Elephants

By MICHAEL KREMER AND CHARLES MORCOM*

Many open-access resources, such as elephants, are used to produce storable goods. Anticipated future scarcity of these resources will increase current prices and poaching. This implies that, for given initial conditions, there may be rational expectations equilibria leading to both extinction and survival. The cheapest way for governments to eliminate extinction equilibria may be to commit to tough antipoaching measures if the population falls below a threshold. For governments without credibility, the cheapest way to eliminate extinction equilibria may be to accumulate a sufficient stockpile of the storable good and threaten to sell it should the population fall. (*JEL* Q20)

Most models of open-access resources assume that the good is nonstorable (H. Scott Gordon, 1954; M. B. Schaefer, 1957; Colin Whitcomb Clark, 1976). While this may be a reasonable assumption for fish, it is inappropriate for many other species threatened by overharvesting, as illustrated in Table 1. Although 30 percent of threatened mammals are hunted for presumably nonstorable meat, 20 percent are hunted for fur or hides, which are presumably storable, and approximately 10 percent are threatened by the live trade (Brian Goombridge, 1992).[1]

African elephants are a prime example of an open-access resource which is used to produce a storable good. From 1981 to 1989, Africa's elephant population fell from approximately 1.2 million to just over 600,000 (Edward B. Barbier et al., 1990). Dealers in Hong Kong stockpiled large amounts of ivory (Jane Perlez, 1990). As the elephant population decreased, the constant-dollar price of uncarved elephant tusks rose from \$7 a pound in 1969 to \$52 per pound in 1978, and \$66 a pound in 1989 (Randy T. Simmons and Urs. P. Kreuter, 1989). These higher prices presumably increased incentives for poaching.

Since the late 1980's, governments have toughened enforcement efforts, with a ban on the ivory trade, shooting of poachers on sight, strengthened measures against corruption of game wardens, and the highly publicized destruction of confiscated ivory.[2] This crackdown on poaching has been accompanied by decreases in the price of elephant tusks (Raymond Bonner, 1993), as well as a revival of the population. Since these policy changes reduce short-run ivory supply as well as demand, it is not clear that the fall in price would have been predicted under a static model, and indeed most economists did not predict this decline. However, the fall in price is consistent with the dynamic model set forth in this paper, under which improved antipoaching enforcement may increase long-run ivory supply by allowing the elephant population to recover.

Under the model, anticipated future scarcity of open-access storable resources leads to higher current prices, and therefore to more

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[1] The others are threatened by factors aside from overharvesting, such as loss of habitat.

[2] In September 1988, Kenya's president ordered that poachers be shot on sight, and in April 1989 Richard Leakey took over Kenya's wildlife department.

TABLE 1—SOME SPECIES USED FOR STORABLE GOODS, OR BY COLLECTORS

Bears	**Lizards**	**Medicinal Plants**
Giant Panda	Horned Lizard	species of *Dioscorea*
Asiatic Black Bear	L. A. Spectacled Caiman	species of *Ephedra*
Grizzly Bear	Common Caiman	*Dioscorea deltoidea*
S. A. Spectacled Bear	Tegus Lizard	*Rauvolfia serpentina*
Malayan Sun Bear	Monitor Lizard	*Curcuma spp.*
Himalayan Sloth Bear	**Snakes**	*Parkia roxburghii*
Cats	Python	*Voacanga gradifolia*
Tiger	Boa Constrictor	*Orthosiphon aristasus*
Cheetah	Rat Snake	species of *Aconitum*
Lynx	Dog-faced Water Snake	**Trees**
Canada Lynx	Sea Snakes	*Astronium urundeuva*
Ocelot	**Butterflies**	*Aspidosperma polyneuron*
Little Spotted Cat	Schaus Swallowtail	*Ilex paraguaiensis*
Margay	Homerus Swallowtail	*Didymopanax morotoni*
Geoffroy's Cat	Queen Alexandra's Bird-wing	*Araucaria hunsteinii*
Leopard Cat	**Orchids**	*Zehyera tuberculose*
Other Mammals	*Dendrobium aphyllum*	*Cordia milleni*
Black Rhino	*D. bellatulum*	*Atriplex repanda*
Amur Leopard	*D. chrysotoxum*	*Cupressus atlantica*
Caucasian Leopard	*D. farmeri*	*Cupressus dupreziana*
Markhor Goat	*D. scabrilingue*	*Diospyros hemiteles*
Saiga Antelope	*D. senile*	*Aniba duckei*
Cape Fur Bull Seal	*D. thrysiflorum*	*Ocotea porosa*
Sea Otter	*D. unicum*	*Bertholetia excelsa*
African Elephant	**Rattan**	*Dipterix alata*
Chimpanzees	*Calamus caesius*	*Abies guatemalensis*
Toads	*C. manan*	*Tectona hamiltoniana*
Colorado River Toad	*C. optimus*	Mahogany
Turtles		**Other Plants**
Hawksbill Sea Turtle		Himalayan Yew
Egyptian Tortoise		
American Box Turtle		
Birds		
Red and Blue Lorry		
Parrots		
Quetzal		
Roseate Spoonbill		
Macaws		

Sources: Sean Kelly (1991, 1992); David Sanger (1991); John Balzar (1992); William Booth (1992); Goombridge (1992); *New York Times* (1992); Sharon Begley (1993); Robert Johnson (1993); Bill Keller (1993); Ian Mander (1993); Robert M. Press (1993); Lena Sun (1993); Paul Taylor (1993); John Ward Anderson (1994); Timothy Egan (1994); Laura Galloway (1994); *Life Magazine* (1994); Gautam Naik (1994); Bill Richards (1994); William K. Stevens (1994).

intensive current exploitation. For example, elephant poaching can lead to expected future shortages of ivory, and thus raise future ivory prices. Since ivory is a storable good, current ivory prices therefore rise, and this increases incentives for poaching today. Because poaching creates its own incentives, there may be multiple rational expectations paths of ivory prices and the elephant population.

In order to gain intuition for why there may be multiple rational expectations equilibria, it is useful to consider the following two-period example, for which we thank Martin L. Weitzman. Suppose that each year there is a breeding season during which population grows by an amount $B(x)$ given an initial population of x. Following the breeding season, an amount h is harvested. Denote the elephant population at the beginning of the harvest season in year one as x. Then the population at the end of the harvest in year one will be $x - h_1$, and the population at the end of the harvest in year two will be $x - h_1 + B(x - h_1) - h_2$. To keep the model as simple as possible, we assume that the world

TABLE 2—TIME LINE FOR TWO-PERIOD EXAMPLE

Time	Population
Initial (year 1)	x_0
After harvest, h_1, in year 1	$x_0 - h_1$
After breeding in year 2	$x_0 - h_1 + B(x_0 - h_1)$
After harvest, h_2, in year 2 (end of world)	$x_0 - h_1 + B(x_0 - h_1) - h_2$

ends after two years. Table 2 shows the time line.

Let c denote the cost of harvesting an animal, and denote the amount of the good demanded at a price of p as $D(p)$. Assume $D' < 0$ and $D(\infty) = 0$. The interest rate, which is assumed to be the only cost of storage, is denoted r.

There will be an equilibrium in which the animal is hunted to extinction in year 1 if the initial population is less than enough to satisfy demand during the first year at a price of c, plus demand during the second year at a price of $(1 + r)c$. Algebraically, this extinction condition can be written as: $x < D(c) + D((1 + r)c)$.

There will be an equilibrium in which the species survives if the initial population, minus the amount required to satisfy first-year demand at price c, plus the births in the breeding season, can more than satisfy second-period demand at price c. This will be the case if $x - D(c) + B(x - D(c)) > D(c)$.

If both conditions hold, then there will be both a survival equilibrium and an equilibrium in which the price is high enough that the population is eliminated in the first period, and the breeding that would have satisfied second-period demand never takes place. There will be multiple equilibria if the initial population is in the range $[2D(c) - B(x - D(c)), D((1 + r)c) + B(c)]$.

Note that as storage costs, r, rise, there will be an extinction equilibrium for a diminishing range of initial population levels. For sufficiently high storage costs, there will only be a single equilibrium path of population for any initial stock, just as in standard models of nonstorable fish.

The model may help explain the sudden destruction of bison populations in the nineteenth century. There had been a gradual acceleration in bison killings before 1870, but in the next four years, over four million bison were killed for their hides on the southern Great Plains alone, and by 1883, the bison were nearly extinct. This followed an improvement in the tanning process for buffalo hides, which presumably increased their storability.

In the example above, we assume that the good was destroyed when it was consumed. For example, rhino horn is consumed in traditional Asian medicines. Multiple equilibria can also arise for durable goods, which are not used up when they are consumed, as long as either the good depreciates, or demand for the good grows over time. Both conditions are often fulfilled: ivory yellows with age, and pieces break or are lost, and rapid population and income growth in East Asia are increasing demand for goods made from endangered species. In a previous, unpublished version of the paper, we derive conditions for multiple equilibria in a two-period model with durable goods.

In any case, in practice, few goods are completely durable. For example, ivory is often considered an example of a durable good, but new and old ivory are not perfect substitutes, since ivory yellows with age, and there is constant demand for uncarved ivory for personalized seals. To the extent that there is demand for new ivory, there may be multiple equilibria in the absence of demand growth or depreciation.

In the remainder of the paper we use a continuous time, infinite-horizon model, which allows us to solve for steady-state population and prices; to examine cases in which extinction is not immediate following a shift in expectations, or the path of population and prices is stochastic; and to examine policy. We will focus on the case of goods which are storable, but not durable, such as rhino horn, but, except for the analysis of stockpiles in Section VI, the intuition should carry over to the case of durable goods as well.

We focus on the case of a purely open-access resource. However, we also discuss the case in which it becomes profitable to protect the resource as private property at a sufficiently high price. It is expensive to protect elephants as private property, since they naturally range over huge territories and ordinary fences cannot contain them (Bonner, 1993). However, in a few parts of Africa, with proximity to tourist facilities, it has become profitable to protect elephants as private property. If a species can be protected as private property above a certain price, then there may be one equilibrium in which the species survives as a plentiful open-access resource at a low price, and another equilibrium in which it survives only as a scarce private resource at a high price.

The model carries several policy implications. Under the Gordon-Schaefer model, if the population is steady, or rising, the species will survive. In contrast, this model suggests that a species with stable or rising population could still be vulnerable to a switch to an extinction equilibrium.

The model suggests that expectations of future conservation policy influence current poaching equilibria. Announcements that the government will permanently toughen antipoaching enforcement in the sufficiently distant future may lead to a rush to poach now, and even the extinction of the species. Governments may be able to eliminate the extinction equilibrium, and thus coordinate on the high population equilibrium, merely by credibly promising to implement tough antipoaching measures if the population falls below a threshold. This provides a potential justification for laws which mandate protection of endangered species with little or no regard to cost.[3] If the commitment is credible, the government will never actually have to spend the resources to increase antipoaching enforcement.

Some governments, however, may not be able to credibly commit to protect endangered species. In the case of animals used to produce nondurable but storable goods, it may be possible to eliminate extinction equilibria by building sufficient stockpiles of the storable good, and threatening to sell the stockpile if the animal becomes endangered or the price rises beyond a threshold. This is somewhat analogous to central banks using foreign-exchange reserves to defend an exchange rate.

Several previous papers find multiple equilibria in models of open-access resources with small numbers of players (Kelvin Lancaster, 1973; David Levhari and Leonard J. Mirman, 1980; Jennifer F. Reinganum and Nancy L. Stokey, 1985; Alain Haurie and Matti Pohjohla, 1987; Jess Benhabib and Roy Radner, 1992). In these models, each player prefers to grab resources immediately if others are going to do so, but to leave resources in place, where they will grow more quickly if others will not consume them immediately. Aaron Tornell and Andres Velasco (1992) introduce the possibility of storage into this type of model. Gerard Gaudet et al. (1998) examine the case of nonrenewable resources such as a common pool of oil. This paper is also related to those of Vernon L. Smith (1968), who sets forth a dynamic fisheries model, and of Peter Berck and Jeffrey M. Perloff (1984), who explore rational expectations in an open-access fishery.

The effects examined in the previous papers are unlikely to lead to multiple equilibria if there are many potential poachers, each of whom assumes that his or her actions have only an infinitesimal effect on future resource stocks, and on the actions chosen by other players. In contrast, this paper argues there may nonetheless be multiple equilibria for open-access renewable resources used in the production of storable goods, because if others poach, the animal will become scarce, and this will increase the price of the good, making poaching more attractive.

Because poaching transforms an open-access renewable resource into a private exhaustible resource, this paper can be seen as helping unify the Gordon-Schaefer analysis of open-access renewable resources with the Harold Hotelling (1931) analysis of optimal extraction of private nonrenewable resources.

The remainder of the paper is organized as follows. Section I lays out an analogue of the Gordon-Schaefer fisheries model which allows for storage. Section II uses zero-profit conditions in poaching and storage to derive *local equilibrium conditions* on the possible rational expectations equilibrium paths. Section III uses the local equilibrium conditions to derive differential equations that apply during those portions of equilibrium paths in which poaching takes place at a finite, but positive, rate. It then represents these subpaths using phase diagrams

[3] Note, however, that this would not provide a justification for why these laws would apply to species used to produce nonstorable goods, or species threatened by causes other than overharvesting, such as habitat destruction.

in population-stores space. Section IV examines how, given arbitrary initial population and stores, the system can reach these subpaths via an instantaneous initial cull or a period in which there is no poaching. Section V examines stochastic rational expectation paths. Section VI discusses policy implications and directions for future work.

I. A "Fisheries" Model with Storage

This section introduces the possibility of storage into a Gordon-Schaefer type model of open-access resources. We assume that the cost of storage is a pure interest cost, with rate r, and that there is free entry into storage, so that storage yields zero profits.

A. *The Animal Population*

We model the population following the standard Gordon-Schaefer model, as set forth and developed by Clark (1976),

$$\frac{dx}{dt} = B(x) - h, \tag{1}$$

where x denotes the population, h is the harvesting rate, and B, the net births function, is the rate of population increase in the absence of harvesting. $B(0) = 0$, since if the population is extinct, no more animals can be born. We assume that given the available habitat, the population has some natural carrying capacity, beyond which deaths would exceed births even in the absence of harvesting. We will measure the population in units of carrying capacity, so $B(1) = 0$, and $B(x)$ is strictly negative for $x > 1$. B is strictly positive if population is positive and less than 1. This implies that, without harvesting, the unique stable steady state for the population is 1. We assume that B is continuously differentiable.

B. *Poaching*

The rate of harvest will depend on the demand and the marginal cost faced by poachers. The marginal cost of poaching, c, is a decreasing, continuously differentiable, function of the population x, so that $c = c(x)$, with $c'(x) < 0$. We assume that $c'(x)$ is bounded and that there is a maximum poaching marginal cost of c_m, so that $c(0) = c_m$.[4]

C. *Consumer Demand*

Given price, p, consumer demand is $D(p)$, where D is continuous and continuously differentiable, decreasing in p, and zero at and above a maximum price p_m. We will restrict ourselves to the case in which $p_m > c_m$, so that some poaching will be profitable, no matter how small the population. This condition is necessary for extinction to be a stable steady state.

D. *Private Property*

Although most of the analysis is concerned with the case of a purely open-access resource, it is also possible to examine the case in which it becomes profitable to protect the animal as private property at a high price. We will occasionally consider the case in which there are a few animals in zoos, which are never harvested, and at some price π, where $c_m < \pi < p_m$, it becomes profitable to breed and protect these animals as private property.[5] We assume that an unlimited amount of the resource can be produced at this price.

E. *The Benchmark Model Without Storage*

It is useful to first consider the benchmark case in which the good cannot be stored. In this case, since the good is open access, its

[4] Note that we assume that the marginal cost of poaching is a function only of the population, and not of the instantaneous rate of harvest. In a previous version of the paper, we showed that multiple equilibria could also arise if the marginal cost of poaching depended on instantaneous rate of poaching, rather than on the population.

[5] We assume that any privately grown animals are bred from a stock in zoos in order to avoid considering how demand for live animals to save for breeding stock would affect the equilibrium in the model.

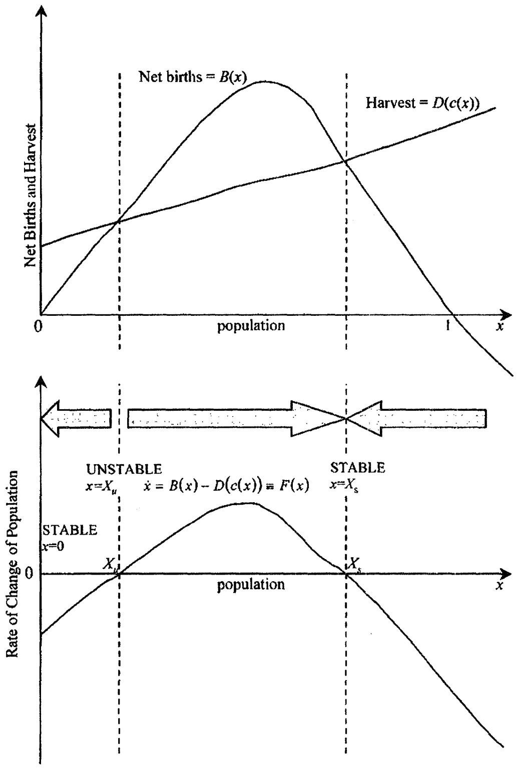

FIGURE 1. DYNAMICS OF THE GORDON-SCHAEFER MODEL WITH NO STORAGE

price must be equal to the marginal poaching cost. Algebraically, $p = c(x)$, where x is the open-access population. Moreover, the harvest must be exactly equal to consumer demand, so $h = D(c(x))$. The evolution of the system in which storage is impossible is thus described by:

$$\text{(2)} \qquad \frac{dx}{dt} = B(x) - D(c(x)) \equiv F(x).$$

Since $B(0) = 0$, and $p_m > c_m$, $D(c(0)) > 0$, so that $F(0) < 0$, as illustrated in Figure 1. Thus, zero is a stable steady state of equation 2. $F(1) < 0$ since $B(1) = 0$, and $D(c(1)) > 0$. (If π is greater than p_m, the species will become extinct, whereas if $p_m > \pi > c_m$, then the animal will become extinct as an open-access resource but a small stock of the resource will be preserved as private property.) We will consider the case in which F is positive at some point in (0, 1), so that extinction is not inevitable. Assuming that F is single peaked, there will generically be points X_S and X_U so that F is negative and increasing on $(0, X_U)$, positive on (X_U, X_S), and negative and decreasing on $(X_S, 1]$.[6] Hence, if population is between 0 and X_U, it will go to zero, whereas if it starts above X_U, it will tend to the high steady state, X_S. Thus, if storage is impossible, there will be multiple steady states, but a unique equilibrium given initial population.

II. Local Equilibrium and Feasability Conditions

We will look for rational expectations equilibria, or paths of population, stores, and price. (We focus on perfect foresight equilibria, but briefly consider stochastic rational expectations equilibria in Section V.) We show that although the possibility of storage does not affect the steady states of the system,[7] it dramatically alters the equilibrium transition paths to those steady states, sometimes creating multiple equilibria leading to different steady states.

Any rational expectations equilibrium path must satisfy the following local equilibria and feasibility conditions set forth below.

A. *The Storage Condition*

As in Hotelling (1931), free entry into storage implies that

$$\frac{dp}{dt} \begin{cases} = rp & \text{if } s > 0, \\ \leq rp & \text{if } s = 0, \end{cases}$$

where s denotes the amount of the good that is stored. People will not hold stores if the price rises less quickly, and if the price were rising

[6] Single-peakedness implies a unique positive stable steady state. Our propositions can be generalized to cover much more general models in which there are many stable steady states, or extinction is not stable.

[7] The stable steady states actually comprise the entire stable limit set of the system with storage (i.e., there are no cycles or chaotic attractors).

more quickly, people would hold on to their stores, or poach more.

B. *The Poaching Condition*

Free entry into poaching implies that the price of the good must be less than or equal to the marginal cost of poaching another unit of the good. The "poaching condition" is therefore:

$$p\begin{cases} = c(x), & \text{if there is poaching} \\ < c(x), & \text{if there is no poaching} \end{cases}$$

where x is the open-access population. We will call the storage and poaching conditions *slack* if stores and poaching respectively equal zero.

In addition to the local equilibrium conditions above, there are some feasibility conditions, as described below.

C. *"Conservation of Animals"*

At all times, the increase in stores plus the increase in population must equal net births minus the amount consumed, or

$$\dot{s} + \dot{x} = B(x) - D(p). \tag{3}$$

Note that this condition applies to nondurable goods, such as rhino horn, which are destroyed when they are consumed, rather than durable goods, such as ivory. Note also that we assume that animals which die naturally cannot be turned into the storable good.[8]

Finally, both population, x, and stores, s, must be nonnegative at all times on a feasible path.

The above conditions imply that, once on a rational expectations equilibrium path, population, stores, and price must be a continuous function of time, as demonstrated in the Appendix, Proposition A1. To see the intuition, note that jumps up in price would be anticipated and arbitraged. This implies that population cannot be anticipated to jump down. As we discuss below, there may be an initial jump down to get to the equilibrium path. The underlying biology does not allow population to jump up.

Since the storage and poaching conditions can each either be satisfied with equality, or with inequality, there are four possible ways that the equilibrium conditions can be satisfied. We call each of these a subpath. The four subpaths are: Poaching Without Storage, Poaching and Storage, Storage Without Poaching, and Neither Storage nor Poaching.

D. *The Poaching Without Storage Subpath*

In this subpath, the zero-profit condition for poaching implies that $p = c(x)$. The storage condition restricts the rate at which the price can rise and not induce storage ($\dot{p} \leq rp$). Because the price is inversely related to the population, it is possible to translate this condition that prices may not rise too fast into a condition that the population may not fall too fast: taking logarithms of $p = c(x)$ and differentiating with respect to time implies that if $\dot{p} < rp$, then

$$\frac{dx}{dt} \geq r\,\frac{c(x)}{c'(x)}. \tag{4}$$

In the Poaching Without Storage Subpath, the dynamics are the same as in the standard Gordon-Schaefer model, in which storage is impossible:

$$\dot{x} = B(x) - D(c(x)) \tag{5}$$

$$s = 0$$

$$p = c(x).$$

E. *The Poaching and Storage Subpath*

Since in this subpath, stores are positive and there is poaching, $dp/dt = rp$, and $p = c(x)$. Here, the exponential path of the price translates into a differential equation for population: taking logarithms of $p = c(x)$ and differentiating with respect to time implies that if $\dot{p} = rp$, then $\dot{x} = rc(x)/c'(x)$. Given the path of population and, hence, price and consumption, the dynam-

[8] We write the conservation condition as an equality. Because the price is positive, no one would throw the good away voluntarily.

ics of stores are determined by "conservation of animals," $\dot{s} = B(x) - D(p) - \dot{x}$, and we can express all the local equilibrium dynamics in terms of the population, x:

$$(6)\qquad \dot{x} = r\,\frac{c(x)}{c'(x)}$$

$$\dot{s} = B(x) - D(c(x)) - \dot{x}$$

$$\dot{p} = rc(x).$$

As discussed in Section IV, we will also need to determine starting values for x, s, and p, but for now we will focus on laws of motion, rather than boundary conditions.

F. *The Storage Without Poaching Subpath*

In this subpath, the rate of change of population is just the net birth rate, since there is no poaching. All demand is being satisfied from stores, so stores must be falling at a rate equal to instantaneous demand. For stores to be positive, price must be rising exponentially at rate r. The dynamics can thus be summarized by:

$$(7)\qquad \dot{x} = B(x)$$

$$\dot{s} = -D(p)$$

$$\dot{p} = rp.$$

Note that since there is no poaching, $p \leq c(x)$.

G. *Neither Storage nor Poaching*

If the open-access population is positive, there must be either storage or poaching, since if demand is being satisfied neither by stores nor poaching, the price will be greater than c_m, the maximum marginal cost of poaching. If the open-access population is zero, demand may be satisfied from private farms if $\pi < p_m$.

H. *Steady States*

To be in steady state, defined as a situation in which population, stores, and prices are all constant, stores must be zero because, if stores are positive, price must be rising exponentially. If stores are zero, then the system will have the same steady states as in the case in which storage is impossible, i.e., $x = 0$ and X_S. This implies that there are only two stable steady states: what we will call the "high steady state," in which the open-access population is X_S, stores are zero, and price is $c(X_S)$; and extinction as an open-access resource (which for convenience, we will refer to as the extinction equilibrium), in which population and stores are zero.

III. Dynamics Within Subpaths with Poaching

In order to solve for the equilibrium paths, we will first look at equilibrium subpaths, and then examine the circumstances under which an equilibrium path can move from one subpath to another subpath. We will begin by looking at the two subpaths in which there is poaching: Poaching Without Storage and Poaching and Storage.

A. *The Poaching Without Storage Subpath and the Poaching and Storage Subpath*

For the system to be in the Poaching Without Storage Subpath, people must not want to hold positive stores, so the price must not be rising faster than rp. Since the price is determined by the population, $p = c(x)$, the storage condition implies that the population cannot fall too fast. Specifically, from equations (4) and (5),

$$(8)\qquad B(x) - D(c(x)) \geq r\,\frac{c(x)}{c'(x)},$$

when $s = 0$.

As is clear from Figure 2, equation (8) will hold if and only if $x \in [X_U^*, X_S^*]$, where X_S^* and X_U^* are the two critical points at which the storage condition is just binding, i.e., $B - D = rc/c'$ (or $X_U^* = 0$ if it never binds). Moreover, $0 \leq X_U^* < X_U < X_S < X_S^*$. We will only consider the case $X_U^* > 0$ in what follows.

If the system starts with population in $(X_U, X_S^*]$ and no stores, then it is an equilibrium to follow the Poaching Without Storage Subpath dynamics to X_S, the stable steady state. If the system starts with no stores and a population of

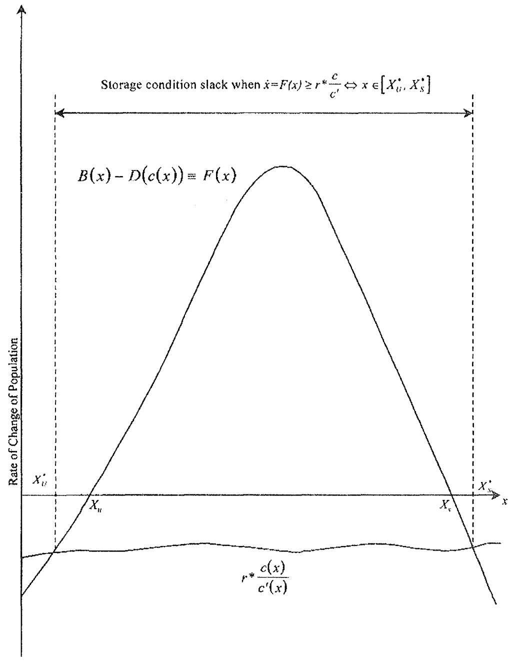

FIGURE 2. THE STORAGE CONDITION IN THE NO-STORAGE REGIME
DEFINITION OF X_U^* AND X_S^*

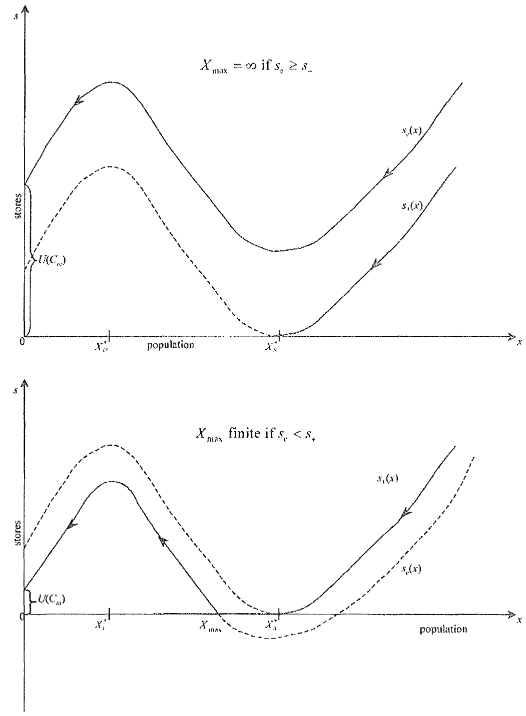

FIGURE 3. STORAGE REGIME EQUILIBRIUM PATHS, X_{MAX}

exactly X_U, the unstable steady state, the system will stay there. Here, as elsewhere, for the sake of clarity, we shall not discuss measure zero cases like this in any detail.

If the system starts with no stores and with population in (X_U^*, X_U), then the Poaching Without Storage dynamics will eventually take population to a point less than X_U^*. At some point, therefore, the system must leave the Poaching Without Storage Subpath and enter the Poaching and Storage Subpath. We discuss this after we have found the equilibrium Poaching and Storage Subpath.

B. *The Poaching and Storage Subpath*

In the Poaching and Storage Subpath, the dynamics of population are determined by the price, which is rising exponentially. The dynamics of stores are determined by "conservation of animals": what is harvested and not consumed must be stored. We may rewrite equation (6) as an equation for the phase trajectory of stores, s, in terms of x:

$$(9) \quad \frac{ds}{dx} = \frac{c'(x)}{rc(x)} \left\{ B(x) - D(c(x)) - r \frac{c(x)}{c'(x)} \right\}.$$

dx/dt is still just $rc(x)/c'(x)$, which is strictly negative, and bounded above.

Equation (9) implies that rational expectations trajectories in population-stores space must have stores increasing as a function of population, x, if $x < X_U^*$, or $x > X_S^*$. Stores must be a decreasing function of population if $x \in (X_U^*, X_S^*)$. There is a maximum of stores at X_U^*, and a minimum at X_S^*. Thus there will be a Poaching and Storage Subpath of the type depicted in Figure 3, in which for populations greater than X_S^*, population and stores decline; for population $\in (X_U^*, X_S^*)$ population rises and stores decline; and for population less than X_U^*, population and stores decline. To see the intuition for this, note that if population is very high or very low, population would tend to fall rapidly without stores, and as may be seen from Figure 2, in the absence of stores, population would fall rapidly enough that price would be rising faster than rate r. In order to prevent population from falling too rapidly to satisfy the storage condition, part of demand must be satisfied out of stores, which implies that there must be stores and that stores must decrease with time. X^*_U and X^*_S are the points at which, in the absence of stores, the population would fall just fast enough that price would rise at rate r. Between X^*_U and X^*_S, the price would rise more slowly than rate r with no storage. Therefore, on a subpath with stores in this population range, more than current demand must be being harvested and stores must increase to make the population fall fast enough so that price rises at exactly rate r.

Equation (9) is the differential equation for the trajectories of equilibria in population-stores space while in the Poaching and Storage Subpath. To tie down the equilibria, we need boundary conditions. One possibility is that stores run out while population is still positive, and the system enters the Poaching Without Storage Subpath. The only place at which this can possibly happen is where population is exactly X^*_S. To see why, consider the following: to be in the Poaching Without Storage Subpath, $x \in [\chi^*_U, \chi^*_S]$. Because population, stores, and price are continuous in equilibrium, the system must leave the Poaching and Storage Subpath at the same point at which it enters the Poaching Without Storage Subpath. As explained above, stores are *decreasing* as a function of x, so strictly *increasing* as a function of time (x is falling) if $x \in (X^*_U, X^*_S)$, and at a maximum at $x = X^*_U$. But stores have to run out at the point of transition from the Poaching and Storage Subpath to the Poaching Without Storage Subpath, so stores must have been falling, (or at least not increasing or at a maximum) immediately before the transition. The only point remaining at which stores could run out is, therefore, X^*_S.

The other possible boundary condition is that open-access population becomes extinct before stores run out. Since x is decreasing at a rate which is bounded below while stores are positive, the population must become extinct in finite time if stores do not run out. After that, stores will be consumed until they reach zero as well.

PROPOSITION 1: *If the initial population is at least $U(c_m)$, (defined below) then along a rational expectations path in which the population becomes extinct, the quantity of stores remaining when the population becomes extinct is*

$$\int_0^{(1/r)\ln(\min\{p_m,\pi\}/c_m)} D(c_m e^{rt})\, dt, \tag{10}$$

where π is the price at which it becomes profitable to protect the resource as private property.

PROOF:

If $p_m < \pi$, then the price charged for the last unit of stores must be p_m, or a storer would profit by waiting momentarily to sell his or her stock. Zero profits in poaching imply that along a rational expectations path leading to extinction, the price when the population becomes extinct must be $c(0) = c_m$. Price is rising exponentially while stores are positive, so the amount consumed from the time when price is c_m until price reaches p_m is:

$$U(c_m) = \int_0^{(1/r)\ln(p_m/c_m)} D(c_m e^{rt})\, dt.$$

If $c_m < \pi < p_m$, so that in the absence of poaching there would still be demand for the good from private sources, then, if the open-access population becomes extinct, eventually the price for the good must be equal to the cost of private production, π. Since the price would then be constant, it cannot be worthwhile to store the good. As above, stores must run out precisely at the point where $p = \pi$. Therefore stores at the time the open-access good becomes extinct must be $\int_0^{(1/r)\ln(\pi/c_m)} D(c_m e^{rt})dt < U(c_m)$.

As we show below, the smaller $U(c_m)$, the harder it is to sustain an extinction equilibrium. Thus the possibility of cultivating the animal as private property makes extinction less likely. If, however, the cost of producing the good privately is high enough, there will still be an equilibrium in which the open-access population is extinct.

We have shown that there can only be two equilibrium Poaching and Storage Subpaths (see Figure 3).

1. *High Steady-State Storage Equilibrium* —In this equilibrium, population starts at $x > X_S^*$. The system evolves until stores run out when population is X_S^*, and then enters the Poaching Without Storage Subpath. The equations $p = c(x)$, and $dp/dt = rp$ determine the path of population and price. Stores are given by $s = s_+(x)$, where:

$$(11)\quad s_+(x) = \int_{X_S^*}^{x} \frac{c'(q)}{rc(q)} \left\{ B(q) - D(c(q)) - r\frac{c(q)}{c'(q)} \right\} dq.$$

2. *Extinction Storage Equilibrium.*—In this equilibrium, population becomes extinct, and at that moment, stores $= U(c_m)$. The equations $p = c(x)$ and $dp/dt = rp$ determine the path of population and price. Stores are given by $s = s_e(x)$, where:

$$(12)\quad s_e(x) = U(c_m) + \int_{0}^{x} \frac{c'(q)}{rc(q)} \left\{ B(q) - D(c(q)) - r\frac{c(q)}{c'(q)} \right\} dq.$$

For this to be an equilibrium, stores must stay positive at all times along this path. If stores would have to become negative at some point in the future, this path is not an equilibrium. If $s_e(x)$ is ever negative, we define $X_{\max}$ to be the smallest positive root of $s_e(x)$. If there is none such, we say that $X_{\max} = \infty$. To be an equilibrium, the starting population must be less than $X_{\max}$.

$s_e(x)$ and $s_+(x)$ are parallel. Both have a minimum at X_S^*. It is clear from Figure 3 that $X_{\max}$ is finite if and only if $s_e(x)$ lies below $s_+(x)$. If $X_{\max}$ is finite, it must lie between X_U^* and X_S^*.

C. *Transitions from the Poaching Without Storage Subpath to the Poaching and Storage Subpath*

We now examine under which circumstances an equilibrium path can move from the Poaching Without Storage Subpath to the Poaching and Storage Subpath. If the initial population is small enough, an equilibrium path can move to the Poaching and Storage Subpath and thence to extinction. If $X_U^* > 0$ and the system starts in the Poaching Without Storage Subpath with population less than X_U, then the system must eventually move to the Poaching and Storage Subpath because if it did not, the population would fall fast enough to violate the storage condition once population was less than X_U^*. If the system starts with zero stores and population greater than X_U but less than $X_{\max}$, then it can go to the high steady state via the Poaching Without Storage Subpath, or to extinction with stores. By continuity of stores, the system can only make the transition from the Poaching Without Storage to the Poaching and Storage Subpath where $s_e(x) = 0$, i.e., at $X_{\max}$.[9] If $X_{\max} \in [X_U^*, X_S^*]$, then the system can move to the Poaching and Storage Subpath s_e leading to extinction. At such a transition, the rates of change of population, stores, and price will jump, but the storage and poaching conditions are not violated, because the levels will not jump.

D. *Summary*

We may thus define two sets of points in the Poaching and Storage Subpath and the Poaching Without Storage Subpath, A_e, the set of points leading to extinction, and A_+, the set of points leading to the high steady state, as illustrated in Figure 4. The top panel shows the case when $X_{\max} = \infty$. In this case, there will be a set of points A_e leading to extinction, along a Poaching and Storage Subpath, from any initial population level. For populations between X_U and X_S^* there will be a Poaching Without Storage Subpath leading to the steady state X_S, and for populations greater than X_S^*, there will be a Poaching and Storage Subpath leading to this Poaching Without Storage Subpath, and from there on to the stable steady state X_S.

The second panel illustrates the case when $X_U \leq X_{\max} \leq X_S$. In this case the set of points leading to survival, A_+, is the same as in the top

[9] The transition cannot happen at X_S^*, because the population would be falling there in the Poaching Without Storage Subpath, so the system could never reach that point.

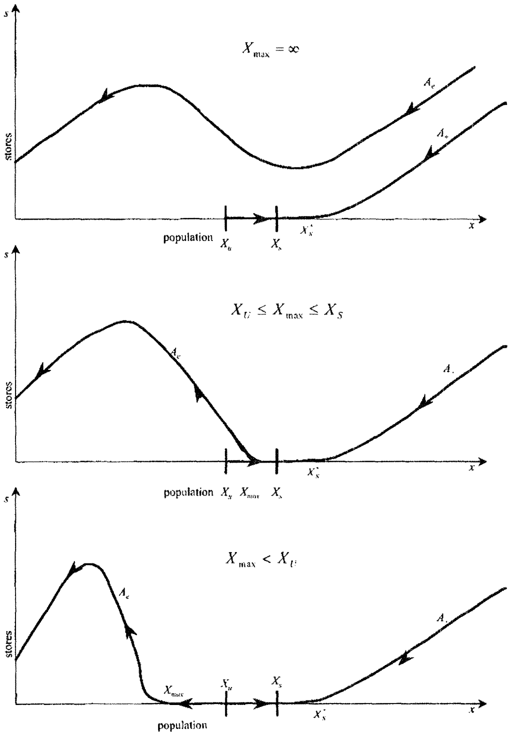

FIGURE 4. STORAGE AND NO-STORAGE REGIME EQUILIBRIUM SETS A_+ AND A_E

panel, but for high enough initial populations, there will be no set of points leading to extinction.

For the case in which $X_s < X_{\max} \leq X_s^*$ the situation is similar to the top panel ($X_{\max} = \infty$), but the A_e and A_+ paths are coincident above $X_{\max}$. Much as in the $X_U \leq X_{\max} \leq X_s$ case, the point ($X_{\max}$, 0) is a branch point, where the system can continue on A_e or A_+.

The bottom panel of Figure 4 illustrates the case when $X_{\max} < X_U$. As before, the survival set, A_+, is unchanged. The set leading to extinction consists of the Poaching and Storage Subpaths beginning with population $X_{\max}$ and zero stores, and the Poaching Without Storage Subpaths leading up to it.

The system must end up on one of these subpaths, A_+ or A_e. The next section explains how the system will reach these paths from an initial point with arbitrary values of population and stores (x_0, s_0).

IV. Moving to a Subpath with Finite and Positive Poaching

If the initial population and stores are not on either the A_+ or A_e paths identified above, then one of two things will happen. If the initial point in population-stores space is below an A_e or A_+ path, then the system may jump instantaneously to the corresponding equilibrium path *via* a cull. If the initial point is above an A_e or A_+ trajectory, demand may be temporarily satisfied from stores with no poaching until the path meets A_e or A_+.

A. *Culling*

If the system starts below an A_e or A_+ path, there may be an instantaneous harvest, which we will call a "cull."[10] Although anticipated jumps up in price are inconsistent with rational expectations, such jumps are possible at the "beginning of time," as in this case. We will distinguish between "initial" values of population and stores and "starting" values, which are the values just after the initial cull. When we need to indicate this, we will write (x_0, s_0) for initial population and stores, and $(x(0), s(0))$ to denote starting (i.e., at time 0 on the equilibrium path) values.

In a cull, live animals are killed and turned into dead animals one to one. This means that, in population-stores space, the system moves up a downward-sloping diagonal, and the total quantity of animals, dead or alive, is conserved. We call this quantity $Q = x + s$. For a cull to be rational, it must take the system to a point on one of the subpaths we identified above, A_e or A_+.

It is possible to cull to reach the A_e subpath from points below $s_e(x)$.[11] There may also be

[10] Realistically, of course, poaching could not kill animals at an infinite rate. If the marginal cost of poaching rose sufficiently with the instantaneous rate of poaching, the harvest would take place over time, rather than instantaneously. Structurally, though, there is little real difference in the two approaches: the rational expectations equilibria are determined by the boundary conditions (where people anticipate the system must end up), and these are essentially the same in both cases.

[11] Technically, if $Q < U(c_m)$, then the system can move immediately to extinction via a cull; it does not go to extinction via the A_e subpath.

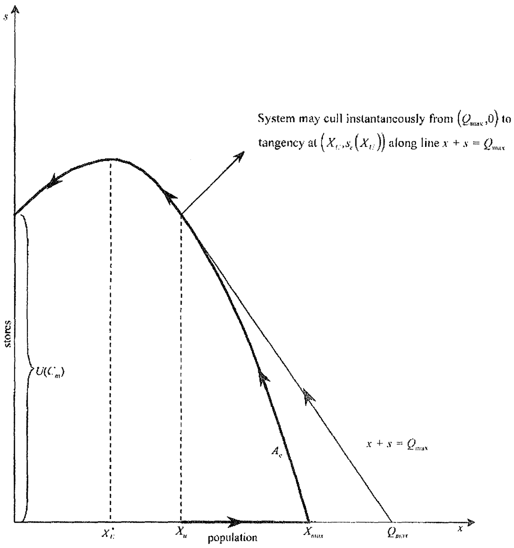

FIGURE 5. DEFINITION OF Q_{MAX}

other points from which this is feasible. In particular, if the curve $s = s_e(x)$ has a tangent of gradient -1, then, as illustrated in Figure 5, it is possible to reach the subpath from points above the curve, but below the tangent, by culling. A quick look at equation (9) shows that the points at which $s_e(x)$ has gradient -1 are X_U and X_S, but only the tangent at X_U can lie above the curve, if $X_{max} > X_U$. The value of Q at this tangency, Q_{max}, is the maximum value Q may have (while $x_0 > X_u$) so that the A_e subpath may be reached *via* culling. If $X_{max} < X_U$, then this tangency does not exist, and the A_e subpath can be reached by culling only from points below s_e. Note that if $Q_{max} > X_S$, then even starting from the high steady state with no stores, the population will be vulnerable to coordination on the extinction equilibrium. To get to the high steady-state equilibrium path by culling, the point corresponding to initial population and stores must lie below the curve $s = s_+(x)$, and $x_0 > X_S^*$.

B. *Storage Without Poaching*

If there are sufficient initial stores, there will be equilibria in which the starting price is below $c(x)$, and there is no poaching for a time while demand is satisfied out of stores. Eventually poaching must resume, at a point on A_e or A_+. While there is no poaching, population will be rising, and stores falling as they are consumed. The price will rise exponentially, at rate r. In population-stores space, trajectories with no poaching must be downward sloping and population must be increasing so long as population is less than one, the carrying capacity.

When poaching resumes at a point on one of the A_i paths, price, population, and stores are all determined. Given the end point, there is a unique, downward-sloping no-poaching trajectory leading to it.[12] In order for storing without poaching to be rational, and for an initial point to end up on one of the A_i, the initial point must lie on one of these trajectories (Figure 6). To get to the path leading to the high steady state, the initial point must lie to the right of the boundary of the set of points on trajectories leading to A_+, which we denote L_+, and above the curve $s = s_+(x)$. To get to the path leading to extinction, the initial point must lie to the left of the boundary of the set of points on trajectories leading to points on A_e, which we denote L_e. We include a more formal treatment of this in the Appendix, Proposition A3.

C. *Summary of Perfect Foresight Equilibria*

We have now found all the possible perfect foresight equilibria of the model. As illustrated in Figure 7, population-stores space may be divided into at most three regions depending on whether there exist equilibria leading to extinction, the high steady state, or both. The middle panel in Figure 7, corresponding to the case $X_U < Q_{max} < \infty$, illustrates a situation in which all three regions exist. In the first, unshaded, region, initial population and stores are high enough that there is no equilibrium path leading to extinction. In this region, speculators who killed animals and stored their parts until the species became extinct would have to hold the parts long enough that they would lose money. In the second, darkly shaded, region, population

[12] However, from a single initial level of population and stores, there may be equilibrium paths leading to different parts of the A_i trajectory. This is because the no-poaching trajectories leading to different endpoints on the A_i trajectory may cross. At the points where the trajectories cross, there will be multiple equilibria. For more technical details, we refer to Kremer and Morcom, 1996.

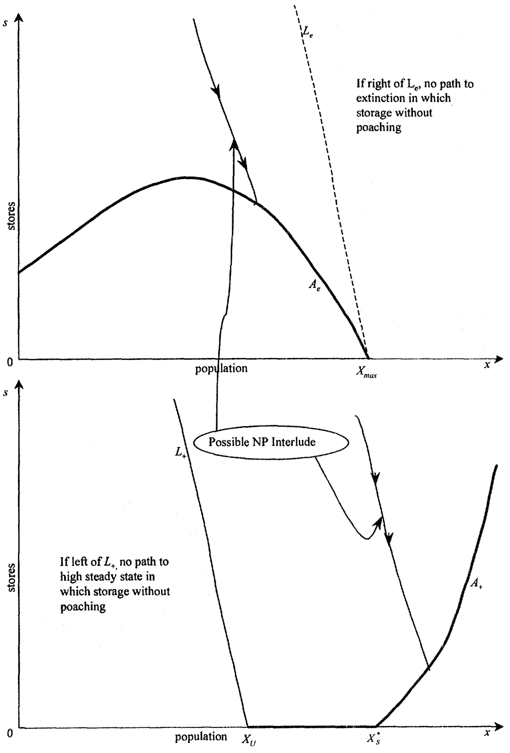

FIGURE 6. EQUILIBRIA WITH NO POACHING

and stores are low enough that, even if poaching temporarily ceased and demand were satisfied from stores until the stores were exhausted, the population could not recover enough for the species to survive. Thus there is no equilibrium path leading to the high steady state. In the third, lightly shaded, region, there are multiple possible equilibria, some to extinction, and some to the high steady state. In this region, expectations about which equilibrium will be chosen are self-fulfilling.

Depending on parameter values, some of these regions may be empty. It is possible that there will be no region in which survival is assured. If $X_{\max}$ is infinite, as depicted in the top panel of Figure 7, a trajectory beginning at any point can get to the extinction set A_e, either through a cull if it lies below s_e, or by an interlude with no poaching if it lies above s_e.

If, on the other hand, $X_{\max}$ is small enough (less than X_U), as depicted in the bottom panel of Figure 7, then there will be no region of multiple equilibria, and the fate of the system will be entirely determined by its initial point, and not by expectations.

It turns out that $X_{\max}$ and $Q_{\max}$ are both decreasing in r, the storage cost. For proofs, see the Appendix, Proposition A2. This should not come as a surprise. $Q_{\max}$ tells us the largest population can be and still reach extinction *via* culling and a storage equilibrium path. The larger the population, the longer stores have to be held before extinction. This is less desirable with higher storage costs. Increasing the storage cost thus always reduces the region of phase space from which extinction is possible. Governments could increase storage costs by threatening prosecution of anybody found to be storing the good. The international ban on ivory trade may have had this effect.

For sufficiently large r, $X_{\max}$ will be less than X_U, and there will be no region of multiple equilibria at all; the ultimate fate of the species is the same as in the model in which storage is impossible, given the same initial conditions. In this sense, our model converges to the standard Gordon-Schaefer model as storage cost rises.

V. Nondeterministic Equilibria

So far, we have focused on perfect foresight equilibria, in which all agents believe that the economy will follow a deterministic path. In this section, we discuss a broader class of rational expectations equilibria in which agents may attach positive probability to a number of future possible paths of the economy. One reason to consider this broader class of equilibria is that the perfect foresight equilibrium concept has the uncomfortable property that there may be a path from A to B, and from B to C, but not from A to C. For example, if $Q_{\max}$ is greater than X_S, then for sufficient initial population, the only equilibrium will lead to the high steady state. For a system that starts in the high steady state, however, an extinction storage equilibrium beginning with a cull would also be possible.

Note also that the concept of a Storage Without Poaching Subpath is also much more relevant when stochastic paths are admissible, since in order to have a subpath with no poaching, there must be stores, and the only way stores can be generated within the model is through a Poaching and Storage Subpath. However, within the limited class of perfect foresight

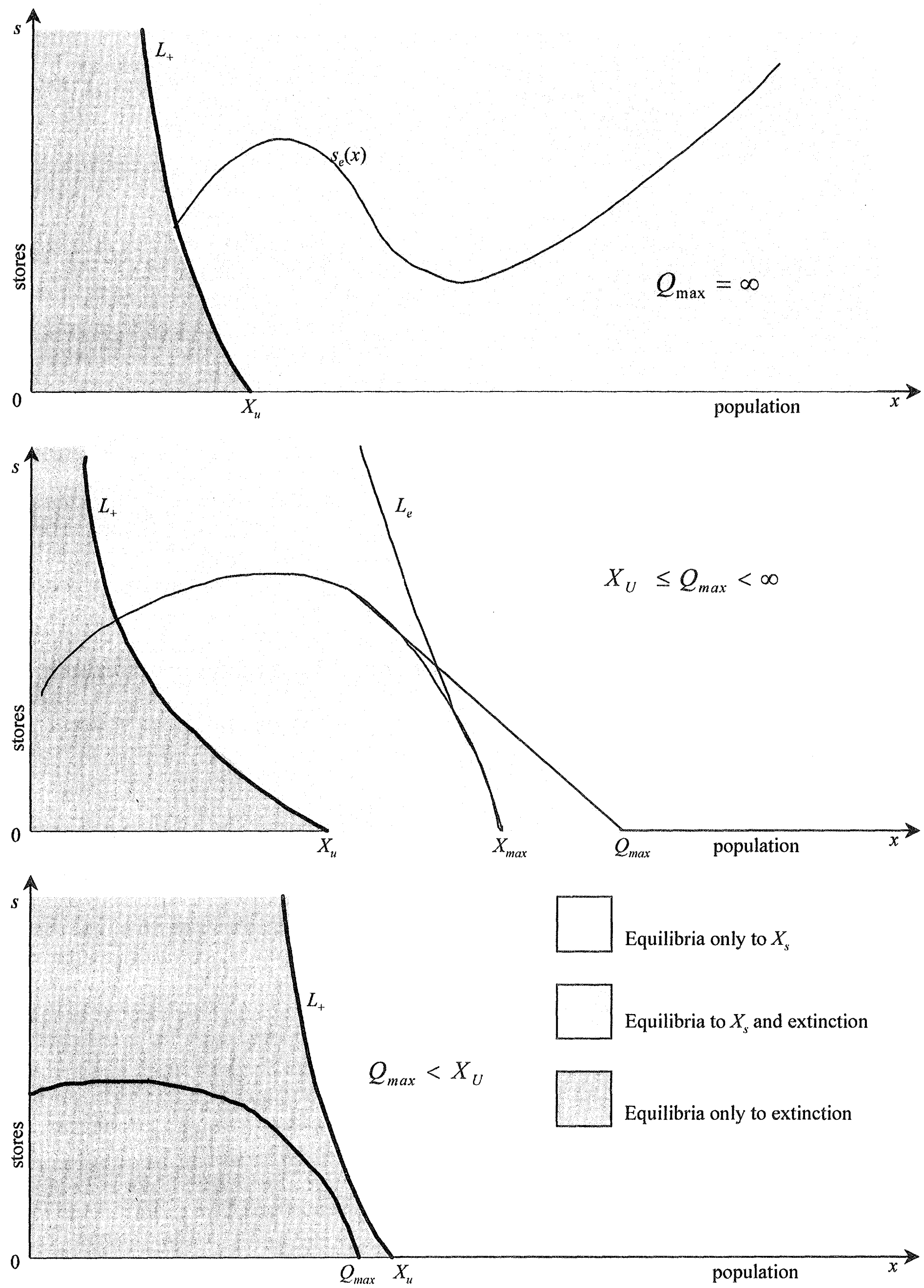

FIGURE 7. COEXISTENCE OF EQUILIBRIA

equilibria, people must assign zero weight to the possibility that there might be a switch from a Poaching and Storage Subpath to a Storage Without Poaching Subpath.

While we have not fully categorized the extremely broad class of equilibria with stochastic rational expectations paths, we have been able to describe a subclass of such equilibria, which we conjecture illustrates some more general aspects of behavior.

We consider equilibria in which agents believe there is a constant hazard that a sunspot will appear and that, when this happens, the economy will switch to the extinction storage equilibrium, with no possibility of any further switches.

Here, we present some results without proof. Those interested should refer to Section III, and Appendix B of National Bureau of Economic Research Working Paper No. 5674 (Kremer and Morcom, 1996), an earlier version of this paper, in which we discuss equilibria with uncertainty in greater detail.

If the constant hazard of a switch to an extinction equilibrium is below a certain threshold, π_h, an analogue of the high steady-state equilibrium exists before the sunspot. If the switching hazard is above this level, then a high steady-state equilibrium is not sustainable.

For low values of the switching hazard, there will be no stores held in steady state before the sunspot, and the steady state looks exactly like that without uncertainty: stores are zero, and population, $x = X_S$.

For moderate values of the switching hazard, positive stores will be held in the pre-sunspot steady state, as the switching hazard is high enough that it is worth holding stores to speculate on the price jump which occurs when there is a switch to the extinction equilibrium. The pre-sunspot steady-state population is still X_S. The quantity of stores held in the pre-sunspot steady state increases with the switching hazard up to π_h.

VI. Policy Implications and Directions for Future Work

Previous sections examined the equilibrium path of population and prices given the cost of poaching, and thus implicitly given the pattern of antipoaching enforcement. Section VI, subsection A, argues that expectations of future government antipoaching enforcement will affect current poaching. In most models of optimal management of open-access resources, the government maximizes the sum of producer and consumer surplus. This assumption may be appropriate for fish, but it is less appropriate for elephants or rhinos. We will assume that governments do not value the welfare of consumers or poachers, but instead seek to avoid extinction at minimum cost in expenditures on game wardens, helicopters, and other antipoaching efforts. Section VI, subsection B, argues that credible governments may be able to most cheaply eliminate extinction equilibria by committing to impose strong antipoaching policies if the species becomes endangered. Some governments may not be able to credibly commit to strong antipoaching policies. The cheapest way for these governments to eliminate extinction equilibria may be to maintain stockpiles, and threaten to sell them if the population falls below a threshold.

A. *Expectations of Antipoaching Policy*

Suppose that the cost of poaching is $c(x, E)$, where E is antipoaching expenditure, $c(x, \infty) > p_m$, and $D[c(x, 0)] > B(x)$ for all x. (These assumptions imply that with sufficiently weak enforcement, the species will be driven to extinction, and that with sufficiently strong enforcement, nothing will be harvested.) The dynamic analysis in this paper implies that expected future adoption of either very tough or very weak antipoaching measures may reduce long-run supply and therefore increase current poaching and storage, as demonstrated in the following propositions.

PROPOSITION 2: *Suppose* $c(X, E) = c_1(X) + c_2(E)$. *Suppose also that at date 0, the population is at the high steady state, and both a survival and extinction equilibrium exist. Finally, suppose that the government announces that at some date T, it will eliminate antipoaching enforcement. If T is small enough, there will be an immediate cull.*

PROOF:

See the Appendix.

PROPOSITION 3: *Suppose that at date 0, antipoaching expenditure is E, the population is at the high steady state, and both a survival and extinction equilibrium exist. Suppose also that the government announces that at date T it will increase the cost of poaching above* p_m, *thus eliminating poaching. Then (i) if T is small enough, the announcement will lead to an* *instantaneous cull, which will make the species extinct if $X_s < U(c_m)$ and (ii) if T is great enough, then the survival equilibrium may be eliminated even if $X_s > U(c_m)$.*

PROOF:

See the Appendix.

B. *Policies to Eliminate the Extinction Equilibrium*

Just as an expected shift to an antipoaching policy which reduces the long-run harvest may lead to an immediate cull, an expected shift towards an antipoaching policy which increases the long-run harvest may lead to a temporary cessation of poaching as the economy switches to a storage without poaching subpath. In particular, governments may be able to coordinate on survival equilibria by committing to follow certain policies. It is beyond the scope of this paper to fully specify optimal antipoaching expenditure and stockpile purchases and sales as functions of x, or more generally, the history of x. However, it is possible to show that for low enough interest rates, the optimal long-run policy involves committing to implement draconian antipoaching policies if the population falls below a threshold, or if this commitment would not be credible, building up stockpiles and threatening to sell them if the population falls below the threshold.

As is clear from Figure 1, if one takes the available habitat as given, the minimum antipoaching expenditure such that there is a steady state with positive population is E_{MIN} such that $D(c(x, E_{\text{MIN}}))$ is tangent to $B(x)$. Let x_{MIN} denote the steady-state population associated with antipoaching expenditures of E_{MIN}. Consider the case in which $x_{\text{MIN}} > U(c_m)$.

The steady-state cost of eliminating the extinction equilibrium is minimized by spending E_{MIN} on antipoaching efforts and committing that if x falls below some threshold, the government will temporarily implement tough antipoaching measures that raise c above p_m until the population recovers to x_{MIN}. This threshold can be any level of population less than x_{MIN}. (In this model, the population is not subject to stochastic shocks, and hence the exact threshold is irrelevant, since in equilibrium, the population never falls below x_{MIN}.[13]) To see that this policy minimizes the steady-state cost of eliminating the extinction equilibrium note first that there is no extinction equilibrium under this policy, since the cost of poaching is above p_m when x is below the threshold. The population cannot be eliminated instantaneously in a cull before the government has an opportunity to raise the cost of poaching above p_m since $x_{\text{MIN}} > U(c_m)$. Note also that no policy with lower expenditure is consistent with survival, since the population cannot survive indefinitely with antipoaching expenditures of less than E_{MIN}.

In general, optimal long-run policy may not minimize steady-state costs because moving to this policy would entail transition costs. To take an extreme example, if the initial population is small enough, assuring species survival will be so costly that the government will allow extinction. However, as the discount rate approaches zero, the optimal long-run policy will approach the policy which minimizes steady-state costs (assuming that these costs are less than the flow value the government attaches to eliminating extinction equilibria).

The model suggests that if a government or international organization could credibly commit to spend a large amount on elephant protection if the herd fell below a certain critical size, it would never actually have to spend the money. This provides a potential rationale for endangered species laws that extend little protection to a species until it is endangered, and then provide extensive protection with little regard to cost.

Note that the policy which minimizes the steady-state cost of eliminating the extinction equilibrium may leave the population very close to extinction. Some additional margin of safety would likely be optimal in a more realistic model in which the population was subject to stochastic shocks.

Some governments with open-access resources may not be able to credibly commit to spend heavily on antipoaching enforcement if the population falls below a threshold, since this

[13] Note that x_{MIN} would be an unstable steady state if the government maintained constant expenditure of E_{MIN}, instead of letting expenditure depend on x.

policy will be time inconsistent if the cost of imposing tough antipoaching enforcement is sufficiently high. In the case of goods used to produce storable, nondurable goods, we argue below that the cheapest way for such governments to eliminate the extinction equilibrium may involve maintaining a stockpile and threatening to sell it if the population falls below a threshold or becomes extinct. Promises to sell stockpiles, unlike promises to increase antipoaching expenditure, are likely to be time consistent, since there is no reason not to sell stores if a species is becoming extinct anyway. (It is important to note that while stockpiles can help protect animals which are killed for goods which are storable but not durable, such as rhino horn, stockpiles will not help protect species which are used to produce durable goods, i.e., goods which are not destroyed when they are consumed.[14])

To see why stockpiles may be useful in eliminating extinction equilibria, note that the smaller the initial population level, the greater the transition costs of antipoaching enforcement needed to make the population survive. For a small enough initial population, the government will find it optimal to allow extinction. Denote this minimum population as $\hat{x}$. There will be a set of population levels at which poachers will find it profitable to cull immediately to $\hat{x}$ or a lower level, knowing that the government will then allow the species to go extinct.[15] The upper boundary of this set will be a level at which poachers will be just indifferent between culling and not culling, if they believe other poachers will cull. Denote this level of population as x_{NC} for the no-commitment level of population. x_{NC} is analogous to $Q_{\max}$, but whereas $Q_{\max}$ is calculated taking the $c(x)$ function as exogenous, x_{NC} is calculated based on $c(x, E^*(x))$ where $E^*(x)$ is the government's optimal antipoaching expenditure, given a population x.

Suppose that x_{NC} is greater than x_{MIN}, so the possibility of a switch to the extinction equilibrium will exist in steady state if $x_s = x_{\text{MIN}}$ and the government does not hold stockpiles. In order to prevent an extinction equilibrium, the government could either maintain a live population of x_{NC} or maintain a steady-state live population of x_{MIN} and a stockpile of $x_{NC} - x_{\text{MIN}}$, which it promises to sell if the population falls below a threshold. To see why holding $x_{NC} - x_{\text{MIN}}$ either as live population or stores will eliminate the extinction equilibrium, note that a cull could only move the system along a 45-degree line extending "northwest" from the initial point in population-stores space. If it is impossible to reach an extinction path along this line, then there will be no extinction equilibrium.

To compare the cost of holding $x_{NC} - x_{\text{MIN}}$ as live population and stores, denote the steady-state cost of antipoaching enforcement and other conservation activity needed to maintain the population at x_{NC} as $\underline{E}(x_{NC})$. Note that if x_{NC} is beyond the carrying capacity, 1, even the complete elimination of poaching will be insufficient to maintain the population at x_{NC}. Food will have to be brought in for the animals, and as overcrowding increases, disease may become more and more of a problem. We assume that, at least for large enough x, the expenditure needed to maintain a population of x, denoted $\underline{E}(x)$, increases at least linearly in x, i.e., $\underline{E}''(x) \geq 0$.

Suppose that there are initially x_{NC} animals. The discounted cost of supporting the animal population at x_{NC} indefinitely is $\underline{E}(x_{NC})/r$. Denote the cost of culling from a population of x_{NC} to a population of x_{MIN} as $c^*(x_{NC}, x_{\text{MIN}})$. We assume that c^* increases less than linearly with x_{NC}, since it is presumably easier to cull animals when there are more of them. The discounted cost of sustaining a population of x_{MIN} and a stockpile of $x_{NC} - x_{\text{MIN}}$ is thus $c^*(x_{NC}, x_{\text{MIN}}) + E_{\text{MIN}}/r$. The cost advantage of stockpiling is thus $(\underline{E}(x_{NC}) - E_{\text{MIN}})/r - c^*(x_{NC}, x_{\text{MIN}})$. This is positive if $rc^*(x_{NC}, x_{\text{MIN}}) < \underline{E}(x_{NC}) - E_{\text{MIN}}$. For small enough r, this will be the case. To see this, note that x_{MIN} (and hence E_{MIN}) do not depend on r, since x_{MIN}

[14] The government has no reason to store durable goods, since private agents will store any durable goods sold on the market. As noted in the introduction, however, few goods are completely durable.

[15] This discussion assumes that the poachers can conduct an instantaneous cull before the government can react. However, a similar phenomenon would occur even if the government could raise E as soon as the population hit a threshold. If poachers believed that the government would eventually give up protecting the animal, they would keep forcing the population back to the threshold, and this could cause the government to spend so much on antipoaching enforcement that the government would in fact prefer to let the species go extinct.

depends only on the $D(p)$ and $B(x)$ functions, each of which depends on current and not future variables. As r approaches zero, x_{NC} [and hence $\underline{E}(x_{NC})$] grow without bound, since $\hat{x}$ is bounded below by 0, and $x_{NC} - \hat{x}$ will grow without bound as r falls, because as r approaches zero poachers will become willing to hold stockpiles for arbitrarily long periods prior to selling at p_m. Under the assumptions that for large enough x, $\underline{E}''(x) \geq 0$, and $\partial^2 c^*/\partial x_{NC}^2 < 0$, L'Hopital's rule implies that in the limit as r approaches zero, $rc^*(x_{NC}, x_{\text{MIN}})$ will grow less quickly than $E(x_{NC}) - E_{\text{MIN}}$ and, hence, stockpiling will be cheaper than maintaining the animal population at x_{NC} indefinitely.

Note that if it is optimal to hold stockpiles if the initial population is x_{NC}, then the cheapest policy that eliminates the risk of extinction must involve holding stockpiles in the long run, no matter what the initial level of population. To see this, note that the cheapest long-run policy that eliminates the risk of extinction without stockpiles is to maintain a population of x_{NC}, but that once population had reached x_{NC}, it would be cheaper to cull to create a stockpile of $x_{NC} - x_{\text{MIN}}$ and to maintain a population of x_{MIN} than to maintain the population at x_{NC}. Hence the cheapest long-run policy that eliminates the risk of extinction must involve holding stockpiles.

While holding sufficient stockpiles will eliminate the extinction equilibrium, the process of building the stockpiles changes the survival equilibrium path. In the perfect foresight model of Sections I–IV,[16] merely *holding* stockpiles does not affect the survival steady state, since it changes neither demand nor supply.

Whereas under the perfect foresight model the survival steady state is the same with and without stores, under the stochastic model of Section V, government stockpiles affect the survival steady state, as well as the equilibrium transition path. In the absence of government stockpiles, private agents will hold sufficient stores in the pre-sunspot steady state that the expected profits in case of a switch to the extinction equilibria just offset the storage costs if no sunspot appears. Government accumulation of stores will crowd out private stores, until private agents no longer hold any stores. Once the government accumulates sufficient stores, the extinction equilibrium will disappear.

Ted Bergstrom (1990) has suggested that confiscated contraband should be sold onto the market. Many conservationists oppose selling confiscated animal products on the market, fearing that it would legitimize the animal products trade. Using confiscated contraband to build stores helps avoid this problem. Stores could potentially be held until scientists develop ways of marking or identifying "legitimately" sold animal products so they can be distinguished from illegitimate products. We have assumed that the only cost of holding stores is the interest cost, but if governments confiscate contraband, they will increase the cost of holding stores, and this will reduce the scope for extinction equilibria.

Our model does not allow for stochastic shocks to population, nor does it differentiate animals by health, age, or sex, but in more realistic models there may be ways of building stores that do not reduce the live population one for one. Stores could be built up by harvesting sick animals, or harvesting animals during periods when population is temporarily above its steady state, due for example to a run of good weather. In future work, we plan to explicitly model the potential of stockpiles to smooth stochastic shocks to population, for example due to weather and disease.

It is worth noting that stockpiles could be built up not only by the government of the country where the species lives, but also by conservation organizations or foreign governments. A further analysis of this case would have to consider strategic interaction between the conservation organization and the government.

We have assumed that the government knows the parameters of the model, but of course this is unlikely to be the case in practice. It is worth noting that inferences about parameters based on the Gordon-Schaefer model may lead to a false optimism if the good is storable. Under the Gordon-Schaefer model of nonstorable open-access resources, stability of the population could be interpreted as indicating that parameters are such that the species will survive. In

[16] Perfect foresight is a somewhat strange context to examine stockpiles, because under perfect foresight, all agents either assign probability one to extinction, in which case it is too late for government stockpiles to prevent extinction, or they assign probability zero to extinction, in which case it is not clear why government stockpiles would be necessary.

contrast, under the model outlined here, species with constant population may be vulnerable to a switch to an extinction equilibrium. One should not become complacent even if the population is increasing, since along Storage Without Poaching Subpaths, the population may temporarily increase above its steady-state level, even if the ultimate steady state is extinction.

Although we have focused on how governments could coordinate on the survival equilibrium, and assumed that poachers are atomistic and take prices as given, it is worth noting that a "George Soros" of poaching who held large stores and had access to sufficient capital could try to coordinate on the extinction equilibrium simply by offering to buy enough of the good at a high enough price. (In practice such an offer could provoke a government reaction.)

Stepping further outside the model, we speculate that if the population of live animals were very small, poachers and storers might not take prices as given, and would instead take into account that killing animals could raise prices. This may help explain why rhinos in Zimbabwe which had been de-horned by game wardens to protect them from poachers were nonetheless killed by poachers. The *New York Times* (1994), quotes a wildlife official as explaining the poachers' behavior by saying: "If Zimbabwe is to lose its entire rhino population, such news would increase the values of stockpiles internationally."[17] It is plausible that traders holding even modest stores would order poachers to kill de-horned rhinos, since rhino populations are small, and once poachers have found a rhino and realized that its horn has been removed, killing the rhino only costs a bullet.

Appendix

PROOF OF PROPOSITION 2:

After period T there will be no steady state with positive population and hence the system must follow an extinction equilibrium path. The date-T extinction path will have a higher starting price and lower starting population for any initial Q than the date-0 extinction path, because poaching will be greater for any population level, and hence the species will become extinct faster, so the price will reach p_m more quickly. [Both the date-T and date-0 extinction trajectories will pass through $[0, U(c_m)]$, but otherwise the date-T extinction trajectory will lie above the date-0 extinction trajectory in s-x space.] Since no jumps in price can be anticipated, if T is small enough, poachers will cull immediately.

PROOF OF PROPOSITION 3:

(i) Consider first the extreme case in which T is infinitesimal. Following a cull of $\Phi < X_s$ at time 0, stores will be Φ and the price will be $c[(X_s - \Phi), E]$. Along a rational expectations equilibrium path in which some animals survive, stores at time T must be $U(c[(X_s - \Phi), E])$ so that the stores will be exactly consumed during the time it takes the price to rise to p_m. Thus the equilibrium $\Phi = U(c[(X_s - \Phi), E])$. If $X_s < U(c_m)$, there is no Φ satisfying this equation. To see this, note that $c(x)$ is decreasing in x and $U(c)$ is increasing in c. Therefore, for $X_s - \Phi \geq 0$, $c(X_s - \Phi) \leq c_m$, and $U(c(X_s - \Phi)) \geq U(c_m) > X_s \geq \Phi$. Since Φ cannot be less than X_s there will be a unique equilibrium in which the entire population is culled at time 0 and the price rises above c_m.

(ii) Now consider the case in which T is greater than the time until population and stores equal zero on any path with a cull. In this case, no equilibrium with a cull is consistent with survival. If there is no immediate cull, then the price must be continuous. If the price is continuous, then if maximum stores along the path satisfying the differential equation for equilibrium trajectories, (9), and passing through the point $(X_s, 0)$ are less than $U(c_m)$, there can be no survival equilibrium. To see this, note that for the animal to survive, the price at time T must be less than c_m. If the price at time T is less than c_m, then on a rational expectations path, stores at time T must be greater than $U(c_m)$.

PROPOSITION A1: *The path of population, x, stores, s, and price, p, is continuous on an equilibrium path.*

PROOF:

Together, the storage and poaching conditions imply that the equilibrium price path must

[17] It is also possible that the poachers killed the rhinos to obtain the stumps of their horns, or to make rhino poaching easier in the future.

be continuous in time. A jump up in price would violate the storage condition, and a jump down in price would imply an instantaneous infinite growth rate of the population, which is impossible.

While there is poaching, $p = c(x)$, which is continuous and monotonic, so population, x, must be continuous. In the no-poaching subpath, population develops as equation (7), so is continuous. Population cannot jump suddenly across subpath changes, either, as that would require a jump in price so there is an instantaneous harvest. Population is thus continuous. Stores are differentiable within subpaths, and so are continuous. For there to be a jump in stores across subpaths, there would have to be an instantaneous harvest, which would require a jump in price, which is impossible. Hence stores are continuous.

PROPOSITION A2: *(i) The maximum initial value of population plus stores the system may have and still get to the Poaching and Storage Equilibrium Subpath $A_e(x)$ via culling is $Q_{\max}$, where*

$$\text{(A1)} \quad Q_{\max} = \max\{X_{\max}, s_e(X_U) + X_U\}.$$

(ii) If finite, $Q_{\max}$ is decreasing in storage cost, r.

PROOF:

(i) $Q_{\max}$ must either be $X_{\max}$, if $s_e(X_U) < 0$, or the point lying on the x axis and the tangent to $s_e(x)$ of gradient -1. These tangencies occur at X_U or X_S. $s_e(x)$ is convex at X_S, so $Q_{\max}$ cannot be associated with X_S.

(ii) If $Q_{\max} = X_{\max}$, then $s_e(Q_{\max}(r), r) = 0$, where we make explicit the dependence of both the function $s_e(x)$ and the point $X_{\max}$ on r. By the implicit function theorem,

$$\frac{dQ_{\max}}{dr} = -\left.\frac{\partial s_e/\partial r}{\partial s_e/\partial Q_{\max}}\right|_{x=Q_{\max}} = -\left.\frac{\partial s_e/\partial r}{s_e'(x)}\right|_{x=Q_{\max}}.$$

Note that the application of the theorem is allowed because, at $X_{\max}$, $s_e'(x)$ is strictly negative. To determine the sign of the numerator of (A1), recall that

$$s_e(x) = U(c_m) + \int_0^x \left(\frac{c'(q)}{rc(q)} F(q) - 1\right) dq, \text{ and}$$

$$U(c_m) = \int_0^{(1/r)\ln(p_m/c_m)} D(c_m e^{rt})\, dt = \int_{c_m}^{p_m} \frac{1}{r} \frac{D(z)}{z}\, dz.$$

Since c_m, p_m do not depend on r,

$$\frac{dU(c_m)}{dr} = -\int_{c_m}^{p_m} \frac{1}{r^2} \frac{D(z)}{z}\, dz = -\frac{1}{r} U(c_m).$$

Likewise,

$$\frac{\partial}{\partial r}\left[\int_0^x \frac{c'(q)}{rc(q)} F(q)\, dq - x\right] = -\frac{1}{r}\int_0^x \frac{c'(q)}{rc(q)} F(q)\, dq.$$

Summing up,

$$\left.\frac{\partial s_e}{\partial r}\right|_{x=X_{\max}} = -\frac{1}{r}(s_e(X_{\max}) + X_{\max}) = -\frac{X_{\max}}{r} < 0.$$

If $Q_{\max} = s_e(X_U) + X_U$, then, because X_U is independent of r, the result also follows straightforwardly, as

$$dQ_{\max}/dr = \partial s_e(x)/\partial r|_{x=X_U} = -\frac{1}{r}(s_e(X_U) + X_U) < 0.$$

PROPOSITION A3: *Consider the system of differential equations (7) for the no-poaching phase:*

$$\text{(A2)} \qquad \dot{x} = B(x)$$

$$\dot{s} = D(p_0 e^{rt})$$

with a set of border conditions of a Cauchy problem for some p_0:

$$\text{(A3)} \qquad x(0) = x_0$$

$$s(0) = s_0.$$

If there exist p_0 *and* t_p *such that for the solution of (A2), (A3),* $s(t_p) = s_i(x(t_p))$, $i \in \{e, +\}$, *this will be a no-poaching interlude leading to the equilibrium path* A_i *with the initial price* p_0. *The duration of this interlude will be exactly* t_p. *We will denote the sets of initial conditions for which such* p_0 *exists by* E_i:

$$\text{(A4)} \quad E_i = \{(x_0, s_0) \mid \exists\, p_0, t_p : s(t_p) = s_i(x(t_p))$$

$$\text{for } s(t), x(t)\text{—solutions to (A2), (A3)}\}.$$

The equilibrium subpath on which the system may end up is not, in general, unique. There may be equilibria leading to A_e *and* A_+. *There may also be cases where* $E_+ \cap E_e \neq \emptyset$. *If there are multiple equilibria from the same point* (x_0, s_0), *then the one with the lower starting price must have a steeper trajectory in* (x, s) *space, since stores will be consumed faster with a lower price.*

In other words, there is a no Storage Without Poaching Subpath ultimately leading to the steady state X_S *if and only if* $L_+(x_0) < s_0$ *and* $s_0 > s_+(x_0)$, *where* L_+ *is the left boundary of the set* E_+ *defined in equation (A4). Likewise, there is a Storage Without Poaching Subpath leading to extinction if and only if* $L_e(x_0) < s_0$, *and* $s_0 > s_e(x_0)$ *(see Figure 6).* L_i *are downward sloping.* L_e *and* L_+ *will be the same line if* $X_{\max} \leq X_U$.

PROOF:

By Figure 6, L_i are downward sloping, because they are possible no-poaching paths, and so stores are decreasing, while population is increasing.

REFERENCES

Anderson, John Ward. "Poachers Felling World's Tigers, Rhinos." *Washington Post,* November 29, 1994, p. A1.

Balzar, John. "Man May Prove to Be More than the Grizzly Can Bear." *Los Angeles Times,* April 15, 1992, p. A1.

Barbier, Edward B.; Burgess, Joanne C.; Swanson, Timothy M. and Pearce, David W. *Elephants, economics, and ivory.* London: Earthscan Publications, 1990.

Begley, Sharon. "Killed by Kindness." *Newsweek,* April 12, 1993, pp. 50–56.

Benhabib, Jess and Radner, Roy. "Joint Exploitation of a Productive Asset: A Game-Theoretic Approach." *Economic Theory,* April 1992, *2*(2), pp. 155–90.

Berck, Peter and Perloff, Jeffrey M. "An Open-Access Fishery with Rational Expectations." *Econometrica,* March 1984, *52*(2), pp. 489–506.

Bergstrom, Ted. "Puzzles on the Economics of Crime and Confiscation." *Journal of Economic Perspectives,* Summer 1990, *4*(3), pp. 171–78.

Bonner, Raymond. *At the hand of man: Peril and hope for Africa's wildlife.* New York: Vintage Books, 1993.

Booth, William. "A Rescue Mission for One of the Rarest Creatures in the World." *Washington Post,* October 19, 1992, p. A3.

Clark, Colin Whitcomb. *Mathematical bioeconomics: The optimal management of renewable resources.* New York: Wiley, 1976.

Egan, Timothy. "Protecting Prey of Humans Sets Two Against a Vast World." *New York Times,* May 2, 1994, p. A1.

Galloway, Laura. "On Their Last Legs?" *Los Angeles Times,* November 15, 1994, p. C5.

Gaudet, Gerard; Moreaux, Michel and Salant, Stephen W. "Private Storage of Common Property." Unpublished manuscript, 1998.

Goombridge, Brian, ed. *Global biodiversity: Status of the Earth's living resources.* New York: Chapman and Hall, 1992.

Gordon, H. Scott. "Economic Theory of a Common Property Resource." *Journal of*

Political Economy, April 1954, *62*(2), pp. 121–42.

Haurie, Alain and Pohjohla, Matti. "Efficient Equilibria in a Differential Game of Capitalism." *Journal of Economic Dynamics and Control*, March 1987, *11*(1), pp. 65–78.

Hotelling, Harold. "The Economics of Exhaustible Resources." *Journal of Political Economy*, April 1931, *39*(2), pp. 137–75.

Johnson, Robert. "Imagine If We Lost the Horned Lizard or the Crested Toad." *Wall Street Journal,* July 8, 1993, p. A1.

Keller, Bill. "Cheetah's Race with Fate: U.S. Couple to Rescue." *New York Times,* May 17, 1993, p. A4.

Kelly, Sean. "Wildlife Biology: Poaching Bears for Asian Market." *Washington Post,* August 26, 1991, p. A2.

______. "Setting CITES on Gallbladder Trade." *Washington Post,* March 2, 1992, p. A2.

Kremer, Michael and Morcom, Charles. "Elephants." National Bureau of Economic Research (Cambridge, MA) Working Paper No. 5674, 1996.

Lancaster, Kelvin. "The Dynamic Inefficiency of Capitalism." *Journal of Political Economy.* September–October 1973, *81*(5), pp. 1092–109.

Levhari, David and Mirman, Leonard J. "The Great Fish War: An Example Using a Dynamic Cournot-Nash Solution." *Bell Journal of Economics*, Spring 1980, *11*(1), pp. 322–34.

Life Magazine. "The Endangered 100." September 1994, pp. 50–63.

Mander, Ian. "Sea Otter Population Booms Amid Hunting Ban." *Los Angeles Times,* July 25, 1993, p. B5.

Naik, Gautam. "Herpetologists Think Poacher of Rattlers Is Lower than a Snake." *Wall Street Journal*, November 15, 1994, p. A1.

New York Times. "Butterfly Farms May Be Last Hope of Rare Hunted Species." April 14, 1992, p. C4.

______. "Zimbabwe's Rhino Efforts Faulted." July 12, 1994, p. C4.

Perlez, Jane. "Devaluing the Tusk." *New York Times Magazine*, January 1, 1990, p. 30; January 7, 1990, pp. 28–33.

Press, Robert M. "Conservationists Rally Round the Rhino." *Christian Science Monitor,* June 2, 1993, p. 1.

Reinganum, Jennifer F. and Stokey, Nancy L. "Oligopoly Extraction of a Common Property Natural Resource: The Importance of the Period of Commitment in Dynamic Games." *International Economic Review*, February 1985, *26*(1), pp. 161–73.

Richards, Bill. "Toad-Smoking Gains on Toad-Licking among Drug Users." *Wall Street Journal,* March 7, 1994, p. A1.

Sanger, David. "Japan, Backing Down, Plans Ban on Rare Turtle Import." *New York Times,* June 20, 1991, p. D6.

Schaefer, M. B. "Some Considerations of Population Dynamics and Economics in Relation to the Management of Marine Fisheries." *Journal of the Fisheries Research Board of Canada,* 1957, *14,* pp. 669–81.

Simmons, Randy T. and Kreuto, Urs P. "Herd Mentality." *Journal of Policy Review*, Fall 1989, *50*, p. 46.

Smith, Vernon L. "Economics of Production from Natural Resources." *American Economic Review*, June 1968, *58*(3), pp. 409–31.

Stevens, William K. "American Box Turtles Decline, Perishing Cruelly in Foreign Lands." *New York Times,* May 10, 1994, p. C1.

Sun, Lena. "Can Giant Pandas Survive the Effort to Save Them?" *Washington Post,* December 27, 1993, p. A1.

Taylor, Paul. "Slaughter on the Beach." *Washington Post,* August 27, 1993, p. A3.

Tornell, Aaron and Velasco, Andres. "The Tragedy of the Commons and Economic Growth: Why Does Capital Flow From Poor to Rich Countries?" *Journal of Political Economy*, December 1992, *100*(6), pp. 1208–31.