^2
1	10	-48	2304	460.8
2	30	-28	784	156.8
3	50	-8	64	12.8
4	80	22	484	96.8
5	120	62	3844	768.8
Sum:	290		7480	1496
Mean:	58.00			
Standard Deviation:	38.68			38.68

The problem with using the standard deviation is that it does not permit ready comparisons of the degree of risk among populations with different sample sizes and means. For this reason, use of the coefficient of variation is preferred, since it provides a relative measure of risk.

B. Coefficient of Variation

The coefficient of variation is simply the standard deviation divided by the mean. Based on the preceding example, the value thus is: 0.66686

Let us now look at an example with two different populations:

Group A		Group B	
1	7.20	1	3.60
2	9.60	2	10.80
3	4.80	3	4.50
4	1.20	4	1.80
5	-7.20	5	-9.00
6	4.80	6	4.50
7	6.00	7	3.60
8	2.40	8	1.80
Sum:	28.80		21.60
Mean:	3.60		2.70
Std.Deviation:	4.76		5.15
C.Variation:	1.32		1.91

The question for an economic agent is whether Group A is preferred to Group B. If one is risk averse, Group A is clearly preferable to Group B since it has a higher mean value and a lower degree of relative risk. Rank ordering alternative choices is a central function in the selection of optimal risk management strategies. The problem is that higher mean values generally correspond with higher coefficients of variation, in which case the optimal choice is the determination of what mean value is necessary to compensate for the relative degree of risk.

Deriving the optimal choice of two or more prospective outcomes depends on one's attitude toward risk. We can first characterize the outcome-risk relationship in terms of a statistical distribution from which we extract the corresponding expected value and coefficient of variation. While the Gini coefficient and the Lorenz curve represent one way of characterizing a statistical distribution, another way is through a normal distribution. To obtain such a function we first define the cumulative probability distribution as:

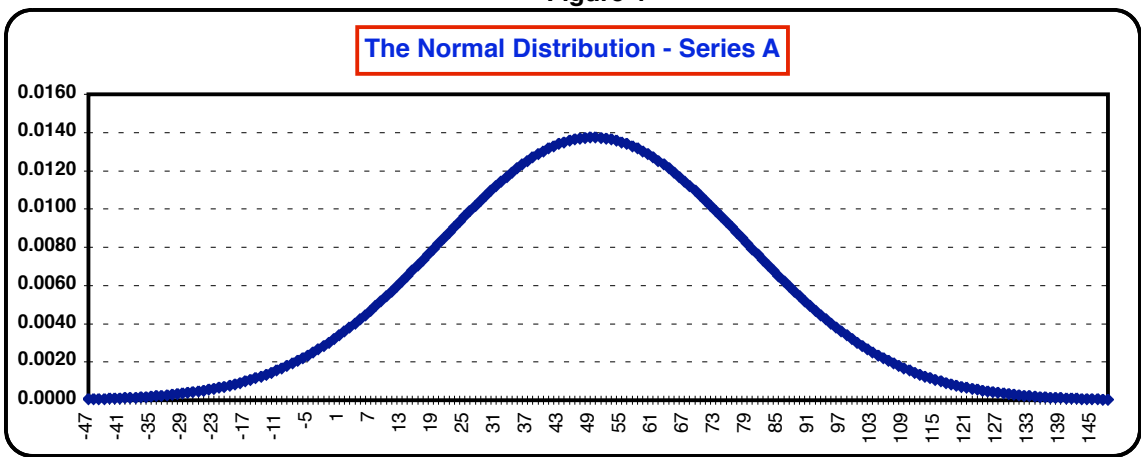
(1). $$P(u) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{u} e^{-\frac{1}{2}(\frac{u-\mu}{\sigma})^2} du$$ where:

π = the ratio of the circumference of a circle to its diameter, or 3.1416
e = the exponential base, or 2.71828
u = is the value of a random variable
μ = the normally distributed mean of a random variable u.
σ = the standard deviation.

From the cumulative probability density function, if we differentiate with respect to u, we obtain the normal, or classically bell-shaped normal distribution, whose formula is given as:

(2). $$p(u) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{u-\mu}{\sigma})^2}$$

Figure 4



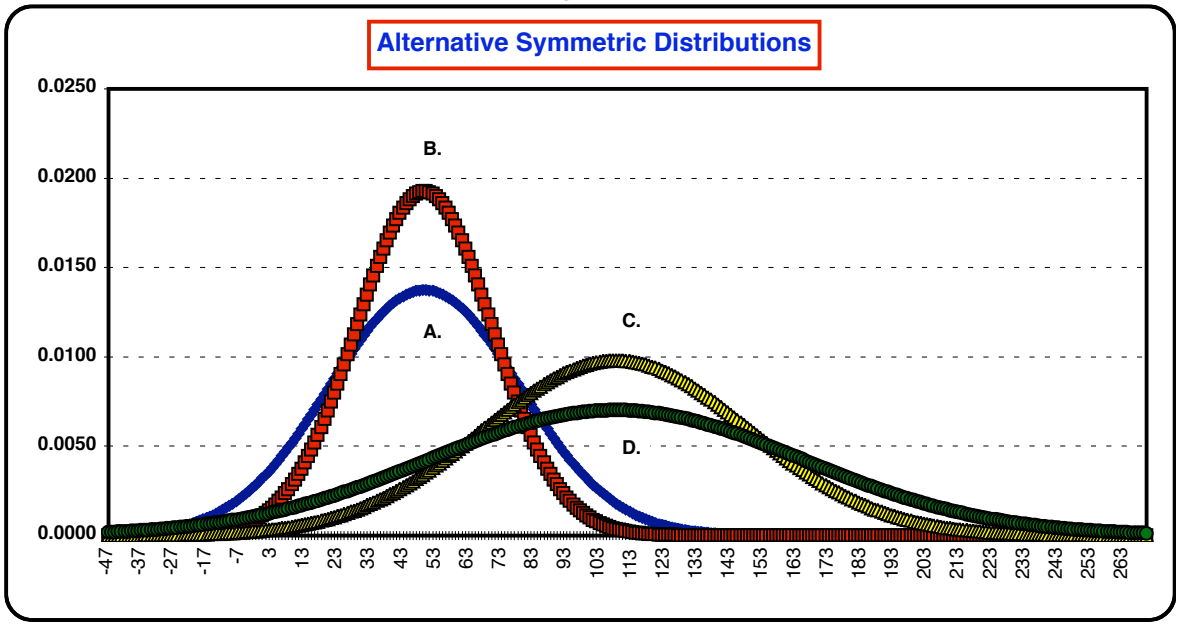
As with our preceding investment example, while we can characterize the mean and standard deviation from a normal distribution, most decisions involving risk are more complicated. What makes decisions more complicated is that one often is making comparisons among a variety of alternatives, in which case one needs a relative measure of risk and return. Again, the coefficient of variation serves as a useful measure of the relative degree of risk.

With a normal distribution shown as series A, alternatives are shown in the figure below::

Table 4

	Alternative Distributions			
	A. Blue	B. Red	C. Yellow	D. Green
π = Pi	3.14159	3.14159	3.14159	3.14159
e = Natural Log.	2.71828	2.71828	2.71828	2.71828
μ = Arithmetic Mean:	49.50	49.50	108.03	108.03
σ = Std. Deviation:	29.01	20.72	40.63	56.90
c.v. = Coeff.Variation:	0.5861	0.4185	0.3761	0.5267
Cumulative area:	0.9996	1.0000	0.9999	0.9947
Kurtosis:	0.0349	1.6616	-0.9726	-1.4245
Champernowne Inequality:	0.1224	0.2806	0.3756	0.1645

Figure 5



The basic normal distribution, as shown in series A, suggests that for an integer series ranging between 1 and 100, and with a mean of 50.5 and a standard deviation of 29.01, half of the population will have values above the mean and half will have values below. This statistically normal distribution may or may not be characteristic of any given income distribution. Thus under various alternatives, we have B, which has the same mean value as in A, but with a smaller standard deviation, is actually more unequally distributed than in A. Alternatively, we also can consider two other distributions, C and D, each of which have the same mean, but which is higher than in A or B, but in which there are different standard deviations. Thus, D, which has the same mean as C, is more equally distributed.

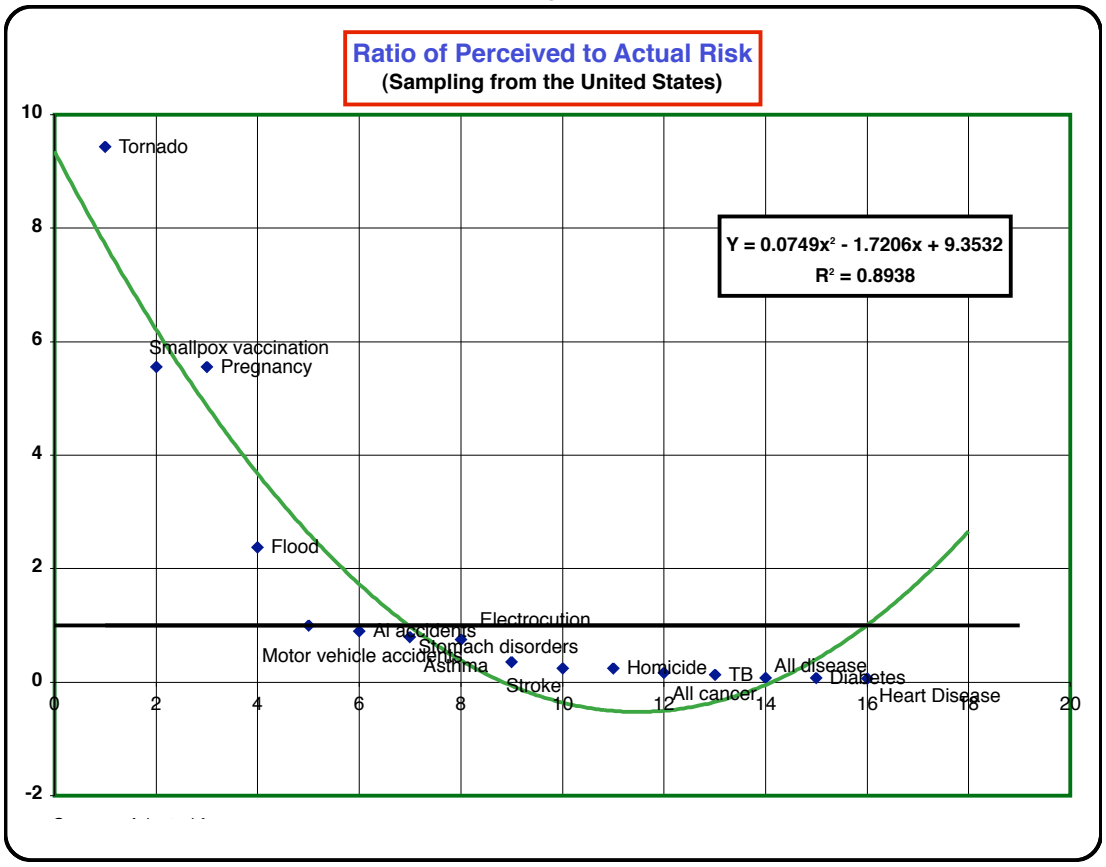
Table 5

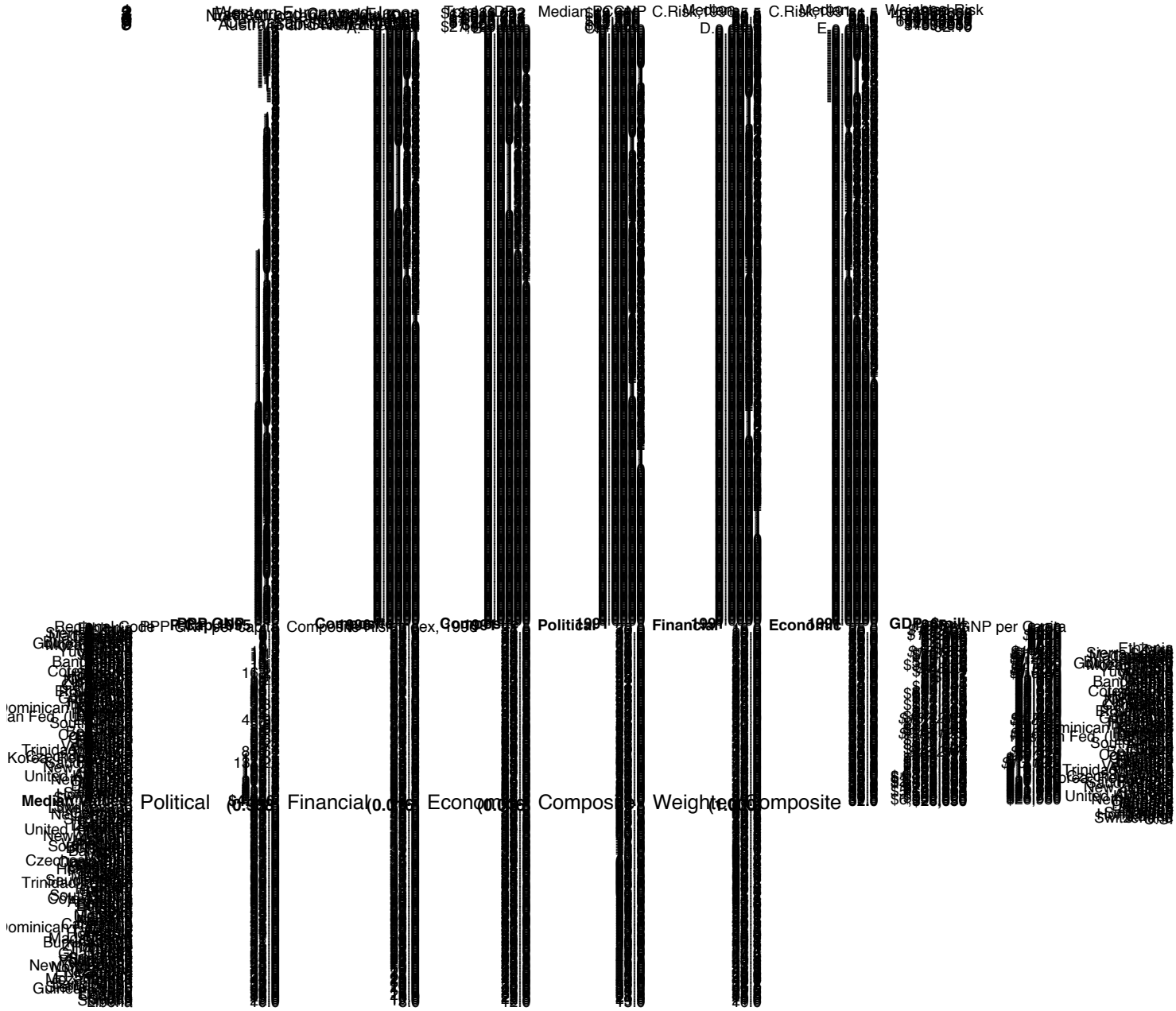
Risk Beliefs and Statistical Evidence

	Judgment Estimate	Statistical Estimate
Smallpox vaccination	50	9
Electrocution	750	1,000
Asthma	900	2,500
TB	1,000	8,000
Diabetes	5000	70,000
Homicide	10,000	40,000
Stomach disorders	80,000	100,000
Stroke	100,000	400,000
Heart disease	50,000	900,000
All cancer	80,000	500,000
All disease	100,000	1,200,000
All accidents	90,000	100,000
Motor vehicle accidents	90,000	90,000
Pregnancy	5,000	900
Flood	950	400
Tornado	850	90
Botulism	500	5

Source: Baruch Fischoff, *et.al* . *Acceptable Risk* (Cambridge: Cambridge U. Press, 1981)

Figure 6




Political
Financial
Economic
Composite
Weighted Composite