Guides to Inventory Policy: Functions and Lot Sizes

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"Why are we always out of stock?" So goes the complaint of great numbers of businessmen faced with the dilemmas and frustrations of attempting simultaneously to maintain stable production operations, provide customers with adequate service, and keep investment in stocks and equipment at reasonable levels.

But this is only one of the characteristic problems business managers face in dealing with production planning, scheduling, keeping inventories in hand, and expediting. Other questions just as perplexing and baffling when managers approach them on the basis of intuition and pencil work alone-are: How often should we reorder, or how should we adjust production, when sales are uncertain? What capacity levels should we set for job shop operations? How do we plan production and procurement for seasonal sales? And so on, and so on.

In this series of articles, I will describe some of the technical developments which aim at giving the business manager better control over inventory and scheduling policy. While these techniques sometimes employ concepts and language foreign to the line executive, they are far from being either academic exercises or mere clerical devices. They are designed to help the business manager make better policy decisions and get his people to follow policy more closely.

As such, these techniques are worth some time and thought, commensurate with the central importance of production planning and inventory policy in business operations. Indeed, many companies have found that analysis of the functions of inventories, measurement of the proper level of stocks, and development of inventory and production control systems based on the sorts of techniques described in this and following sections can be very profitable. For example:

- Johnson & Johnson has used these techniques for studying inventory requirements for products with seasonally changing demand, and also to set economical inventory goals balancing investment requirements against additional training and overtime costs.
- The American Thread Company, as a supplier to the fashion goods industry, plagued with large in-process inventories, day-to-day imbalances among production departments, labor turnover, and customer service difficulties, found these methods the key to improved scheduling and control procedures. Now these improved procedures help keep an inventory of tens of thousands of items in balance and smooth out production operations even in the face of demand showing extremely erratic fluctuations due to fashion changes.
- The Lamp Division of the General Electric Company has reported using these methods to survey its finished inventory functions and stock requirements in view of operating conditions and costs. This survey indicated how an improved warehouse reorder system would yield inventory cuts at both factories and warehouses, and pointed to the reorder system characteristics that were needed; it led to the installation of a new reorder and stock control system offering substantial opportunities for stock reduction. The analytic

approach can also be used to show clearly what the cost in inventory investment and schedule changes is to achieve a given level of customer service.

- An industrial equipment manufacturer used these methods to investigate inventory and scheduling practices and to clear up policy ambiguities in this area, as a prelude to installing an electronic computer system to handle inventory control, scheduling, and purchase requisitions. In general, the analytic approach has proved a valuable help in bringing disagreements over inventory policy into the open, helping each side to recognize its own and the others' hidden assumptions, and to reach a common agreement more quickly.
- The Procter & Gamble Company recently described how analysis of its factory inventory functions and requirements, using these methods, has pointed out means for improved scheduling and more efficient use of finished stock. The analysis indicated how the company could take advantage of certain particular characteristics of its factories to cut stocks needed to meet sales fluctuations while still maintaining its long-standing policy of guaranteed annual employment.

These are only a few instances of applications. Numerous others could be drawn from the experience of companies ranging from moderate to large size, selling consumer goods or industrial products, with thousands of items or only a few, and distribution in highly stable, predictable markets or in erratically changing and unpredictable circumstances.

In the present article major attention will be devoted to (a) the conceptual framework of the analytic approach, including the definition of inventory function and the measurement of operational costs; and (b) the problem of optimum lot size, with a detailed case illustration showing how the techniques are applied.

This case reveals that the appropriate order quantity and the average inventory maintained do not vary directly with sales, and that a good answer to the lot-size question can be obtained with fairly crude cost data, provided that a sound analytical approach is used. The case also shows that the businessman does not need calculus to solve many inventory problems (although use has to be made of it when certain complications arise).

INVENTORY PROBLEMS

The question before management is: How big should inventories be? The answer to this is obvious-they should be just big enough. But what is big enough?

This question is made more difficult by the fact that generally each individual within a management group tends to answer the question from his own point of view. He fails to recognize costs outside his usual framework. He tends to think of inventories in isolation from other operations. The sales manager commonly says that the company must never make a customer wait; the production manager says there must be long manufacturing runs for lower costs and steady employment; the treasurer says that large inventories are draining off cash which could be used to make a profit.

Such a situation occurs all the time. The task of all production planning, scheduling, or control functions, in fact, is typically to balance conflicting objectives such as those of minimum purchase or production cost, minimum inventory investment, minimum storage and distribution cost, and maximum service to customers.

Production vs. Time - Often businessmen blame their inventory and scheduling difficulties on small orders and product diversity: "You can't keep track of 100,000 items. Forecasts mean nothing. We're just a job shop." Many businessmen seem to feel that their problems in this respect are unusual, whereas actually the problems faced by a moderate-size manufacturer with a widely diversified product line are almost typical of business today.

The fact is, simply, that under present methods of organization the costs of paper work, setup, and control, in view of the diversity of products sold, represent an extremely heavy drain on many a company's profit and a severe cost to its customers. The superficial variety of output has often blinded management to the opportunities for more systematic production flow and for the elimination of many of the curses of job-shop operation by better organization and planning.

The problem of planning and scheduling production or inventories pervades all operations concerned with the matter of production versus time-i.e., the interaction between production, distribution, and the location and size of physical stocks. It occurs at almost every step in the production process: purchasing, production of in-process materials, finished production, distribution of finished product, and service to customers. In multiplant operations, the problem becomes compounded because decisions must be made with reference to the amount of each item to be produced in each factory; management must also specify how the warehouses should be served by the plants.

Action vs. Analysis - The questions businessmen raise in connection with management and control of inventories are basically aimed at action, not at arriving at answers. The questions are stated, unsurprisingly, in the characteristic terms of decisions to be made: "Where shall we maintain how much stock?" "Who will be responsible for it?" "What shall we do to control balances or set proper schedules?" A manager necessarily thinks of problems in production planning in terms of centers of responsibility.

However, action questions are not enough by themselves. In order to get at the answers to these questions as a basis for taking action, it is necessary to back off and ask some rather different kinds of questions: "Why do we have inventories?" "What affects the inventory balances we maintain?" "How do these effects take place?" From these questions, a picture of the inventory problem can be built up which shows the influence on inventories and costs of the various alternative decisions, which the management may ultimately want to consider.

This type of analytic or functional question has been answered intuitively by businessmen with considerable success in the past. Consequently, most of the effort toward improved inventory management has been spent in other directions; it has been aimed at better means for recording, filing, or displaying information and at better ways of doing the necessary clerical work. This is all to the good, for efficient data-handling helps. However, it does not lessen the need for a more systematic approach to inventory problems that can take the place of, or at least supplement, intuition.

As business has grown, it has become more complex, and as business executives have become more and more specialized in their jobs or farther removed from direct operations, the task of achieving an economical balance intuitively has become increasingly difficult. That is why more businessmen are finding the concepts and mathematics of the growing field of inventory theory to be of direct practical help.

One of the principal difficulties in the intuitive approach is that the types and definitions of cost, which influence appropriate inventory policy are not those characteristically found on the books of a company. Many costs, such as setup or purchasing costs, are hidden in the accounting records. Others, such as inventory capital costs, may never appear at all. Each cost may be clear to the operating head primarily responsible for its control; since it is a "hidden" cost, however, its importance may not be clear at all to other operating executives concerned. The resulting confusion may make it difficult to arrive at anything like a consistent policy.

In the last five years in particular, operations research teams have succeeded in using techniques of research scientists to develop a practical analytic approach to inventory questions, despite growing business size, complexity, and division of management responsibility.

INVENTORY FUNCTIONS

To understand the principles of the analytic approach, we must have some idea of the basic functions of inventories.

Fundamentally, inventories serve to uncouple successive operations in the process of making a product and getting it to consumers. For example, inventories make it possible to process a product at a distance from customers or from raw material supplies or to do two operations at a distance from one another (perhaps only across the plant). Inventories make it unnecessary to gear production directly to consumption or, alternatively, to force consumption to adapt to the necessities of production. In these and similar ways, inventories free one stage in the production-distribution process from the next, permitting each to operate more economically.

The essential question is: At what point does the uncoupling function of inventory stop earning enough advantage to justify the investment required? To arrive at a satisfactory answer we must first distinguish between (a) inventories necessary because it takes time to complete an operation and to move the product from one stage to another; and (b) inventories employed for organizational reasons, i.e., to let one unit schedule its operations more or less independently of another.

(1) *Movement Inventories* - Inventory balances needed because of the time required to move stocks from one place to another are often not recognized, or are confused with inventories resulting from other needs - e.g., economical shipping quantities (to be discussed in a later section).

The average amount of movement inventory can be determined from the mathematical expression $I = S \ X \ T$ in which S represents the average sales rate, T the transit time from one stage to the next, and I the movement inventory needed. For example, if it takes two weeks to move materials from the plant to a warehouse, and the warehouse sells 100 units per week, the average inventory in movement is 100 units per week times 2 weeks, or 200 units. From a different point of view, when a unit is manufactured and ready for use at the plant, it must sit idle for two weeks while being moved to the next station (the warehouse); so, on the average, stocks equal to two weeks' sales will be in movement.

Movement inventories are usually thought of in connection with movement between distant points-plant to warehouse. However, any plant may contain substantial stocks in movement from one operation to another-for example, the product moving along an assembly line. Movement stock is one component of the "float" or in-process inventory in a manufacturing operation.

The amount of movement stock changes only when sales or the time in transit is changed. Time in transit is largely a result of method of transportation, although improvements in loading or dispatching practices may cut transit time by eliminating unnecessary delays. Other somewhat more subtle influences of time in transit on total inventories will be described in connection with safety stocks.

(2) *Organization Inventories* - Management's most difficult problems are with the inventories that "buy" organization in the sense that the more of them management carries between stages in the manufacturing-distribution process, the less coordination is required to keep the process running smoothly. Contrariwise, if inventories are already being used efficiently, they can be cut only at the expense of greater organization effort - e.g., greater scheduling effort to keep successive stages in balance, and greater expediting effort to work out of the difficulties which unforeseen disruptions at one point or another may cause in the whole process.

Despite superficial differences among businesses in the nature and characteristics of the organization inventory they maintain, the following three functions are basic: business. They are maintained wherever the user makes or purchases material in larger lots than are needed for his immediate purposes. For example, it is common practice to buy raw materials in relatively large quantities to order to obtain quantity price discounts, keep shipping costs in balance, and hold down clerical costs connected with making out requisitions, checking receipts, and handling accounts payable. Similar reasons lead to long production runs on equipment calling for expensive setup, or to sizable replenishment orders placed on factories by field warehouses.

(3) *Fluctuation stocks*, also very common in business, are held to cushion the shocks arising basically from unpredictable fluctuations in consumer demand. For example, warehouses and retail outlets maintain stocks to be able to supply consumers on demand, even when the rate of consumer demand may show quite irregular and unpredictable fluctuations. In turn, factories maintain stocks to be in a position to replenish retail and field warehouse stocks in line with customer demands.

Short-term fluctuations in the mix of orders on a plant often make it necessary to carry stocks of parts of subassemblies, in order to give assembly operations flexibility in meeting orders as they arise while freeing earlier operations (e.g., machining) from the need to make momentary adjustments in schedules to meet assembly requirements. Fluctuation stocks may also be carried in semi-finished form in order to balance out the load among manufacturing departments when orders received during the current day, week, or month may put a load on individual departments which is out of balance with long-run requirements.

In most cases, anticipating all fluctuations is uneconomical, if not impossible. But a business cannot get along without some fluctuation stocks unless it is willing and able always to make its customers wait until the material needed can be purchased conveniently or until their orders can be scheduled into production conveniently. Fluctuation stocks are part of the price we pay for our general business philosophy of serving the consumers' wants (and whims) rather than having them take what they can get. The queues before Russian retail stores illustrate a different point of view.

(4) *Anticipation stocks* are needed where goods or materials are consumed on a predictable but changing pattern through the year, and where it is desirable to absorb some of these changes by building and depleting inventories rather than by changing production rates with attendant fluctuations in employment and additional capital capacity requirements. For example, inventories may be built up in anticipation of a special sale or to fill needs during a plant shutdown.

The need for seasonal stocks may also arise where materials (e.g., agricultural products) are produced at seasonally fluctuating rates but where consumption is reasonably uniform; here the problems connected with producing and storing tomato catsup are a prime example.[1]

Striking a Balance - The joker is that the gains which these organization inventories achieve in the way of less need for coordination and planning, less clerical effort to handle orders, and greater economies in manufacturing and shipping are not in direct proportion to the size of inventory. Even if the additional stocks are kept well balanced and properly located, the gains become smaller, while at the same time the warehouse, obsolescence, and capital costs associated with maintaining inventories rise in proportion to, or perhaps even at a faster rate than, the inventories themselves. To illustrate:

Suppose a plant needs 2,000 units of a specially machined part in a year. If these are made in runs of 100 units each, then twenty runs with attendant setup costs will be required each year.

If the production quantity were increased from 100 to 200 units, only ten runs would be required-a 50 percent reduction in setup costs, but a 100 percent increase in the size of a run and in the resulting inventory balance carried.

If the runs were further increased in length to 400 units each only five production runs during the year would be required-only 25 percent more reduction in setup cost's, but 200 percent more increase in run length and inventory balances.

The basic problem of inventory policy connected with the three types of inventories, which "buy" organization, is to strike a balance between the increasing costs and the declining return earned from additional stocks. It is because striking this balance is easier to say than to do, and because it is a problem that defies solution through an intuitive understanding alone, that the new analytical concepts are necessary.

INVENTORY COSTS

This brings us face to face with the question of the costs that influence inventory policy, and the fact, noted earlier, that they are characteristically not those recorded, at least not in directly available form, in the usual industrial accounting system. Accounting costs are derived under principles developed over many years and strongly influenced by tradition. The specific methods and degree of skill and refinement may be better in particular companies, but in all of them the basic objective of accounting procedures is to provide a fair, consistent, and conservative valuation of assets and a picture of the flow of values in the business.

In contrast to the principles and search for consistency underlying accounting costs, the definition of costs for production and inventory control will vary from time to time-even in the same company-according to the circumstances and the length of the period being planned for. The following criteria apply:

(1) *The costs shall represent "out-of-pocket" expenditures, i.e., cash actually paid out or opportunities for profit foregone.* Overtime premium payments are out-of-pocket; depreciation on equipment on hand is not. To the extent that storage space is available and cannot be used for other productive purposes, no out-of-pocket cost of space is incurred; but to the extent that storage space is rented (out-of-pocket) or could be used for other productive purposes (foregone opportunity), a suitable charge is justified. The charge for investment is based on the out-of-pocket investment in inventories or added facilities, not on the "book" or accounting value of the investment.

The rate of interest charged on out-of-pocket investment may be based either on the rate paid banks (out-of-pocket) or on the rate of profit that might reasonably be earned by alternative uses of investment (foregone opportunity), depending on the financial policies of the business. In some cases, a bank rate may be used on short-term seasonal inventories and an internal rate for long-term, minimum requirements.

Obviously, much depends on the time scale in classifying a given item. In the short run, few costs are controllable out-of-pocket costs; in the long run, all are.

(2) *The costs shall represent only those out-of-pocket expenditures or foregone opportunities for profit whose magnitude is affected by the schedule or plan.* Many overhead costs, such as supervision costs, are out-of-pocket, but neither the timing nor the size is affected by the schedule. Normal material and direct labor costs are unaffected in total and so are not considered directly; however, these as well as some components of overhead cost do represent out-of-pocket investments, and accordingly enter the picture indirectly through any charge for capital.

Direct Influence - Among the costs which directly influence inventory policy are (a) costs depending on the amount ordered, (b) production costs, and (c) costs of storing and handling inventory.

Costs that depend on the amount ordered - These include, for example, quantity discounts offered by vendors; setup costs in internal manufacturing operations and clerical costs of making out a purchase order; and, when capacity is pressed, the profit on production lost during downtime for setup. Shipping costs represent another factor to the extent that they influence the quantity of raw materials purchased and resulting raw stock levels, the size of intra-plant or plant-warehouse shipments, or the size and the frequency of shipments to customers.

Production costs - Beyond setup or change-over costs, which are included in the preceding category, there are the abnormal or non-routine costs of production whose size may be affected by the policies or control methods used. (Normal or standard raw material and direct labor costs are not significant in inventory control; these relate to the total quantity sold rather than to the amount stocked.) Overtime, shakedown, hiring, and training represent costs that have a direct bearing on inventory policy.

To illustrate, shakedown or learning costs show up wherever output during the early part of a new run is below standard in quantity or quality.[2] A cost of under capacity operation may also be encountered-for example, where a basic labor force must be maintained regardless of volume (although sometimes this can be looked on as part of the fixed facility cost, despite the fact that it is accounted for as a directly variable labor cost).

Costs of handling and storing inventory - In this group of costs affected by control methods and inventory policies are expenses of handling products in and out of stock, storage costs such as rent and heat, insurance and taxes, obsolescence and spoilage costs, and capital costs (which will receive detailed examination in the next section).

Inventory obsolescence and spoilage costs may take several forms, including. (1) outright spoilage after a more or less fixed period; (2) risk that a particular unit in stock or a particular product number will (a) become technologically unsalable, except perhaps at a discount or as spare parts, (b) go out of style, or (c) spoil.

Certain food and drug products, for example, have specified maximum shelf lives and must either be used within a fixed period of time or be dumped. Some kinds of style goods, such as many lines of toys, Christmas novelties, or women's clothes, may effectively "spoil" at the end of a season, with only reclaim or dump value. Some kinds of technical equipment undergo almost constant engineering change during their production life; thus component stocks may suddenly and unexpectedly be made obsolete.

Capital Investment - Evaluating the effect of inventory and scheduling policy upon capital investment and the worth of capital tied up in inventories is one of the most difficult problems in resolving inventory policy questions.

Think for a moment of the amount of capital invested in inventory. This is the out-of-pocket, or avoidable, cash cost for material, labor, and overhead of goods in inventory (as distinguished from the "book" or accounting value of inventory). For example, raw materials are normally purchased in accordance with production schedules; and if the production of an item can be postponed, buying and paying for raw materials can likewise be put off.

Usually, then, the raw material cost component represents a part of the out-of-pocket inventory investment in finished goods. However, if raw materials must be purchased when available (e.g., agricultural crops) regardless of the production schedule, the raw material component of finished product cost does not represent avoidable investment and therefore should be struck from the computation of inventory value for planning purposes.

As for maintenance and similar factory overhead items, they are usually paid for the year round, regardless of the timing of production scheduled; therefore these elements of burden should not be counted as part of the product investment for planning purposes. (One exception: if, as sometimes happens, the maintenance costs actually vary directly with the production rate as, for example, in the case of supplies, they should of course be included.)

Again, supervision, at least general supervision, is usually a fixed monthly cost which the schedule will not influence, and hence should not be included. Depreciation is another type of burden item representing a charge for equipment and facilities already bought and paid for; the timing of the production schedule cannot influence these past investments and, while they represent a legitimate cost for accounting purposes, they should not be counted as part of the inventory investment for inventory and production planning purposes.

In sum, the rule is this: for production planning and inventory management purposes, the investment value of goods in inventory should be taken as the cash outlay made at the time of production that could have been delayed if the goods were not made then but at a later time, closer to the time of sale.

Cost of Capital Invested. This item is the product of three factors: (a) the capital value of a unit of inventory, (b) the time a unit of product is in inventory, and (c) the charge or imputed interest rate placed against a dollar of invested cash. The first factor was mentioned above. As for the second, it is fixed by management's inventory policy decisions. But these decisions can be made economically only in view of the third factor. This factor depends directly on the financial policy of the business.

Sometimes businessmen make the mistake of thinking that cash tied up in inventories costs nothing, especially if the cash to finance inventory is generated internally through profits and depreciation. However, this implies that the cash in inventories would otherwise sit idle. In fact, the cash could, at least, be invested in government bonds if not in inventories. And if it were really idle, the cash very likely should be released to stockholders for profitable investment elsewhere.

Moreover, it is dangerous to assume that, as a "short-term" investment, inventory is relatively liquid and riskless. Businessmen say, "After all, we turn our inventory investment over six times a year." But, in reality, inventory investment may or may not be short-term and riskless, depending on circumstances. No broad generalization is possible, and each case must be decided on its own merits. For example:

- A great deal of inventory carried in business is as much a part of the permanent investment as the machinery and buildings. The inventory must be maintained to make operations possible as long as the business is a going concern. The cash investment released by the sale of one item from stock must be promptly reinvested in new stock, and the inventory can be liquidated only when the company is closed. How much more riskless is this than other fixed manufacturing assets?
- To take an extreme case, inventory in fashion lines or other types of products having high obsolescence carries a definite risk. Its value depends wholly on the company's ability to sell it. If sales are insufficient to liquidate the inventory built up, considerable losses may result.
- At the other extreme, inventory in stable product lines built up to absorb short-term seasonal fluctuations might be thought of as bearing the least risk, since this type of investment is characteristically short term. But even in these cases there can be losses. Suppose, for instance, that peak seasonal sales do not reach anticipated levels and substantially increased costs of storage and obsolescence have to be incurred before the excess inventory can be liquidated.

Finally, it might be pointed out that the cost of the dollars invested in inventory may be underestimated if bank interest rate is used as the basis, ignoring the risk-bearing or entrepreneur's compensation. How many businessmen are actually satisfied with uses of their companies' capital funds, which do not earn more than a lender's rate of return? In choosing a truly appropriate rate -a matter of financial policy-the executive must answer some questions:

1. Where is the cash coming from-inside earnings or outside financing?
2. What else could we do with the funds, and what could we earn?
3. When can we get the investment back out, if ever?
4. How much risk of sales disappointment and obsolescence is really connected with this inventory?
5. How much of a return do we want, in view of what we could earn elsewhere or in view of the cost of money to us and the risk the inventory investment entails?

Investment in Facilities. Valuation of investment in facilities is generally important only in long-run planning problems-as, for example, when increases in productive or warehouse capacity are being considered. (Where facilities already exist and are not usable for other purposes, and where planning or scheduling do not contemplate changing these existing facilities, investment is not affected.)

Facilities investment may also be important where productive capacity is taxed, and where the form of the plan or schedule will determine the amount of added capacity which must be installed either to meet the plan itself or for alternative uses. In such cases, considerable care is necessary in defining the facilities investment in order to be consistent with the principles noted above: i.e., that facilities investment should represent out-of-pocket investment, or, alternatively, foregone opportunities to make out-of-pocket investment elsewhere.

Customer Service - An important objective in most production planning and inventory control systems is maintenance of reasonable customer service. An evaluation of the worth of customer service, or the loss suffered through poor service, is an important part of the problem of arriving at a reasonable inventory policy. This cost is typically very difficult to arrive at, including as it does the paper-work costs of rehandling back orders and, usually much more important, the effect that dissatisfaction of customers may have on future profits.

In some cases it may be possible to limit consideration to the cost of producing the needed material on overtime or of purchasing it from the outside and losing the contribution to profit which it would have made. On the other hand, sometimes the possible loss of customers and their sales over a substantial tune may outweigh the cost of direct loss in immediate business, and it may be necessary to arrive at a statement of a "reasonable" level of customer service- i.e., the degree of risk of running out of stock, or perhaps the number of times a year the management is willing to run out of an item. In other cases, it may be possible to arrive at a reasonable maximum level of sales, which the company is prepared to meet with 100 percent reliability, being reconciled to have service suffer if sales exceed this level.

One of the uses of the analytic techniques described below is to help management arrive at a realistic view of the cost of poor service, or of the value of building high service, by laying out clearly what the cost in inventory investment and schedule changes is to achieve this degree of customer service. Sometimes when these costs are clearly brought home, even a 100 percent service-minded management is willing to settle for a more realistic, "excellent" service at moderate cost, instead of striving for "perfect" service entailing extreme cost.

OPTIMUM LOT SIZE

Now, with this background, let us examine in some detail one of the inventory problems which plague businessmen the most -that of the optimum size of lot to purchase or produce for stock. This happens also to be one of the oldest problems discussed in the industrial engineering texts-but this does not lessen the fact that it is one of the most profitable for a great many companies to attack today with new analytic techniques.

Common Practices - This problem arises, as mentioned earlier, because of the need to purchase or produce in quantities greater than will be used or sold. Thus, specifically, businessmen buy raw materials in sizable quantities-carloads, or even trainloads -in order to reduce the costs connected with purchasing and control, to obtain a favorable price, and to minimize handling and transportation costs. They replenish factory in-process stocks of parts in sizable quantities to avoid, where possible, the costs of equipment setups and clerical routines. Likewise, finished stocks maintained in warehouses usually come in shipments substantially greater than the typical amount sold at once, the motive again being, in part, to avoid equipment setup and paperwork costs and, in the case of field warehouses, to minimize shipping costs.

Where the same equipment is used for a variety of items, the equipment will be devoted first to one item and then to another in sequence, with the length of the run in any individual item to be chosen, as far as is economically possible, to minimize change-over cost from one item to another and to reduce the production time lost because of clean-out requirements during change-overs. Blocked operations of this sort are seen frequently, for example, in the petroleum industry, on packaging lines, or on assembly lines where change-overs from one model to another may require adjustment in feed speeds and settings and change of components.

In all these cases, the practice of replenishing stocks in sizable quantities compared with the typical usage quantity means that inventory has to be carried; it makes it possible to spread fixed costs (e.g., setup and clerical costs) over many units and thus to reduce the unit cost. However, one can carry this principle only so far, for if the replenishment orders become too large, the resulting inventories get out of line, and the capital and handling costs of carrying these inventories more than offset the possible savings in production, transportation, and clerical costs. Here is the matter, again, of striking a balance between these conflicting considerations.

Even though formulas for selecting the optimum lot size are presented in many industrial engineering texts,[3] few companies make any attempt to arrive at an explicit quantitative balance of inventory and change-over or setup costs. Why?

For one thing, the cost elements which enter into an explicit solution frequently are very difficult to measure, or are only very hazily defined. For example, it may be possible to get a fairly accurate measure of the cost of setting up a particular machine, but it may be almost impossible to derive a precise measure of the cost of making out a new production order. Again, warehouse costs may be accumulated separately on the accounting records, but these rarely show what the cost of housing an additional unit of material may be. In my experience the capital cost, or imputed interest cost, connected with inventory investment never appears on the company's accounting records.

Furthermore, the inventory is traditionally valued in such a way that the true incremental investment is difficult to measure for scheduling purposes. Oftentimes companies therefore attempt to strike only a qualitative balance of these costs to arrive at something like an optimum or minimum-cost reorder quantity.

Despite the difficulty in measuring costs-and indeed because of such difficulty-it is eminently worthwhile to look at the lot-size problem explicitly formulated. The value of an analytic solution does not rest solely on one's ability to plug in precise cost data to get an answer. An analytic solution often helps clarify questions of principle, even with only crude data available for use. Moreover, it appears that many companies today still have not accepted the philosophy of optimum reorder quantities from the over-all company standpoint; instead, decisions are dominated from the standpoint of some particular interest such as production or traffic and transportation. Here too the analytic solution can be of help, even when the cost data are incomplete or imperfect.

Case Example - To illustrate how the lot-size problem can be attacked analytically - and what some of the problems and advantages of such an attack are - let us take a fictitious example. The situation is greatly oversimplified on purpose to get quickly to the heart of the analytic approach.

Elements of the Problem. Brown and Brown, Inc., an automotive parts supplier, produces a simple patented electric switch on long-term contracts. The covering is purchased on the outside at $0.01 each, and 1,000 are used regularly each day, 250 days per year.

The castings are made in a nearby plant, and B. and B. sends its own truck to pick them up. The cost of truck operation, maintenance, and the driver amounts to $10 per trip.

The company can send the truck once a day to bring back 1,000 casings for that day's requirements, but this makes the cost of a casing rather high. The truck can go less frequently, but this means that it has to bring back more than the company needs for its immediate day-to-day purposes.

The characteristic "saw-tooth" inventory pattern which will result in shown in Exhibit 1, where 1,000 Q casings are picked up each trip (Q being whatever number of days' supply is obtained per replenishment trip). These are used up over a period of Q days. When the inventory is depleted again, another trip is made to pick up Q days' supply or 1,000 Q casings once more, and so on.

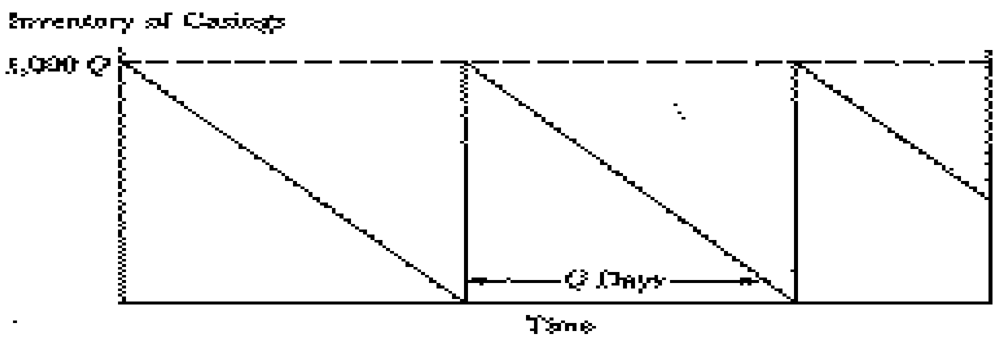

EXHIBIT 1

PATTERN OF INVENTORY BALANCE
(1,000 Q CASINGS OBTAINED PER REPLENISHMENT TRIP;
1,000 CASINGS USED PER DAY)

B. and B. estimates that the cost of storing casings under properly controlled humidity conditions is $1 per 1,000 casings per year. The company wants to obtain a 10 percent return on its inventory investment of $10 (1,000 times $0.01), which means that it should properly charge an additional $1 (10 percent of $10), making a total inventory cost of $2 per 1,000 casings per year.

(Note that, in order to avoid undue complications, the inventory investment charge is made here only against the purchase price of the casings and not against the total delivery cost including transportation. Where transportation is a major component of total cost, it is of course possible and desirable to include it in the base for the inventory charge.)

Graphic Solution. Brown and Brown, Inc., can find what it should do by means of a graph (see Exhibit 2) showing the annual cost of buying, moving, and sorting casings:

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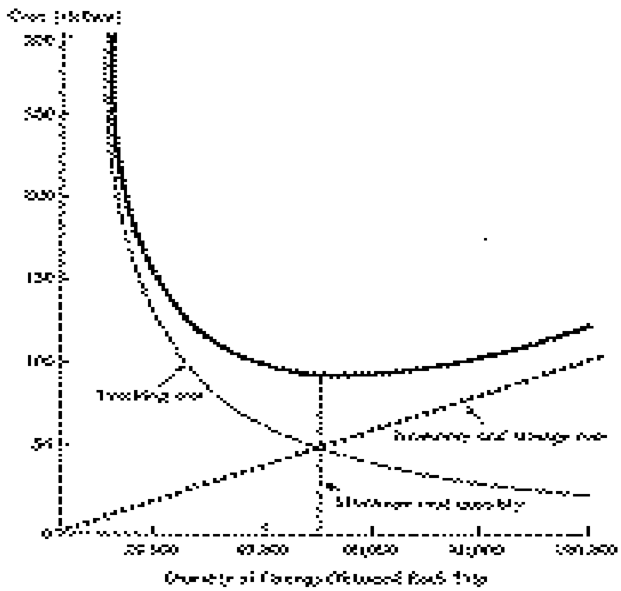

The broken line shows total trucking costs versus the size of the individual purchase quantity:

- If 1,000 casings are purchased at a time, the total cost is $10 times 250 trips, or $2,500 per year.
- If 10,000 casings are purchased at one time, only 25 trips need be made, for a total cost of $250 per year.
- If 100,000 casings are purchased, only 2 1/2 trips, on the average, have to be taken each year, for a total of $25.

The dotted line shows the inventory cost compared with the size of the purchased quantity:

- If 10,000 casings are purchased at one time, the inventory at purchase will contain 10,000, and it will gradually be depleted until none are on hand, when a new purchase will be made. The average inventory on hand will thus be 5,000 casings. The cost per year will be $2 times 5,000 casings, or $10.
- Similarly, if 100,000 casings are purchased at one time, the average inventory will be 50,000 casings, and the total inventory and storage cost will be $100.

The solid line is the total cost, including both trucking and inventory and storage costs. The total cost is at a minimum when 50,000 casings are purchased on each trip and five trips are made each year.

The solution to B. and B. 's problem can be reached algebraically as well as graphically. Exhibit 3 shows how the approach works in this very simple case.

Similar Cases The problem of Brown and Brown, Inc., though artificial, is not too far from the questions many businesses face in fixing reorder quantities.

Despite the simplifications introduced-for example, the assumption that usage is known in advance-the method of solution has been found widely useful in industries ranging from mail order merchandising (replenishing staple lines), through electrical equipment manufacturing (ordering machined parts to replenish stockrooms), to shoe manufacturing (ordering findings and other purchased supplies). In particular, the approach has been found helpful in controlling stocks made up of many low-value items used regularly in large quantities.

A number of realistic complications might have been introduced into the Brown and Brown, Inc., problem. For example:

• In determining the size of a manufacturing run, it sometimes is important to account explicitly for the production and sales rate. In this case, the inventory balance pattern looks like Exhibit 4 instead of the saw-tooth design in Exhibit 1. The maximum inventory point is not equal to the amount produced in an individual run, but to that quantity less the amount sold during the course of the run. The maximum inventory equals $Q (1 - S/P)$ where Q is the amount produced in a single run, and S and P are the daily sales and production rates respectively.

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EXHIBIT 3

EXAMPLE OF ALGEBRAIC SOLUTION OF SAME INVENTORY PROBLEM AS EXHIBIT 2

The total annual cost of supplying casings is equal to the sum of the direct cost of the casings, plus the trucking cost, plus the inventory and storage cost.

Let:

T = total annual cost
b = unit purchase price, $10 per 1,000 casings
s = annual usage, 250,000 casings
A = trucking cost, $10 per trip
N = number of trips per year
i = cost of carrying casings in inventory at the annual rate of $2 per 1,000, or $0.002 per casing
x = size of an individual purchase ($x/2$ = average inventory)

Then the basic equation will be:

$$T = bs + AN + ix/2$$

The problem is to choose the minimum-cost value of x (or, if desired, N). Since x is the same as s/N, N can be expressed as s/x. Substituting s/x for N in the above equation, we get:

$$T = bs + As/x + ix/2$$

From this point on we shall use differential calculus. The derivative of total cost, T, with respect to x will be expressed as:

$$dT/dx = -As/x^2 + i/2$$

And the minimum-cost value of x is that for which the derivative of total cost with respect to x equals zero. This is true when:

$$x = \sqrt{2As/i}$$

Substituting the known values for A, s, and i:

$$x = \sqrt{2 \cdot 10 \cdot 250{,}000/.002} = 50{,}000 \text{ casings}$$

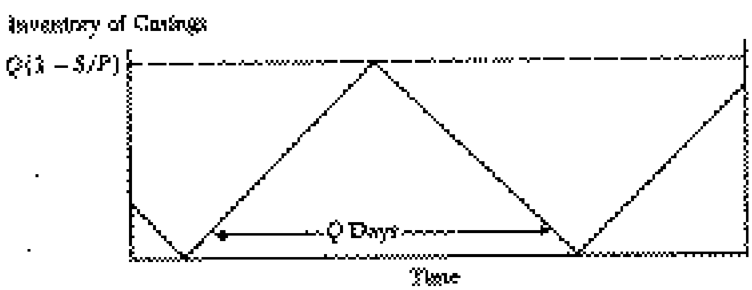

EXHIBIT 4

INFLUENCE OF PRODUCTION AND SALES RATE ON PRODUCTION CYCLE INVENTORY

This refinement can be important, particularly if the sales rate is fairly large compared with the production rate. Thus, if the sales rate is half the production rate, then the maximum inventory is only half the quantity made in one run, and the average inventory equals only one-fourth the individual run quantity. This means that substantially more inventory can be carried-in fact, about 40 percent more.

• When a number of products are made on a regular cycle, one after another, with the sequence in the cycle established by economy in change-over cost, the total cycle length can be obtained in the same ways as described above. Of course, it sometimes happens that there is a periodic breach in the cycle, either to make an occasional run of a product with very low sales or to allow for planned maintenance of equipment; the very simple run-length formulas can be adjusted to allow for this.

• Other kinds of costs can also be included, such as different sorts of handling costs. Or the inventory cost can be defined in such a way as to include transportation, obsolescence, or even capital and storage cost as part of the unit value of the product against which a charge for capital is made. When a charge for capital is included as part of the base value in computing the cost of capital, this is equivalent to requiring that capital earnings be compounded; this can have an important bearing on decisions connected with very low volume items which might be purchased in relatively large, long-lasting quantities.

Complications such as the foregoing, while important in practice, represent changes in arithmetic rather than in basic concept.

Significant Conclusions - When the analytic approach is applied to Brown and Brown's problem and similar cases, it reveals certain relationships which are significant and useful to executives concerned with inventory management:

(1) *The appropriate order quantity and the average inventory maintained do not vary directly with sales.* In fact, both of these quantities vary with the square root of sales. This means that with the same ordering and setup cost characteristics, the larger the volume of sales of an item, the less inventory per unit of sales is required. One of the sources of inefficiency in many inventory control systems is the rigid adoption of a rule for ordering or carrying inventory equivalent to, say, one month's sales.

(2) *The total cost in the neighborhood o f the optimum order quantity is relatively insensitive to moderately small changes in the amount ordered.* Exhibit 2 illustrates this proposition. Thus, all that is needed is just to get in the "right ball park," and a good answer can be obtained even with fairly crude cost data. For example, suppose the company had estimated that its total cost of holding 1,000 casings in inventory for a year was $1 when it actually was $2 (as in our illustration). Working through the same arithmetic, the company would have arrived at an optimum order quantity of 70,000 casings instead of 50,000. Even so, the total cost would have been (using the correct $2 annual carrying cost):

3.6 trips per year @ $10	= $36
35,000 casings average inventory @ $0.002	= 70
Total annual cost	=$106

Thus, an error of a factor of 2 in one cost results in only 6 percent difference in total cost.

In summary, Brown and Brown's problem, despite its oversimplification, provides an introduction to the analytic approach to inventory problems.

In particular, it illustrates the first essential in such an approach - i.e., defining an inventory function. In this case the function is to permit purchase or manufacture in economical order quantities or run lengths; in other cases it may be different. The important point is that this basic function can be identified wherever it may be found-in manufacturing, purchasing, or warehouse operation.

The only way to cut inventories is to organize operations so that they are tied more closely together. For example, a company can cut its raw materials inventory by buying in smaller quantities closer to needs, but it does so at a cost; this cost results from the increased clerical operations needed to tie the purchasing function more closely to manufacturing and to keep it more fully informed of manufacturing's plans and operation. The right inventory level is reached when the cost of maintaining any additional inventory cushion offsets the saving that the additional inventory earns by permitting the plant to operate in a somewhat less fully organized fashion.

B. and B. 's problem also illustrates problems and questions connected with defining and making costs explicit. The inventory capital cost is usually not found on a company's books, but it is implied in some of the disagreements over inventory policy. Here, again, bringing the matter into the open may help each side in a discussion to recognize its own and the others' hidden assumptions, and thus more quickly to reach a common agreement.

E N D N O T E S

[1] See Alexander Henderson and Robert Schlaifer, "Mathematical Programing: Better Information for Better Decision-making," *Harvard Business Review*, May-June 1954, p. 73. [This paper also appears in this volume.Ed.]

[2] See Frank J. Andress, "The Learning Curve as a Production Tool," *Harvard Business Review*, January-February 1954, p. 87

[3] See, for example, Raymond E. Fairfield, *Quantity and Economy in Manufacture* (New York, D. Van Nostrand Company, Inc., 1931).