On Monopoly Welfare Losses

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The American Economic Review, Volume 63, Issue 5 (Dec., 1973), 853-870.

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By Abram Bergson*

Are welfare losses due to monopolistic pricing apt to be consequential in an advanced economy such as that of the United States? Generations of economists have assumed that they are, but in 1954 Arnold Harberger argued that in the United States during the late 1920's, the total of such losses in manufacturing was equivalent to but ". . . a tenth of a percent of national income" (p. 87). Subsequently, David Schwartzman found that in the United States in 1954 welfare losses due to monopolistic pricing in industry were equivalent to ". . . less than $234 million, or less than 0.1 percent of the national income" (pp. 629–30). Still more recently Harvey Leibenstein, relying partly on the findings of Harberger and Schwartzman, has also argued that departures from competitive norms such as result under monopolistic pricing are likely to be relatively inconsequential. Generally, ". . . they hardly seem worth worrying about" (Leibenstein, p. 395).

From such findings, George Stigler was an early dissenter, and further calculations of David Kamerschen indicated that welfare losses due to monopoly pricing may be appreciably greater than Harberger and Schwartzman computed. The contention that such losses are inconsequential nevertheless has apparently gained no little currency. Perhaps that is as it should be, but the underlying methodology still does not seem to have been sufficiently explored. Some of the more basic empirical assumptions may also be more questionable than has been supposed. A brief reexamination stressing those aspects may facilitate appraisal of what is surely one of the more important current issues in economics.

I. Some Methodological Considerations

The evaluations of monopoly welfare losses by Harberger, Schwartzman, and Leibenstein all represent applications of consumer's surplus analysis. As such, they are in a sense rather special, for expressly or by implication, all these writers apply a well-known formula of Harold Hotelling for such surplus. Their calculations may also be viewed, however, as measurements of consumer's surplus without reference to the Hotelling formula. As a preliminary, I briefly reformulate the relevant principles, which still seem somewhat in need of clarification.

What counts for Harberger, Schwartzman, and Leibenstein is efficiency rather than equity. I assume, therefore, as is often done in similar contexts, that there is but one household in the community. The household, however, behaves competitively in purchasing consumer's goods. As will appear, the analyses being considered also assume constant costs. For present purposes it is more convenient to work with the somewhat more restrictive supposition that the community's production possibilities are linear.

As J. R. Hicks taught us long ago, consumer's surplus is susceptible to diverse constructions. The particular construction need not be a practically very important matter, but we may conveniently consider the evaluations that have been made in relation to a concept of surplus corresponding to the compensating variation as understood by Hicks (see pp. 61ff, 69ff),

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i.e., the compensatory change in income needed to assure that a household's utility is unaffected by a change in price.

For Hicks, however, the compensating variation represents a change from an otherwise given income. Such a variation is properly taken to measure welfare losses or gains if income is indeed constant. Our household, however, is subject not only to changes in prices, those being the changes due to the fixing of the prices in monopolistic industries at monopolistic rather than competitive levels, but also to a concomitant variation in income. In the analysis of monopoly welfare losses, the concern is with the impact of misallocation rather than unemployment. Hence, with the prices of monopolistic products at monpolistic rather than competitive levels, resources released from monopolistic industries supposedly are reemployed in competitive ones. That is possible only if household income is higher when prices in the monopolistic industries are at monopolistic levels than when they are at competitive levels. Indeed, income must increase by the amount of monopoly profits, for only in that way will there still be sufficient income to employ all the primary factors previously employed.

In short, we are confronted with variations within a general equilibrium context. In consumer's surplus analysis the usual practice even so is to focus on partial equilibrium, but the analysis here seems facilitated if general equilibrium is put clearly to the fore, and Hicks' compensating variation is viewed accordingly.

Thus, a distinction is made between the *gross* compensating variation (GCV), representing the change in income needed to compensate households for some specified variation in prices, and the *net* compensating variation (NCV), representing the gross change, less the adjustment in income (MIA) required to maintain full employment in the face of such a variation in prices, i.e.,

$$NCV = GCV - MIA \tag{1}$$

The GCV corresponds to what Hicks called compensating variation, but the NCV is of primary interest here.[1]

Of particular concern now is a rise in prices in monopolistic industries to monopolistic from competitive levels. Referring to such price changes, it suffices here if there is but a single competitive industry in the community considered. If for the moment we also assume that there is only one monopolistic industry, the price and income variations of interest may be illustrated graphically. In Figure 1, x_1 is the product of the monopolistic and x_2 the

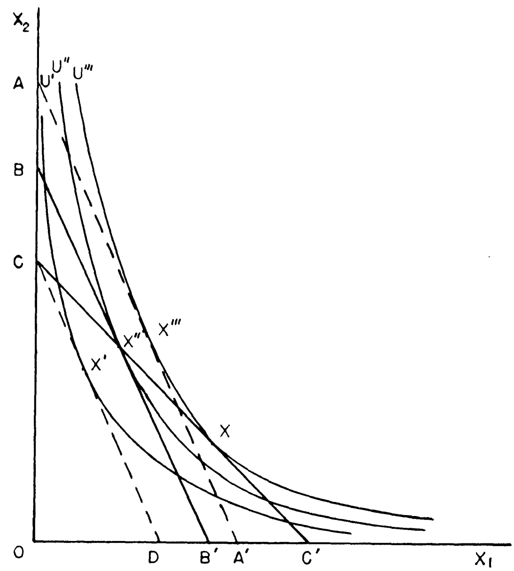


FIGURE 1

[1] For simplicity I assume throughout that monopoly price-cost ratios are unchanged under the impact of the changes in household income in question, but it is easy to see that no essentials would be affected if the ratios should vary. What counts are the monopolistic price-cost ratios when income is again at the full employment level, and the GCV could simply be understood as the addition to the household's initial income level that is needed to compensate for the charging of those price-cost ratios. The MIA is calculated as before as the difference between the two full employment income levels, one where monopolistic products are priced monopolistically and the other where they are priced competitively.

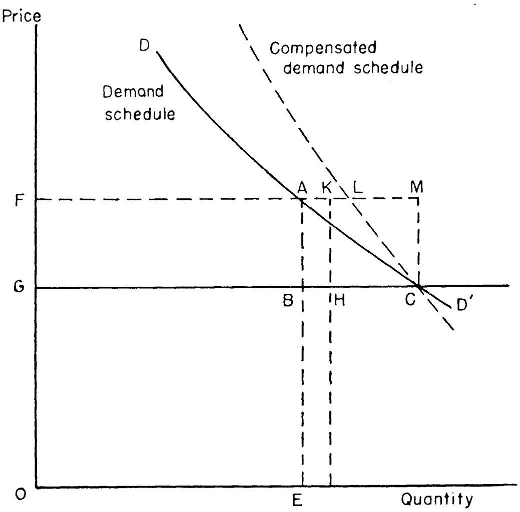
product of the competitive industry, and CC' represents, the community's production possibilities. Several of the household's indifference curves are also shown, and the point x on U''' represents the household's consumption as it would be if there were competitive pricing throughout the economy, i.e., price equals marginal costs in both the monopolistic and the competitive industries. At x' on U' the household's consumption is as it would be if under monopolistic pricing for x_1 the household's income (with x_2 as the numeraire) should be constant at the competitive level OC. With consumption at x', however, there would be unemployed resources, and that is avoided only if income rises by BC to OB, permitting consumption under monopolistic pricing for x_1 to be at x''. That point is again on the production possibilities schedule, and also on U''. The household has still suffered a welfare loss, though, as a result of monopolistic pricing for x_1. The loss is represented by the shift from U''' to U'', but this would be fully offset were there a further increase in income by AB, that would permit consumption to rise to x''' on U'''. Such consumption, of course, is not feasible, but the increase in income by AB still can serve as a hypothetical measure of the welfare loss due to monopoly.

Thus, AB represents the NCV for monopolistic pricing as that has been defined. We also have

$$AB = AC - BC \tag{2}$$

where AC corresponds to the GCV for monopolistic pricing, and BC represents the MIA, or increase in income above the competitive level that is needed to maintain full employment under monopolistic pricing for x_1.

I have delineated the NCV in terms of the household's indifference map. Could it be calculated from a conventional Marshallian demand schedule for the



OF = Monopoly Price
OG = Competitive Price
FK = Monopoly Output After Adjustment in Money Income to Assure Full Employment

Figure 2

monopolized product? Subject to one proviso, it could be, for the NCV then precisely corresponds to the triangle-like area ABC in Figure 2. Here DD' represents the Marshallian demand schedule for x_1 that prevails when household income is at a level that would assure full employment under competitive pricing for x_1. Since costs are constant, such pricing means that x_1 would sell at average cost, equalling OG. Under monopoly, however, the price is increased to OF.

The proviso is the proverbial one that the income elasticity of demand for the product in question be zero. In that case, the budget points x', x'', and x''' in Figure 1 all include the same x_1 and differ only in respect of x_2 Also, the GCV, or AC in that figure, corresponds to the area ABC together with the rectangle $FABG$ in Figure, 2, for "income effects" that might distort the relation between the sum of those areas and the GCV are zero (see Hicks, pp. 68ff). It also follows that the MIA, or BC in Figure 1, corresponds to the rectangle *FABG* in Figure 2. Given that the consumption of x_1 in x' is the same as in x'', the x_1 in x'' corresponds to *FA*. Hence, *FABG* represents monopoly profits at a full employment income level, and also the addition that must be made to income if the resources released from x_1 by the introduction of monopolistic pricing there are to be fully absorbed into x_2. It also follows that the *NCV*, or *AB* in Figure 1, corresponds to the area *ABC* in Figure 2, which is in effect the difference between the *GCV* and *MIA*.

What if the income elasticity of demand is other than zero? As is well known, calculation of the Hicksian compensating variation from a Marshallian demand schedule in that case is subject to error. That must be so also, therefore, for the *GCV*, but of interest here is the *NCV* so we must also consider that the *MIA*, as just calculated, is likewise subject to error. Curiously, the errors tend to be offsetting.

Thus, suppose the monopolized product is a superior one. Then the *GCV* for an increase in the price of that product from the competitive to the monopolistic level now corresponds to the area that is bounded by that price change to the left of the Hicksian or compensated demand schedule, that is, the area *FLCG* in Figure 2. That area exceeds the corresponding area, *FACG*, to the left of the Marshallian or uncompensated demand schedule, which previously represented the *GCV*. On the other hand, monopoly profits at a full employment income level and hence the *MIA* also increase, and are now represented by the area *FKHG* in the figure, rather than the area formerly considered, *FABG*. That follows at once from the fact that *FK* corresponds to x_1 in x'', and is thus intermediate between x_1 in x', or *FA*, and x_1 in x''', or *FL*.

Since the monopolized product must be supposed to be absorbing a large part of the household's income, the first of these income effects should often be rather large. Presumably that would ordinarily be true also of the second. But as indicated the two effects should also be mutually offsetting. How the *NCV*, now given by the area *KLCH* (i.e., the excess of *FLCG* over *FKHG*), compares with the area *ABC*, is nevertheless not an easy subject for generalization. For a superior good, however, it might be supposed that, as given by the *NCV*, the welfare loss would, if anything, be understated by an area such as *ABC* in Figure 2. Curiously that need not be so, for the error in the *GCV* might be more than offset by the further one in the *MIA*. For diverse linear demand schedules it can be shown that that is in fact the case.[2]

Proceeding more formally, we may readily extend the analysis to many monopolized products. Suppose there are n goods that are monopolized, and that a single product, $(n+1)$, is competitive. If we take that as numeraire, there are n prices, $p_1 \ldots p_n$, which vary from $p_1^c \ldots p_n^c$ to $p_1^m \ldots p_n^m$. Household income is I, and in the initial equilibrium,

[2] Reference is to demand schedules for the monopolized product of the form:

$$x = ap + bI + c$$

where x is the consumption of the monopolized product (for convenience, I omit subscripts at this point), p is its price, I is income, and a, b, and c are constants. This formula represents a family of Marshallian or uncompensated demand schedules, each such schedule corresponding to a given level of I. From the Slutsky equation, I derive a formula for the corresponding Hicksian or compensated demand schedules, and on this basis calculate the value of *NCV* corresponding to the area *ABC* and that corresponding to the area *KLCH* in Figure 2 under alternative assumptions as to the uncompensated price elasticity (η_{xp}) and income elasticity (η_{xI}) of demand in the vicinity of the full employment equilibrium with monopolistic pricing for x. With $\eta_{xp}=1.5$ and $\eta_{xI}=1.0$, I find that the *NCV* corresponding to *ABC* is 1.04 and that corresponding to *KLCH*, .77 percent of household income. The corresponding measures of *NCV* when $\eta_{xp}=6.0$ and $\eta_{xI}=3$ are 4.17 and 3.88 percent of household income. I assume that x accounts for one-half the household budget under full employment equilibrium with monopolistic pricing for x, and that with that pricing p is 20 percent above costs.

I^c. Then the household's utility may be written as a function:

$$(3) \qquad U = U(p_1, \ldots, p_n, I)$$

Hence, the GCV is such that

$$(4) \quad U(\overset{c}{p}_1 \ldots \overset{c}{p}_n, \overset{c}{I}) = U(\overset{m}{p}_1 \ldots \overset{m}{p}_n, \overset{c}{I} + GCV)$$

According to a well-known formula due mainly to Hicks (pp. 169ff), we also have

$$(5) \qquad GCV = \sum_{i=1}^{n} gcv_i$$

In (5), GCV is as before, though now changes in many prices together are being compensated for. Also, gcv_i is the gross compensating variation for the increase in the price of the ith of n monopolized products. The gcv_i for those products, however, must be determined sequentially. Thus, if they are treated in the order in which the products are numbered, then for gcv_j reference is to the income increment that would be needed to compensate for the increase in the price of the jth product when prices for all products $l \ldots (j-1)$ have already changed and have been compensated for.[3]

[3] So far as it is understood that in the determination of gcv_j, previous price changes have been compensated for, this formulation seems somewhat novel. That gvc_j must be so construed, however, is seen at once when we consider that with that construction:

$$U(\overset{c}{p}_1, \ldots, \overset{c}{p}_j, \ldots, \overset{c}{p}_n, \overset{c}{I}) = U(\overset{m}{p}_1, \overset{c}{p}_2, \ldots, \overset{c}{p}_j, \ldots, \overset{c}{p}_n, \overset{c}{I} + gcv_1) = \ldots$$

$$U\left(\overset{m}{p}_1, \ldots, \overset{m}{p}_{j-1}, \overset{c}{p}_j, \ldots, \overset{c}{p}_n, \overset{c}{I} + \sum_{i=1}^{j-1} gcv_i\right) = \ldots$$

$$= U\left(\overset{m}{p}_1, \ldots, \overset{m}{p}_n, \overset{c}{I} + \sum_{i=1}^{n} gcv_i\right)$$

Then,

$$U(\overset{m}{p}_1, \ldots, \overset{m}{p}_n, \overset{c}{I} + GCV) = U\left(\overset{m}{p}_1, \ldots, \overset{m}{p}_n, \overset{c}{I} + \sum_{i=1}^{n} gcv_i\right)$$

Since utility is supposed to vary monotonically with I, (5) follows.

It also follows that GCV may still be evaluated from uncompensated demand schedules, but for each gcv_i constituting it reference, strictly speaking, must be to that uncompensated schedule for x_i conforming to the indicated determination of gcv_i. Thus, suppose the demand function for the jth product is given by

$$(6) \qquad x_j = d^j(p_1 \ldots p_j \ldots p_n, I)$$

Then gcv_j is to be calculated from the particular member of that family given by

$$(7) \quad x_j = d^j\left(\overset{m}{p}_1 \ldots \overset{m}{p}_{j-1}, p_j, \overset{c}{p}_{j+1} \ldots \overset{c}{p}_n, \overset{c}{I} + \sum_{i=1}^{j-1} gcv_i\right)$$

Each and every gcv_i as so calculated is precisely correct and all together sum to GCV if income elasticities of demand for all monopolized products are zero, but otherwise calculated gcv_i and the resulting GCV will err in one way or another. Of course, if income elasticities were zero, there would also be no need to be concerned about the precise level at which money income is held constant in each demand schedule, for example, whether at I^c or at I^c plus a sum such as appears in (7).

In order to obtain the NCV, we must deduct from GCV the MIA, the adjustment in income needed to assure full employment in the face of all price changes together. Here

$$(8) \quad MIA = \sum_{j=1}^{n} mia_j = \sum_{j=1}^{n} (\overset{m}{p}_j - \overset{c}{p}_j)\overset{m}{x}_j$$

where x_j^m is the consumption of x_j, when all prices of monopolized goods are at monopoly levels and income has been increased to $(I^c + MIA)$, i.e.,

$$(9) \quad \overset{m}{x}_j = d^j(\overset{m}{p}_1 \ldots \overset{m}{p}_j \ldots \overset{m}{p}_n, \overset{c}{I} + MIA)$$

MIA is determined in principle, along with x_j^m, from (8) together with (9) for $x_j^m, j=1 \ldots n$.

Might we also calculate NCV as so determined simply by summing areas such as ABC in Figure 2? To calculate such areas, reference here presumbaly should be made to demand schedules corresponding to that for x_j in (7) when all prices other than p_j are at competitive levels while I corresponds to I^c, or full employment income when prices are at competitive levels. On that understanding, the answer to the question posed is in the affirmative under two conditions: (i) The income elasticity of demand for every monopolized good is zero. (ii) Cross elasticities of demand among all monopolized products are zero.

Actually, under these conditions it is immaterial at what levels prices of products other than the one considered and income are held constant in the calculation of an area such as ABC, for then all terms other than the price of the product in question drop out of the formulas for the demand for x_j in (7) and (9). The uncompensated demand schedule for any good also coincides with its compensated demand schedule. Hence for any x_i, the areas $FACG$ and $FABG$ in Figure 2 precisely correspond to gvc_j and mia_j. The difference between the aggregate of areas such as $FACG$ and the aggregate of rectangles such as $FABG$, and hence also the resulting aggregate of areas such as ABC, for all commodities, also precisely correspond to the NCV.

What if the stated conditions are not met? In that case, the aggregate of areas such as $FACG$ can only be expected to diverge from the GCV, since for any x_j such an area will differ from the gcv_j. That will be so insofar as: (i) The gcv_j is properly calculated from a demand schedule such as that in (7), where prices for products other than x_j are held constant, some at monopolistic and some at competitive levels, and income is held constant at a level corresponding to that indicated in (7); and (ii) There are income effects of the usual sort, i.e., those manifest in a divergence between the compensated and uncompensated demand schedules for x_j. Similarly, the sum of areas such as $FABG$ can hardly be expected to correspond with the MIA as given by (8) and (9). As in the case of a single monopolized product, errors at different points may sometimes tend to be offsetting. So far as products generally are more often substitutes than complements, that, as easily seen, is apt to be so regarding errors due, on the one hand, to the difference between uncompensated and compensated demand schedules and, on the other, to the difference between the sum of rectangles such as $FABG$ and the MIA as given by (8) and (9). Nevertheless, the relation of the sum of areas such as ABC in Figure 2 to welfare losses as given by the NCV is rather complex. Without a detailed knowledge of demand functions, in all their dimensions, i.e., demand functions such as (7), we cannot really tell how nearly, if at all, the one is approached by the other.

II. The Case for Inconsequentiality

To come to Harberger, Schwartzman, and Leibenstein, their case for the inconsequentiality of welfare losses due to monopolistic pricing is too familiar to need any detailed replication here. Moreover, the chief question that I must raise about their methodology is already evident. Thus, Harberger and Schwartzman both calculate monopoly welfare losses simply by aggregating for different industries areas such as ABC in Figure 2. That is also Leibenstein's procedure, though beyond collating previous findings he limits himself to illustrative calculations. The procedure, to repeat, is also a valid one, provided that the income elasticity of demand for every monopolized product and all cross elasticities of demand among such products are zero. Otherwise, however, the procedure is subject to error at diverse points; an error, moreover, which is difficult to gauge. The practical import of the Harberger, Schwartzman, and Leibenstein findings, therefore, is obscure.

But I have considered their methodology from a more or less conventional standpoint. Is their procedure not more defensible in terms of the Hotelling formula that they apply? That seems doubtful when we consider that Hotelling derives his formula simply by disregarding higher order terms in a Taylor expansion of the utility function. As he properly explains, the formula yields "an approximate measure when deviations from the optimum system are not great" (p. 253). While Harberger, Schwartzman, and Leibenstein consider deviations from the optimum represented by varying but often sizable divergencies of prices from costs, they apparently assume that the Hotelling formula still yields a good approximation. They could be right, but it is difficult to know that without further inquiry.

According to the Hotelling formula, furthermore, welfare losses resulting from a displacement from a competitive equilibrium, and hence optimum, amount in money terms to $\frac{1}{2}\sum dp_i dx_i$. Here dp_i and dx_i are the variations in p_i and x_i that the displacement entails. The summation is taken over all goods, though if, as here, one commodity is taken as numeraire, that necessarily drops out of the formula. Note, however, that dx_i is the change in the consumption of x_i that is induced by a change not only in the price of that good but in the prices of all other goods. Supposedly income also varies in a prescribed manner.[4] Hence complex demand effects of the sort I encountered above in the calculation of the NCV are also encountered here. As already indicated, such effects are nevertheless neglected by Harberger, Schwartzman, and Leibenstein.

Finally, Hotelling takes welfare losses to be represented in money terms by the quotient dU/α, where dU is the increment in utility and α is the marginal utility of money. For variations of any magnitude, however, welfare losses as so understood are unique only if the marginal utility of money is assumed constant. That was Marshall's assumption, but he would not have made it where prices are varying for products constituting a major part of the household's budget. Constancy in the marginal utility of money also entails a further assumption of cardinal measurability of utility, which Hotelling himself apparently preferred to avoid.[5]

[4] In his essay, Hotelling focuses on the impact of the introduction of excise in place of income taxes yielding the same total revenue. Since what counts is money income net of income taxes, the prescribed change in money income is evidently equal to the total revenue from the excise taxes, and so corresponds precisely to the MIA as defined here. As it turns out, that also means that the displacement from the optimum that Hotelling considers, like the one I consider, entails a shift along a plane in which consumption at predisplacement prices is constant. In my analysis, however, that occurs because of the assumption of a linear transformation locus. I find it rather puzzling, therefore, that Hotelling, pp. 255–56, apparently considers his formula as applying even where marginal costs are variable.

[5] Might we not, however, interpret α as representing simply the marginal utility of money at some intermediate point in the range in question, and so avoid in this way the assumption of constancy of the marginal utility of money? If we may judge from a recent discussion by Harberger (1971, pp. 786ff) of a formula similar to Hotelling's, he might now argue that α in the Hotelling formula be construed in just that way. But that apparently would mean that the term

$$(\alpha/2)\sum dp_i dx_i$$

which was derived as a lower order term in a Taylor's expansion, is nevertheless to be interpreted as the remainder term. How that may be done is not clear. Moreover, it is still disconcerting that the measure of welfare losses depends on an unknown point to which reference is made within the range in question. While Harberger applies the Hotelling formula without qualification, it should be observed that Schwartzman, p.

As seen by Harberger, Schwartzman, and Leibenstein, monopoly welfare losses evidently must turn primarily on two empirical factors: price elasticities of demand and price-cost ratios for monopolized products. That is still true according to the alternative methodology that I have elaborated, though Harberger, Schwartzman, and Leibenstein refer to own price elasticities, while as seen here cross and income elasticities also matter (and in the latter case, as seen from (7) and (9), not merely through the usual sort of income effects). It should be observed, therefore, that Harberger assumes throughout a price elasticity of unity for monopolized products. In his view,

> . . . one need only look at the list of industries . . . considered in order to get the feeling that the elasticities in question are probably quite low. The presumption of low elasticity is further strengthened by the fact that what we envisage is not the substitution of one industry's product against all other products, but rather the substitution of one great aggregate of products . . . for another aggregate. [1954, p. 79]

The assumption of a price elasticity of unity, however, was held "objectionable" by Stigler for the reason that "A monopolist does not operate where his marginal revenue is zero. . . . In any event, . . . most industries have long-run demand curves which are elastic" (p. 34). According to Schwartzman, Harberger's assumption is indeed to be thought of as applying to an industry rather than an individual firm: "Moreover, if we are interested in the value of resource misallocation by monopolistic industries as a group, the relevant demand elasticity is less than the average of the individual industry demand elasticities" (pp. 628–29). Schwartzman assumes the price elasticity of demand is no more than 2. For purposes of illustrative calculation, Leibenstein assumes that in monopolized industries the " . . . average elasticity of demand is 1.5" (pp. 395–96).

How elastic the demand for a composite of all monopolized commodities is in respect of price is an interesting question. But for purposes of appraising monopoly welfare losses, it is also not too relevant. All monopolized products could properly be treated here as a single composite product only if the prices of all such products exceed their competitive levels by relatively the same amount. So far as the facts are otherwise, welfare losses originate not only in inordinately high monopoly prices generally, but in the variation for different products in the relation of such prices to costs. Also, what counts is variation not only between industries in any conventional sense, but between products; even between different qualities or models of the same product (for example, different models of Buicks) so far as price-cost ratios vary here as well. In principle, therefore, welfare losses must be calculated by reference to the demand for each and every such monopolistic product, quality or model as the case may be. Only in that way can all such losses be allowed for.[6]

As to how elastic demand is for one or another product, quality or model, that

630, notes that he is neglecting variations in the demand for a product associated with changes in equilibrium magnitudes generally, as distinct from a change in the price of that product alone. He apparently considers such variations unimportant. Leibenstein does not refer to Hotelling, but he accepts Harberger's methodology. Leibenstein, p. 396, n. 3, refers, however, to the error due to neglect of income effects originating in the divergence of uncompensated from compensated demand schedules, and concludes that such effects are apt to be negligible. He may be right but as indicated income effects are seen here to be much more complex than he thus assumes.

[6] As explained, Harberger, Schwartzman, and Leibenstein all assume constant costs. In this essay, I have fallen in with that assumption, but note that so far as costs vary even uniform monopoly price-cost ratios would not mean that monopoly prices are all in the same relation to those that would prevail under competitive pricing.

must depend on the item. Presumably it also depends on the prices of other goods and household income. That is a matter of some importance, since in the calculation of gcv_i reference is to demand schedules which vary from item to item in respect of the levels at which prices of related goods and income are held constant. Any a priori assumption must be a gross oversimplification in this sphere, but I doubt that an elasticity of unity or even of 2 can be nearly large enough to encompass the interesting range of possibilities for the purposes of a calculation such as in question.

In this connection, it is pertinent to recall the relation familiar in monopoly theory that is alluded to by Stigler:

$$\eta_{xp}^{i} = p_i/(p_i - MC_i) \qquad (10)$$

Here η_{xp}^{i} is the price elasticity of demand for the ith product, p_i is its price and MC_i, its marginal cost. No one supposes that (10) can be more than very broadly applicable to real world monoplies, but Leibenstein, for example, assumes a monopoly price-cost ratio of 1.2. It seems illuminating that with that ratio (10) would imply an elasticity of demand as high as 6. Leibenstein considers that the monopolistic price-cost ratios must in fact average less than 20 percent, but for a ratio of, say 10 percent, (10) yields an elasticity of 11. To repeat, though, (10) hardly applies exactly in practice, and it may be just as well to note here that I myself will allow later for possible divergencies from it in either direction. This assumption is in order when we consider such aspects as the limitations in a firm's knowledge of its cost and demand schedules; the fact that because of price leadership, collusion and other adaptations to oligopolistic interdependence the firm may calculate in terms of a demand schedule that is less elastic than the one relevant here (i.e., that where prices of all other products, including even close substitutes, are unchanged), and so on.

What of the price-cost ratios? Harberger deduced these from data on profit rates for different industries. Stigler, pp. 34–35, held, however, that monopoly profits are often capitalized, and in diverse ways not always sufficiently allowed for by Harberger, with the result that the indicated price-cost ratios may often be too low. Still other forces could operate to produce the same result. As Harberger is aware, monopoly prices reflect not only monopoly profits but monopolistic advertising, but, even for the early period he considers, the resulting additional misallocation cannot be dismissed as he dismisses it simply on the ground that advertising outlays were ". . . well under 2 percent of sales for all the industries . . ." (1954, pp. 85–86) studied. The industries studied include competitive as well as monopolistic ones, so the corresponding ratio for monopolistic industries alone must be higher. What counts here, moreover, is not only average relations but their variation between industries and products.

Harberger treats intermediates as if they were consumers' goods. The relevant monopoly profits for any consumer's good, however, are not those prevailing in the industry in question, but those profits, together with the additional profits accruing on intermediate products used in that industry. That is so even though the concern is, as in the studies in question, only with misallocation between consumers' goods industries, and not with further losses due to monopolistic distortions in factor mixes. (I return to that later.) If, as a practical expedient, reference is to be made only to "direct" as distinct from "direct and indirect" monopoly profits, it should be closer to the mark to relate such profits to value-added rather than to sales, as Harberger does.

In his calculation Schwartzman seeks to determine the impact of monopoly on *U.S.* industrial prices from a comparison of ratios of sales and variable costs in similar concentrated and unconcentrated industries in the United States and Canada. In this way, the average "monopoly effect" is estimated at 8.3 percent of average variable costs. Schwartzman avowedly seeks to meet objections such as Stigler's to the Harberger calculation. To what extent he succeeds in doing so in his involved calculation is difficult to judge from his very brief exposition, and further questions such as were raised above regarding the Harberger calculation do not seem clearly disposed of. Schwartzman takes a step back from Harberger by assuming a uniform monopoly effect in all monopolized industries. He thus fails to allow for the misallocation due to variations in that effect between products. Monopoly price-cost ratios are a complex matter on which there will have to be many more studies before we can confidently narrow the possibilities.

I referred at the outset to a study of Kamerschen indicating that monopoly losses may be appreciably greater than Harberger and Schwartzman calculated. Kamerschen proceeds for the most part as Harberger did, so his findings too are difficult to construe, but he refers to profits data for a later period, 1956–57 to 1960–61. He also attempts to extend the calculations to embrace nonmanufacturing business and enterprises other than corporations, and to allow more systematically than Harberger could for capitalized monopoly profits and monopolistic distortions in costs in the form of intangible assets, royalties, and advertising expense. While intermediate products are again treated as final, however, monopoly profits are still related to sales rather than value added. Monopoly welfare losses are estimated at 1.03 to 1.87 percent of the national income depending on which of a number of variants is considered. These magnitudes, while modest, are, of course, far larger than Harberger's. They result from the assumption of a unitary elasticity of demand. Alternative calculations apparently assuming elasticities in different industries such as conform to (10) yield still much higher figures: 3.87 to 6.82 percent of national income.[7]

III. An Alternative Approach

In inquiring into the methodology of Harberger, Schwartzman, and Liebenstein, I have elaborated an alternative that is akin to theirs so far as monopoly welfare losses are still calculated from uncompensated demand schedules. But while the alternative methodology served as a basis for appraising that of Harberger, Schwartzman, and Leibenstein, it evidently must be difficult to apply when monopolized products are at all numerous. I now explore still another approach to the calculation of welfare losses due to monopoly pricing. Resting on the assumption of a special type of household indifference map, this approach can only yield hypothetical measures, but these may illuminate further what might already have been surmised: the marked sensitivity of the calculated welfare losses to the elasticity of demand for monopolized products and to the level and distribution of the monopoly price-cost ratios.

The household indifference map in question is given by equation (11):

$$U(x_1, \ldots, x_{n+1}) = \sum_{i=1}^{n+1} A_i x_i^{(1-1/\sigma)} \tag{11}$$

As before, I assume that there are $n+1$ industries producing a corresponding number of commodities, that of these industries one, $n+1$, is competitive and that all others are monopolistic. The output and

[7] I refer to calculations based on after-tax profits. Further calculations based on before-tax profits often yield even higher figures.

hence also the household's consumption of the ith good is x_i.

In (11), A_i is a constant, and so too is σ. If x_i stood for employment of a productive input rather than consumption, therefore, (11) would simply represent on the $x_1 \ldots x_{n+1}$ hyperplane a *CES* production function such as lately has become familiar in productivity analysis. The term σ would then represent the elasticity of substitution between any two factors, employment of all others being given.

I refer here nevertheless to consumption, but as Paul Samuelson, p. 787, has pointed out, the *CES* production function itself corresponds to a function of the same form that I derived long ago (1936, reprinted 1966) for a household's indifference map in an analysis of Ragnar Frisch's methods of measuring marginal utility. In any event, (11) may be conceived of as applying to consumption no less than to production, and for present purposes it has the distinct merit that by varying σ, now representing the elasticity of substitution between any two consumer's goods, we may allow different degrees of substitutability between products, and so ultimately for varying elasticities of demand for one or another of our monopolized goods. In the absence of empirical data on the comparative income elasticities of monopolized and competitive goods, it may be more of a virtue than a limitation of (11) that it implies unitary income elasticities of demand for all products alike.

While I shall refer to $U(x_1, \ldots, x_{n+1})$ as a utility function, our concern is only with the indifference map that the function defines. Where appropriate, utility might be envisaged as being represented by some function of $U(x_1, \ldots, x_{n+1})$, rather than by that function itself. The utility dimension, therefore, need not be representable by a linear homogeneous function such as is usually considered in productivity analysis. Even on that understanding, (11) is admittedly rather restrictive, but it still embraces an interesting range of possibilities. I use a function of the form of (11) here at the suggestion of Samuelson, who has also been helpful at other points.[8]

As before, the monopolized products, $x_1 \ldots x_n$, sell at prices $p_1 \ldots p_n$, which when fixed monopolistically have the values $p_1^m \ldots p_n^m$. The competitive good, x_{n+1}, serves as numeraire. I again designate by I^c the household's income as it would be if $x_1 \ldots x_n$ were to sell at competitive prices, $p_1^c \ldots p_n^c$, and there were no unemployment. Then

$$GCV = I^* - I^c \tag{12}$$

where

$$U(p_1^m, \ldots, p_n^m, I^*) = U(p_1^c, \ldots, p_n^c, I^c) \tag{13}$$

In other words, I^* is the income that the household would require if, when $x_1 \ldots x_n$ sell at $p_1^m \ldots p_n^m$, it is to enjoy the same utility as it would when $x_1 \ldots x_n$ sell at $p_1^c \ldots p_n^c$ and its income is I^c.

We also have

$$MIA = I^m - I^c \tag{14}$$

where I^m is the income that the household

[8] As he has also pointed out to me, the usual *CES* variant of (11), i.e., the one that is linear homogeneous, might itself be properly taken as a measure of real income. Moreover, for shifts along a linear production possibility schedule such as I have been assuming and will continue to assume here, real income as so understood and the net compensating variation turn out to be essentially the same metric. Thus, I express the *NCV* below as a coefficient, the decisive term being the ratio of two magnitudes of household income, one representing the income needed to assure, under monopoly pricing in monopolistic industries, the same utility as might have been enjoyed under competitive pricing in all industries, and the other representing simply the income needed to assure full employment under monopoly pricing in monopolistic industries. That ratio, as is not difficult to see, corresponds precisely to the ratio of real income in equilibrium under competitive pricing in all industries to real income in equilibrium under monopoly pricing in monopolistic industries.

must have if employment is to continue to be full when prices are $p_1^m \ldots p_n^m$ rather than $p_1^c \ldots p_n^c$. Finally,

$$(15) \quad NCV = (I^* - I^c) - (I^m - I^c) = I^* - I^m$$

While NCV is of interest, we may more readily relate our analysis to previous ones if I focus on a corresponding coefficient:

$$(16) \quad CNCV = (I^* - I^m)/I^m$$

In effect, this expresses the NCV as a fraction of the national income that would be produced in the full employment equilibrium under monopoly pricing.

If $x_i, i=1 \ldots n$, is measured in units such that $p_i^c=1$, production possibilities, which are again linear, are readily expressed in terms of a familiar formula convenient here:

$$(17) \quad \sum_{i=1}^{n+1} x_i = k$$

From (11), (16), (17) and the usual conditions for household equilibrium, it can be proven that:

$$(18) \quad CNCV = \left(\sum_{i=1}^{n+1} \gamma_i \lambda_i^{\sigma-1}\right)^{1/(\sigma-1)} \cdot \left(\sum_{i=1}^{n+1} \gamma_i \lambda_i^{-1}\right) - 1$$

Here, for $i=1 \ldots n$, λ_i is p_i^m when the unit of x_i is such that $p_i^c=1$, and also represents the monopolistic price-cost ratio. The term λ_{n+1} equals unity. Also,

$$(19) \quad \gamma_i = (p_i^m x_i^m)/I^m$$

or since $p_i^m=\lambda_i$

$$(20) \quad \gamma_i = \lambda_i x_i^m / I^m$$

where x^m represents the output of x_i when p_i, $i=1 \ldots n$ is fixed monopolistically, and there is full employment equilibrium, i.e., income is I^m. Thus γ_i is the income share that is accounted for in such an equilibrium by the household's expenditure on x_i. A proof of (18) is given in the Appendix.

From (18), the $CNCV$ depends on: (i) γ_i, the income share devoted to x_i, $i=1 \ldots n+1$, in the full employment equilibrium with monoply prices; (ii) λ_i, $i=1 \ldots n$, the monopoly price-cost ratio for each and every product, it being understood that λ_{n+1}, the price-cost ratio for the competitive product, is unity; and (iii) σ the elasticity of substitution. In order to see how the $CNCV$ might vary in dependence on these parameters, let us consider first the case where there is but one monopolistic product, x_1, and the competitive product is accordingly x_2. Our parameters, then, consist simply of γ_1, λ_1, and σ. In Table 1, I show in percentages the magnitudes of the $CNCV$ indicated by alternative hypothetical values of γ_1, λ_1, and σ. In all cases, γ_1 is taken equal to 0.5, so the household divides its income equally between the monopolized and competitive products. For λ_1, the monopoly price-cost ratio, I consider three alternative values: 1.1,

TABLE 1— VALUES OF THE COEFFICIENT OF NET COMPENSATING VARIATION ($CNCV$) FOR A TWO-GOOD ECONOMY AND ALTERNATIVE λ_1 AND σ

	$CNCV$[a]		
σ	$\lambda_1=1.1$	$\lambda_1=1.2$	$\lambda_1=1.3$
(1)	(2)	(3)	(4)
.50	.06	.21	.43
1.00	.11	.41	.86
2.00	.23	.83	1.73
4.00	.45	1.66	3.43
8.00	.90	3.20	6.38
16.00	1.70	5.47	9.95
32.00	2.85	7.58	12.46
64.00	3.86	8.80	13.74

[a] Shown in percent.

TABLE 2—VALUES OF THE COÊFFICIENT OF NET COMPENSATING VARIATION (*CNCV*) FOR A THREE-GOOD ECONOMY FOR ALTERNATIVE $M_m(\lambda)$, λ_1, λ_2, AND σ

	$CNCV^a$					
	$M_m(\lambda)=1.1$		$M_m(\lambda)=1.2$		$M_m(\lambda)=1.3$	
σ	$\lambda_1=1.05$ $\lambda_2=1.15$	$\lambda_1=1.05$ $\lambda_2=1.20$	$\lambda_1=1.1$ $\lambda_2=1.3$	$\lambda_1=1.1$ $\lambda_2=1.4$	$\lambda_1=1.15$ $\lambda_2=1.45$	$\lambda_1=1.15$ $\lambda_2=1.60$
	(1)	(2)	(3)	(4)	(5)	(6)
.50	.08	.10	.28	.33	.54	.62
1.00	.17	.20	.56	.70	1.11	1.29
2.00	.33	.42	1.15	1.43	2.33	2.81
4.00	.67	.88	2.43	3.18	4.98	6.48
8.00	1.41	1.97	5.15	7.39	10.43	15.09
16.00	2.93	4.46	9.82	15.10	17.93	27.08
32.00	5.36	8.46	14.34	21.85	23.38	35.01
64.00	7.50	11.54	16.94	25.47	26.21	39.03

[a] Shown in percent.

1.2, and 1.3. The elasticity of substitution, σ, varies between .50 and 64.[9]

The *CNCV* evidently varies widely in dependence on both these parameters. Thus, for $\lambda_1=1.1$, the *CNCV* varies from less than a tenth of a percent to 3.9 percent as σ varies from .50 to 64. For $\lambda_1=1.2$, the corresponding variation in *CNCV* is from two-tenths of a percent to 8.8 percent, and for $\lambda_1=1.3$, from four-tenths of a percent to 13.7 percent. For the single household economy in question, as indicated, reference in each case is in effect to the percentage relation of monopoly welfare losses to the national income. The *CNCV* is also affected by γ_1, though often not very markedly. Interestingly, depending on σ, a deviation of γ_1 in either direction from 0.5 may raise or lower the *CNCV*.[10]

Turning to many monopolized products, from the calculations just discussed, the *CNCV* evidently must depend on the average λ_i in monopolized industries. By merely increasing the number of monopolized products to two, we see that the *CNCV* is also sensitive to the distribution of λ_i about their mean.

Thus, in Table 2, I assume that there are two monopolized industries, x_1 and x_2, and one competitive one, x_3. The corresponding price-cost ratios are λ_1, λ_2, and λ_3, where $\lambda_3=1$. For the weighted mean of λ_1 and λ_2, we have

$$(21)\quad M_m(\lambda) = \left(\sum_{i=1}^{n} \gamma_i\lambda_i\right) \Big/ \left(\sum_{i=1}^{n} \gamma_i\right)$$

In the table, I consider, for $n=2$, three alternative values of $M_m(\lambda)$ corresponding to the three values of λ_1 that were considered previously, that is, 1.1, 1.2, 1.3. For each $M_m(\lambda)$ I refer to two variants, one in which λ_1 and λ_2 are distributed symmetrically about their mean, and the other in which their distribution about their mean is asymmetric, with λ_2 ex-

[9] Where $\sigma=1$, formula (11) gives way to the familiar Cobb-Douglas variant. In that case it is easy to show by reasoning such as is used to prove (18), that that formula is supplanted by:

$$CNVC = \left(\prod_{i=1}^{n+1} \lambda_i^{a_i}\right)\left(\sum_{i=1}^{n+1} a_i\lambda_i^{-1}\right) - 1$$

[10] For example, for $\lambda_1=1.2$, and σ taken in turn to be 2.0, 8.0, 16.0, and 46.0, I find that for $\gamma_1=.33$ the *CNVC* has these values: .74, 3.21, 6.19 and 11.37. For $\gamma_1=.67$, the corresponding figures are .74, 2.56, 4.04, and 5.98.

ceeding $M_m(\lambda)$ by more than λ_1 falls short of it. The specific λ_1 and λ_2 assumed in each variant are as indicated in the table. In all cases, γ_i for the competitive good—now γ_3—is 0.5, so as with the single monopolized good considered in Table 1, the two monopolized goods account for half the household's income. For the symmetric distributions, then, $\gamma_1=\gamma_2=0.25$. For the asymmetric distributions, $M_m(\lambda)$ is as assumed for $\gamma_1=.33$ and $\gamma_2=.17$.

With the inclusion of a second monopolized good in the economy the *CNCV* apparently increases markedly. That is so for all the assumed $M_m(\lambda)$ and for all the symmetric distributions of λ_1 and λ_2. It is even more so if λ_1 and λ_2 are distributed asymmetrically. For example, for $\sigma=8$, a monopoly price-cost ratio of 1.2 implied previously a *CNCV* of 3.2 percent. With a mean price-cost ratio for the two monopolized industries of 1.2, the *CNCV* now rises to 5.15 percent for the symmetric distribution. For the asymmetric distribution, the coefficient is still greater: 7.39 percent.

Further calculations assuming that the number of monopolistic products is indefinitely large but with price-cost ratios distributed in a more or less skew way, as might be expected, yield for a wide range of σ measures of the *CNVC* very similar to those obtained from the asymmetric distribution in Table 2.[11]

In selecting values of parameters for evaluation of the *CNCV*, I have tried to take account of related magnitudes considered in previous analyses, though I also refer to quite other possibilities as well, though perhaps not always very realistic ones. One of the parameters considered, however, is σ, the elasticity of substitution. That is, of course, not the same thing as the elasticity of demand considered in previous analyses, but for the utility function being considered the two aspects are related by a very simple formula:[12]

$$\eta^i_{xp} = \sigma - \gamma_i(\sigma - 1) \tag{22}$$

Here, η^i_{xp}, as before, is the negative of $(p_i/x_i)(\partial x_i/\partial p_i)$, the partial derivative being understood to have the usual sort of subscripts. Evidently, if $\sigma<1$, then $\eta^i_{xp}>\sigma$, but otherwise η^i_{xp} is no greater than σ, and is in fact less than σ whenever $\sigma>1$. The shortfall in the latter case is the greater the larger is σ and the larger is γ_i, the share of the commodity in question in the household's budget. While theoretically the divergence between η^i_{qp} and σ could thus be marked, presumably γ_i would tend to be small so far as there are many commodities. For practical purposes, therefore, η^i_{xp} should usually closely approximate σ, and my calculations may be read accordingly.

I have assumed that production possibilities are linear. That is more or less the counterpart of the assumption of constant costs made in previous analyses. Where costs are not constant but increasing, it has been held (see Harberger (1954, p. 82)) that other things equal, monopoly welfare losses will be less than where costs are constant. The appropriate translation of increasing costs here is a curvilinear production possibilities schedule that is concave from below. With such a schedule, the structural shift in outputs induced by

[11] The calculations assume as before that the competitive product accounts for one-half of income. The remaining half is distributed among monopolistic products on the assumption that the share of income represented by price cost ratios within a small interval $\Delta\lambda$ embracing any particular ratio λ is given by the product $\Delta\lambda$ and a linear function $\gamma(\lambda)=a\lambda+b$ which slopes downward to the right and reaches zero from above at $\lambda=h\equiv-(b/a)$. The constants a and b are evaluated so as to assure that $M_m(\lambda)$, the mean monopoly λ, corresponds to those considered previously: 1.1, 1.2, and 1.3. To cite one or two examples for $\sigma=8.00$ the *CNCV* for $M_m(\lambda)=1.1$ is 1.96; for $M_m(\lambda)=1.2$, 7.39; for $M_m(\lambda)=1.3$, 15.19.

[12] See the Appendix.

monopoly pricing is necessarily dampened, and one might suppose that as a result monopoly welfare losses would indeed be reduced. That could be so, but I believe it need not be. I leave this matter, however, to separate inquiry. Of course, so far as there is monopoly, we cannot at all exclude that in the vicinity of the equilibrium costs will be decreasing or that production possibilities will be convex from below.

I have also assumed that the elasticity of substitution is the same between any and all pairs of goods. As seen from (22) that need not mean that the elasticity of demand is also the same for all goods, but I have tacitly assumed that monopoly price-cost ratios might vary between goods independently of their price elasticities. As indicated, (10) can hardly be expected to apply systematically, but so far as it applies at all the price-cost ratios must in some degree vary inversely with price elasticities. Given that, one surmises that losses would be less than otherwise, but I leave this, too, to separate inquiry.

VII. Conclusions

I have sought to appraise calculations indicating that welfare losses due to monopoly pricing in a country such as the United States are relatively inconsequential, and have argued that those calculations are open to question on both conceptual and empirical grounds. Under an alternative approach, which does not seem subject to conceptual limitations affecting previous calculations, the coefficient of net compensating variation, which is taken to measure monopoly welfare losses, is quite sensitive to empirical aspects which still seem unsettled, particularly, the elasticity of demand for monopolized products, and the varying magnitudes of price-cost ratios for different monopolized products, together with the shares of household income that those products account for. While I have queried the conclusions of Harberger, Schwartzman, and Leibenstein, they have focused attention in a forceful way on an important question. That is a significant contribution in itself, from which nothing that has been said can detract.

I have joined previous writers in tacitly assuming that in the community considered, supplies of productive factors are given. No account has thus been taken of the adverse effect of monopolistic pricing on choices between leisure and work. The analysis can be extended in familiar ways to embrace such choices, however, so they seem to pose no new question of principle, though since leisure must be treated as a competitive good, its inclusion must affect the share of such goods in household income.[13]

I have also joined previous writers in focusing on welfare losses due to monopolistic pricing as that affects resource allocation as between consumer's goods. Monopolistic pricing, of course, also causes misallocation of resources at other points. Such further misallocation is properly the subject of a separate inquiry, but the calculations in question are sometimes taken to reflect monopoly welfare losses generally. Perhaps I should underscore, therefore, that whatever the losses due to misallocation between consumer goods, they are only compounded by misallocation caused by monopoly prices in other spheres, perhaps chiefly the determination of comparative factor proportions in different industries,[14] and the volume of saving and investment. Related to, but not the same thing as, misallocation due to monopoly pricing is the adverse impact of

[13] Monopolistic pricing also may have an adverse effect on effort, but at least in principle it is not difficult to extend the analysis to that aspect too.

[14] Harberger (1959) has tried to grapple with this matter, too, though reference is to distortions due to not only monopoly prices but other causes.

monopoly on the discovery and introduction of new technologies, though Schumpeter, of course, long ago made it a controversial matter whether monopoly is really always disadvantageous in this sphere. But the point being made is obvious and need not be labored: If consequential welfare losses due to monopoly are not precluded in respect of consumption structure, they hardly are so more generally.

Appendix

This appendix derives equations (18) and (22). To begin with (18), given (11) and that x_{n+1} is numeraire, we derive at once these equilibrium conditions for household consumption:

$$\text{(A1)} \quad U_i/U_{n+1} = (A_i/A_{n+1})(x_{n+1}/x_i)^{1/\sigma} = p_i, \qquad i = 1, \ldots, n$$

Here and elsewhere U_i, $i=1 \ldots n+1$, represents the marginal utility of x_i. Formula (A1) holds whether prices for monopolistic products, i.e., $x_i \ldots x_n$, are at competitive or monopolistic levels. But, given our choice of units for those products, $p_i^c=1$, $i=1 \ldots n$. Hence, when prices are at competitive levels and I is such as to assure full employment at those prices, the consumption of x_i and x_{n+1} conform to the relation:

$$\text{(A2)} \quad \overset{c}{x}_i = (A_i/A_{n+1})^{\sigma} \overset{c}{x}_{n+1}$$

From this and (17),

$$\text{(A3)} \quad \sum_{i=1}^{n+1} (A_i/A_{n+1})^{\sigma} \overset{c}{x}_{n+1} = k$$

It follows that

$$\text{(A4)} \quad \overset{c}{x}_{n+1} = kA_{n+1}^{\sigma} \Big/ \sum_{i=1}^{n+1} A_i^{\sigma}$$

and

$$\text{(A5)} \quad \overset{c}{x}_i = kA_i^{\sigma} \Big/ \sum_{i=1}^{n+1} A_i^{\sigma}, \qquad i = 1, \ldots, n$$

When prices of monopolistic products are at monopoly levels, $\lambda_1 \ldots \lambda_n$, and money income is I^m, which assures full employment at those prices, we see by similar reasoning that the consumption of x_{n+1} is:

$$\text{(A6)} \quad \overset{m}{x}_{n+1} = kA_{n+1}^{\sigma} \Big/ \sum_{i=1}^{n+1} \lambda_i^{-\sigma} A_i^{\sigma}$$

and of x_i,

$$\text{(A7)} \quad \overset{m}{x}_i = k\lambda_i^{-\sigma} A_i^{\sigma} \Big/ \sum_{i=1}^{n+1} \lambda_i^{-\sigma} A_i^{\sigma}, \qquad i = 1, \ldots, n$$

Consider now the still different equilibrium that prevails when prices of monopolistic products are at monopoly levels, but money income is at I^*, as given by (13). Proceeding again much as before, we find that consumption of x_i and x_{n+1} conform to:

$$\text{(A8)} \quad \overset{*}{x}_i = \lambda_i^{-\sigma} (A_i/A_{n+1})^{\sigma} \overset{*}{x}_{n+1}$$

In order to prove (18), I derive from the foregoing relations certain formulas for I^* and I^m. To refer first to I^*, we know that

$$\text{(A9)} \quad I^* = \sum_{i=1}^{n+1} \lambda_i \overset{*}{x}_i$$

Also, as seen from (13),

$$\text{(A10)} \quad U(\overset{*}{x}_1, \ldots, \overset{*}{x}_{n+1}) = U(\overset{c}{x}_1, \ldots, \overset{c}{x}_{n+1})$$

From this and (11),

$$\text{(A11)} \quad \sum_{i=1}^{n+1} A_i(\overset{*}{x}_i)^{(1-1/\sigma)} = \sum_{i=1}^{n+1} A_i(\overset{c}{x}_i)^{(1-1/\sigma)}$$

Using this, (A2), and (A8),

$$\text{(A12)} \quad \overset{*}{x}_{n+1} = \overset{c}{x}_{n+1} \left\{ \sum_{i=1}^{n+1} A_i^{\sigma} \right\}^{\sigma/(\sigma-1)} \Big/ \left\{ \sum_{i=1}^{n+1} \lambda_i^{1-\sigma} A_i^{\sigma} \right\}^{\sigma/(\sigma-1)}$$

and

$$\text{(A13)} \quad \overset{*}{x}_i = \lambda_i^{-\sigma} (A_i/A_{n+1})^{\sigma} \overset{c}{x}_{n+1} \cdot \left\{ \sum_{i=1}^{n+1} A_i^{\sigma} \right\}^{\sigma/(\sigma-1)}$$

$$\Big/ \left\{\sum_{i=1}^{n+1} \lambda_i^{1-\sigma} A_i^{\sigma}\right\}^{\sigma/(\sigma-1)}$$

From (A4), (A9), and (A13),

$$\text{(A14)} \quad I^* = k\left\{\sum_{i=1}^{n+1} A_i^{\sigma}\right\}^{1/(\sigma-1)} \Big/ \left\{\sum_{i=1}^{n+1} \lambda_i^{1-\sigma} A_i^{\sigma}\right\}^{1/(\sigma-1)}$$

Turning to I^m, we have

$$\text{(A15)} \quad I^m = \sum_{i=1}^{n+1} \lambda_i x_i^m$$

Using (A7),

$$\text{(A16)} \quad I^m = k\left\{\sum_{i=1}^{n+1} \lambda_i^{1-\sigma} A_i^{\sigma}\right\} \Big/ \left\{\sum_{i=1}^{n+1} \lambda_i^{-\sigma} A_i^{\sigma}\right\}$$

From (16), (A14), and (A16),

$$\text{(A17)} \quad CNCV = \left[\left\{\sum_{i=1}^{n+1} A_i^{\sigma}\right\}^{1/(\sigma-1)} \left\{\sum_{i=1}^{n+1} \lambda_i^{-\sigma} A_i^{\sigma}\right\} \Big/ \left\{\sum_{i=1}^{n+1} \lambda_i^{1-\sigma} A_i^{\sigma}\right\}^{\sigma/(\sigma-1)}\right] - 1$$

From (A7) and (A16),

$$\text{(A18)} \quad \gamma_i \equiv (\lambda_i x_i^m / I^m) = \left(\lambda_i^{1-\sigma} A_i^{\sigma} \Big/ \sum_{i=1}^{n+1} \lambda_i^{1-\sigma} A_i^{\sigma}\right)$$

Formula (18) follows from (A17) and (A18).

While it may have been useful to prove (18) in the foregoing manner, it should be observed that the same result can be obtained somewhat more quickly if it is considered that from (16) the *CNVC* reduces to $(I^*/I^m - 1)$. Also, as given by (11) the utility function is homogeneous. Hence, equilibrium consumption positions relating to different levels of money income but the same prices all must be on the same ray through the origin. That is true particularly of the equilibrium consumption positions corresponding to I^* and I^m. It also follows that the ratio I^*/I^m corresponds to the comparative consumption of any one good in the two equilibrium positions. Given that, (18) follows from (A13), (A7), (A4), and (A16).

To come to (22), from (A1),

$$\text{(A19)} \quad p_j^{\sigma}(\partial x_j/\partial p_i) - (A_j/A_{n+1})^{\sigma} \cdot (\partial x_{n+1}/\partial p_i) = 0, \qquad j \neq i,$$

and

$$\text{(A20)} \quad p_i^{\sigma}(\partial x_i/\partial p_i) - (A_i/A_{n+1})^{\sigma} \cdot (\partial x_{n+1}/\partial p_i) = -\sigma p_i^{\sigma-1} x_i$$

In these formulas, the derivatives shown are of a partial sort; with appropriate subscripts understood. Also, since

$$\text{(A21)} \quad \sum_{i=1}^{n+1} p_i x_i = I,$$

we have

$$\text{(A22)} \quad \sum_{j=1}^{n+1} p_j(\partial x_j/\partial p_i) = -x_i$$

From (A19), (A20), and (A22),

$$\text{(A23)} \quad (\partial x_{n+1}/\partial p_i) = (\sigma-1) A_{n+1}^{\sigma} x_i \Big/ \sum_{i=1}^{n+1} p_i^{1-\sigma} A_i^{\sigma}$$

And from this and (A20),

$$\text{(A24)} \quad (\partial x_i/\partial p_i) = \left\{(\sigma-1) A_i^{\sigma} x_i / p_i^{\sigma} \sum_{i=1}^{n+1} p_i^{1-\sigma} A_i^{\sigma}\right\} - \sigma x_i/p_i$$

or

$$\text{(A25)} \quad \eta_{xp}^{i} = \sigma - (\sigma-1) p_i^{1-\sigma} A_i^{\sigma} \Big/ \sum_{i=1}^{n+1} p_i^{1-\sigma} A_i^{\sigma}$$

Formula (22) follows at once from (A18) for monopoly equilibrium where $\lambda_i = p_i$, but it is evident that it must hold for any other equilibrium as well, for in (A18) λ_i could be construed as well to represent any p_i, and not merely a monopolistic one.

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