Forming McNemar's Confidence Interval Estimate

A (1 - α) % confidence interval estimate of the differences in related population proportions $(p_{\bullet 1} - p_{1\bullet})$ was given by Marascuilo and McSweeney as

$$(\hat{p}_{\bullet 1} - \hat{p}_{1\bullet}) \pm Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_{\bullet 1}\hat{p}_{\bullet 2}}{n'} + \frac{\hat{p}_{1\bullet}\hat{p}_{2\bullet}}{n'} - \frac{2(\hat{p}_{11} - \hat{p}_{\bullet 1}\hat{p}_{1\bullet})}{n'}}$$

where $\hat{p}_{\bullet 1}, \hat{p}_{1\bullet}, \hat{p}_{\bullet 2}, \hat{p}_{2\bullet}$ are the estimators of the respective parameters $p_{\bullet 1}, p_{1\bullet}, p_{\bullet 2}, p_{2\bullet}$ and the term added and subtracted from $(\hat{p}_{\bullet 1} - \hat{p}_{1\bullet})$ is the survey's margin of error.

To form a 95% confidence interval estimate of the differences in related population proportions $(p_{\bullet 1} - p_{1\bullet})$, representing increase in voter preference for Jones at the expense of Smith following a political debate, the estimates from the hypothetical data in Table 2 shown in the Encyclopedia are summarized as follows:

Before Debate	*After Debate*
Jones: $\hat{p}_{1\bullet} = \frac{380}{1000} = 0.38$	Jones: $\hat{p}_{\bullet 1} = \frac{420}{1000} = 0.42$
Smith: $\hat{p}_{2\bullet} = \frac{620}{1000} = 0.62$	Smith: $\hat{p}_{\bullet 2} = \frac{580}{1000} = 0.58$

For these data

$$(0.42 - 0.38) \pm 1.96\sqrt{\frac{(0.42)(0.58)}{1000} + \frac{(0.38)(0.62)}{1000} - \frac{2[(0.34) - (0.42)(0.38)]}{1000}}$$

and

$$0.019 \le (p_{\bullet 1} - p_{1\bullet}) \le 0.061$$

It can be concluded with 95% confidence that the gain in support for Jones at the expense of Smith as a result of the political debate is between 1.9% and 6.1%. The margin of error around the observed 4.0% difference is 2.1%.

Comments

When comparing differences in two proportions based on related samples, the McNemar procedure should always be used. Failure to do so will often lead to erroneous conclusions. A researcher unaware of the magnitude of the correlated proportions that are accounted for in the margin of error term shown in the McNemar confidence interval formula may erroneously treat the paired responses as independent and thus inflate the margin of error, causing a loss of precision in the confidence interval.

The pedagogical advantage to the confidence interval approach to McNemar’s procedure over the significance testing approach shown in the Encyclopedia derives from the fact that the former makes use of all 1,000 repeated responses, whereas the corresponding hypothesis test statistic is conditioned on the reduced set that contains only the 120 “candidate-switchers” in the cross-classifications table; the fact that the test statistic discards the 880 registered voters unaffected by the treatment intervention (i.e., the political debate) is not palatable to some researchers. On the other hand, the major advantage of the hypothesis test procedure over the confidence interval is its inherent simplicity, be it using the binomial probability distribution for an exact test result or the easy-to-use normal approximation formula for an approximate test result.

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See also Confidence Interval; Margin of Error; Measurement Levels, Nominal/Categorical; Normal Curve Distribution; Qualitative Data; Repeated Measures; Respondents; Sampling, Probability; Standard Error; Survey: Dichotomous Questions; Variables, Categorical; Z Transformation

FURTHER READINGS

Marascuilo, L.A., & McSweeney, M. (1977). *Nonparametric and distribution free methods for the social sciences.* Monterey, CA: Brooks/Cole.

McNemar, Q. (1947). Note on the sampling error of the difference between correlated proportions or percentages. *Psychometrika*, *12*, 153-157.