Econometrica, Vol. 39, No. 5 (September, 1971)

CONGESTION TOLLS FOR COMMERCIAL AIRPORTS

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This paper discusses the role of congestion tolls in increasing the efficiency of use of commercial airports, in terms of a simple model of air transportation. In contrast to previous discussions, users and producers of transportation service (passengers and airlines) are explicitly distinguished. In the first version of the model, ticket prices are assumed—unrealistically—to be flexible and competitively determined; then perfect optimality is attainable by imposing an appropriate toll either on airlines or on passengers. In the more realistic second version of the model, ticket price is fixed above the competitive level. In the absence of a toll, there are two inefficiencies: the level of transportation is non-optimal, and it is produced inefficiently, using partially loaded airplanes. An appropriate toll on airlines can do much to correct both of these inefficiencies, and is always superior to the best toll on passengers.

PREVIOUS DISCUSSIONS of congestion tolls[2] as a means of optimizing the use of given transportation facilities have not distinguished between the *user* and the *producer* of transportation services.[3] The usual example has been road transportation by private automobile, a case in which there is no need for such a distinction. But the distinction may be relevant in other cases. One such case is air transportation, the focus of recent interest because of lengthy airplane delays at several major airports. In arriving at an optimal congestion toll for this case, it may be important to distinguish between airlines and their passengers. In particular, does it matter on whom the toll is imposed?

I shall explore this question, and more generally, the role of congestion tolls at commercial airports, with the help of a simple model of air transportation. In the first version of the model, ticket prices are assumed—unrealistically—to be flexible and competitively determined. In that case, as one would expect, the toll may be applied indifferently to airlines or to passengers. In a second, more realistic version of the model, ticket prices are assumed to be fixed at something above the competitive level; competition among airlines is based on schedules, not price. With ticket price fixed, perfect optimality is not always attainable; all one can hope for in general is the choice of a toll that will effect a second-best allocation.[4] And then it does make a difference whether the toll is collected from airlines or from passengers.

Both versions of the model incorporate many simplifying assumptions. All variables are continuous. There is only a single destination. Traffic is homogeneous;

[1] I should like to thank Stephen Carroll and the referees for their comments. Any views expressed in this paper are my own. They should not be interpreted as reflecting the views of The RAND Corporation or the official opinion or policy of any of its governmental or private research sponsors.

[2] See, for example, Beckmann, McGuire, and Winsten [1], Strotz [8], and Vickrey [9].

[3] The distinction has, of course, been made in other contexts. A referee cites Buchanan [2] and Knight [5].

[4] See Marchand [7] for derivation of the optimal congestion toll in another sort of imperfect environment.

only airlines use the airport, not "general aviation."[5] Variations in demand over time are disregarded. But the simplified model does yield considerable insight into the role of congestion tolls in more complex and realistic situations, as discussed in the concluding section.

1. THE OPTIMUM

In the model, the total value, V, of transportation provided per unit time, is a function of the number of passengers, N, and the level of delay, D:

(1) $$V = V(N, D), \qquad V_N > 0, \qquad V_{NN} < 0, \qquad V_D < 0, \qquad V_{DN} < 0.$$ [6]

That is, the value of an additional trip is positive but smaller the more trips are taken; an increase in delay reduces value, and more greatly reduces value the more trips are affected.

Delay is an increasing function of the number of flights, n:

(2) $$D = D(n), \qquad D' > 0, \qquad D'' > 0.$$ [7]

The cost per flight, c, is an increasing function of delay:

(3) $$c = c(D), \qquad c' > 0, \qquad c'' \geqslant 0.$$

It is convenient to have a symbol for total cost, so I define

(4) $$C \equiv nc.$$

Note that cost is assumed not to depend on the number of passengers, but only on the number of flights.

For an optimum, we seek values of N and n to maximize the surplus, S, of the value of transportation over the cost of providing it, subject to the constraint that the number of passengers per flight cannot exceed the fixed airplane size, $\bar{A}$. That is, maximize

(5) $$S = V[N, D(n)] - nc[D(n)]$$

subject to

	primal constraint	dual variable
(6)	$\bar{A} - N/n \geqslant 0,$	$\alpha.$

Kuhn-Tucker conditions for a maximum[8] are

	dual constraint	primal variable
(7)	$V_N - \alpha/n \leqslant 0,$	$N,$
(8)	$V_D D' - nc'D' - c + \alpha N/n^2 \leqslant 0,$	$n,$

[5] General aviation, consisting primarily of relatively small aircraft, is a category that includes air taxis and business and private planes.

[6] V_N denotes $\partial V/\partial N$, V_{NN} denotes $\partial^2 V/\partial N^2$, etc.

[7] D' denotes dD/dn, D'' denotes d^2D/dn^2, etc.

[8] Kuhn and Tucker [**6**].

and either each constraint holds as an equality, or the corresponding variable is zero. Throughout this paper, the Kuhn-Tucker conditions are both necessary and sufficient for a maximum.[9]

Because N and n are positive in any interesting case, we take (7) and (8) to be equalities. From (7), $\alpha = nV_N$, the value imputed to a marginal increase in airplane size. Since $\alpha > 0$, (6) must hold as an equality; rather obviously, efficiency requires that planes fly fully loaded.

Eliminating α from (7) and (8), we have

$$(9) \qquad V_N = \frac{n}{N}(-V_D D' + nc'D' + c)$$

which, together with (6) as an equality, determines optimal values of N and n. Then (5) gives maximal S. I shall denote these particular values of the variables by appending the section number as a subscript, that is, by N_1, n_1, and S_1. The other variables are determined by (1) through (4). As the need arises, I shall use subscript 1 to denote their optimal values as well.

Note that an allocation is completely determined by specifying N and n, or equivalently, by specifying N and N/n. Optimal conditions (6) as an equality and (9) may be readily interpreted in the latter terms. Condition (6) requires that whatever level of transportation is provided should be provided efficiently, i.e., should make use of fully loaded planes; $N/n = \bar{A}$. Condition (9) then determines the efficient level of transportation, $N = N_1$.

Figure 1 shows the optimum graphically. The curves $V(N, D_1)$, $V(N, D_2)\ldots$ show value as a function of N for specified constant levels of delay. Not all points on these curves are attainable. Maximal attainable value for any particular number of passengers results from minimal delay, which in turn results from using the minimal number of (full) airplanes needed to carry that many passengers. The curve $V(\bar{A})$[10] represents this upper limit on attainable value. Similarly, the lowest attainable cost of transporting N passengers is attained using $N/\bar{A}$ planes; the minimal cost curve is shown as $C(\bar{A})$.[11] Obviously, the maximal surplus occurs where the maximal value and minimal cost curves are farthest apart, at N_1.[12]

2. FLEXIBLE TICKET PRICE, NO TOLL

In the absence of a congestion toll, the competitive equilibrium is not an optimum. Competition among airlines reduces profits to zero (or to a "normal" level included in the definition of c). Denoting ticket price by p, the zero profit condition is

$$(10) \qquad nc = pN.$$

[9] Enthoven [**4**, pp. 389–390].

[10] Shorthand label for the graph of $V[N, D(N/\bar{A})]$.

[11] Shorthand for $(N/\bar{A}) \cdot c[D(N/\bar{A})]$.

[12] The geometry provides an alternative route to optimal condition (9). Along $V(\bar{A})$, $V = V[N, D(n)]$ and $n = N/\bar{A}$. Taking total differentials, $dV = V_N\,dN + V_D D'\,dn$ and $dn = dN/\bar{A}$. Substituting for dn and dividing through by dN, we find the slope of $V(\bar{A})$ to be $dV/dN = V_N + (1/\bar{A})V_D D'$. In a similar manner, we find the slope of $C(\bar{A})$ to be $dC/dN = (1/\bar{A})(c + nc'D')$. Equating dV/dN and dC/dN yields (9).

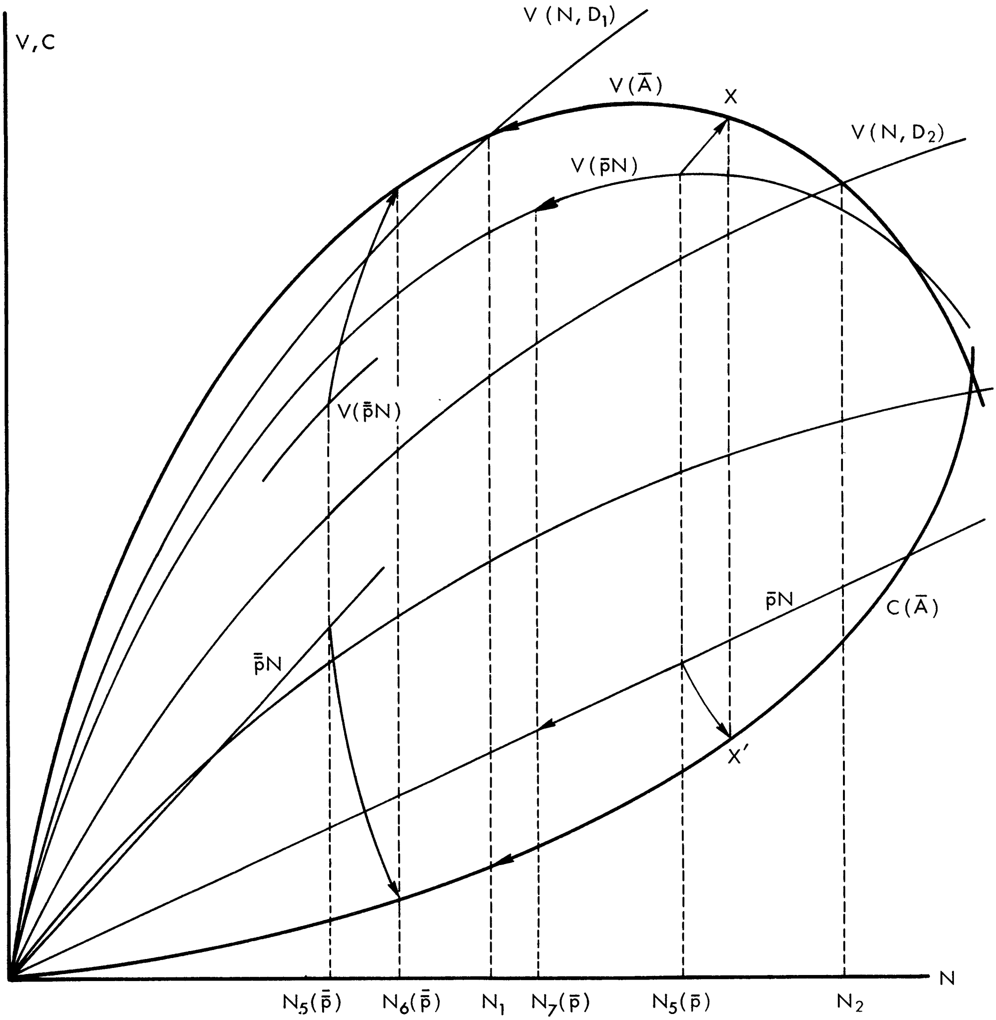

FIGURE 1

The marginal passenger values his trip at V_N. For equilibrium in the passenger market,

(11) $$V_N = p.$$

With flexible ticket price, competition assures that whatever level of transportation service is provided, is provided efficiently in the sense that airplanes fly fully loaded. For otherwise, one airline could lower the ticket price infinitesimally, attract a full planeload of passengers, and make a profit. Competitive airlines would follow suit until the profit opportunity was eliminated when all planes were full. Thus (6) holds as an equality. In Figure 1, value and cost are somewhere on $V(\bar{A})$ and $C(\bar{A})$.

But competition does not assure that the optimal level of transportation is provided. From (10) and (11), we have $V_N = nc/N$, violating condition (9) for an optimum. In fact, a larger than optimal level of transportation is provided because each airline ignores the costs that a marginal flight imposes on others. At N_1, we have from (9), using (1), (2), and (3), that $V_N > nc/N$. In Figure 1, increasing N decreases V_N along $V(\bar{A})$,[13] and increases nc/N along $C(\bar{A})$,[14] until equality is reached at $N_2 > N_1$.

Conditions (10), (11), and (6) as an equality determine competitive equilibrium values of the variables, $N_2 > N_1$, $n_2 = N_2/\bar{A}$, and p_2. Equation (5) then gives $S_2 < S_1$.

3. FLEXIBLE TICKET PRICE, TOLL ON AIRLINES

One way to bring about an optimal level of transportation would be to impose a congestion toll, or flight fee, of F per flight on the airlines. In this case, the zero profit condition becomes

$$nc + nF = pN. \tag{12}$$

To find the appropriate value of F, one may solve the following constrained maximization problem: maximize

$$S = V[N, D(n)] - nc[D(n)] \tag{5}$$

subject to

	primal constraint	dual variable
(12)	$nc[D(n)] + nF - pN = 0,$	$\pi,$
(11)	$V_N - p = 0,$	$\mu,$
(6)	$\bar{A} - N/n \geqslant 0,$	$\alpha.$

The Kuhn-Tucker conditions are

	dual constraint	primal variable
(13)	$V_N - \pi p + \mu V_{NN} - \alpha/n = 0,$	$N,$
(14)	$V_D D' - c - nc'D' + \pi c + \pi nc'D' + \pi F + \mu V_{ND}D' + \alpha N/n^2 = 0,$	$n,$
(15)	$-\pi N - \mu = 0,$	$p,$
(16)	$\pi n = 0,$	$F,$

with equality holding in each case because all primal variables are presumptively positive.

[13] On $V(\bar{A})$, $d(V_N) = V_{NN}dN + V_{ND}D'dn$ and $dn = dN/\bar{A}$, so $d(V_N)/dN = V_{NN} + V_{ND}D'/\bar{A} < 0$ by (1) and (2).

[14] On $C(\bar{A})$, $d(nc/N)/dN = (1/\bar{A})c'D'(1/\bar{A}) > 0$ by (2) and (3).

The constraints can be solved for the optimal toll, F. Using all except (6), we find

(17) $F = -V_D D' + nc'D'.$

This expression for F is precisely what previous discussions of congestion tolls would lead one to expect. The first term represents the reduction in value to passengers of transportation services due to increased delay caused by a marginal flight. The second term represents the increased operating cost imposed on all other flights by a marginal flight. Thus the optimal toll is simply the delay costs imposed on all others—airlines and passengers—by a marginal flight.[15]

With flexible ticket price, the appropriate toll results in perfect optimality. Competition and flexible ticket price assure that planes fly fully loaded, so (6) holds as an equality. We thus have four equations, (6), (12), (11), and (17), to determine equilibrium values N_3, n_3, p_3, and F_3. Since (12), (11), and (17) imply optimal condition (9), $N_3 = N_1$ and $n_3 = N_3/\bar{A}$; hence $S_3 = S_1$.

4. FLEXIBLE TICKET PRICE, TOLL ON PASSENGERS

Alternatively, one might try to encourage an optimal level of transportation by imposing a congestion toll or head tax, H, on passengers. Then the condition for passenger market equilibrium becomes

(18) $V_N = p + H.$

The problem then is to maximize (5) subject to (10), (18), and (6).

The Kuhn-Tucker conditions again make it possible to solve for the optimal head tax; we find

(19) $H = \frac{n}{N}(-V_D D' + nc'D').$

That is, the expression for optimal toll on passengers simply divides the optimal toll per flight given by (17) equally among the passengers per flight.

Again, perfect optimality is attained. Competition and flexible ticket price assure that (6) is an equality. Conditions (6), (10), (18), and (19) determine equilibrium values N_4, n_4, p_4, and H_4. Conditions (10), (18), and (19) imply (9), so $N_4 = N_1$, $n_4 = N_4/\bar{A}$; hence $S_4 = S_1$.[16]

With flexible ticket price, it does not matter whether the toll is imposed on airlines or on passengers. If it is imposed on airlines, competitive ticket price adjustments pass it on to passengers, who end up paying the same in either case; $p_3 = p_4 + H_4$.[17] Also, airline revenue net of any flight fee is the same in either case; $p_3\bar{A} - F_3 = p_4\bar{A}$.[18]

[15] I assume for simplicity that each airline is small enough so that the delay it imposes on its own flights is negligible, compared with the delay it imposes on others.

[16] One also notes that perfect optimality is attainable using any *combination* of flight fee and head tax such that $(N/n)H + F = -V_D D' + nc'D'$.

[17] From (11) and (18), because $(V_N)_3 = (V_N)_4$.

[18] From (12) and (10) using (6) as an equality, because $N_3 = N_4$ and $n_3 = n_4$.

5. FIXED TICKET PRICE, NO TOLL

More realistically, we recognize that adjustments to airline ticket prices are at best a sticky business, and we investigate the situation in which the ticket price is fixed at something above the competitive level. Airlines still compete in this version of the model, but they compete solely through schedules. That is, they add flights until load factors are forced down to break-even levels.

In the model, we simply set ticket price equal to a constant, $p = \bar{p} > p_2$. The zero profit condition is then

$$(20) \qquad nc(n) = \bar{p}N,$$

and the condition for passenger market equilibrium is

$$(21) \qquad V_N = \bar{p}.$$

Conditions (20) and (21) determine equilibrium values N_5 and n_5, whence (5) gives S_5. Necessarily $N_5 < N_2$; within the feasible region on and below $V(\bar{A})$ in Figure 1, it is only to the left of N_2 that one finds $V_N > (V_N)_2$.[19] The value of n_5 may be $\gtreqless n_2$. From (20) and (10), $n_5 c_5 \gtreqless n_2 c_2$ (and hence $n_5 \gtreqless n_2$) as $\bar{p}N_5 \gtreqless p_2 N_2$. That is, if passenger demand is elastic between N_2 and N_5, $n_5 < n_2$; if it is inelastic, $n_5 > n_2$.

In any case, planes fly less than fully loaded if ticket price is fixed above the competitive level.[20] If $n_5 \geqslant n_2$, this follows immediately from $N_5 < N_2$. If $n_5 < n_2$, from (20) and (10) $\bar{p} > p_2$ implies $n_5 c_5/N_5 > n_2 c_2/N_2$, or equivalently $N_5/n_5 < (N_2/n_2)(c_5/c_2)$. Since $c_5 < c_2$ if $n_5 < n_2$, it follows a fortiori that $N_5/n_5 < N_2/n_2 = \bar{A}$.

Interestingly, the surplus S_5 with ticket price fixed above the competitive level may be greater than the surplus S_2 when ticket price is flexible. Although N_5 is not produced efficiently, it may be closer to the optimal level of transportation N_1 than is N_2. The gain from the latter effect may more than offset the loss from the former.

The expression for C_5 is given by (20) as a linear function of N; with ticket price fixed at $\bar{p}$, total cost lies along the line $\bar{p}N$ in Figure 1. (This line lies above $C(\bar{A})$ at N_2 because $\bar{p}N_2 > p_2 N_2 = C_2$.) For each N, (20) determines a corresponding n, hence D, hence V. The value curve corresponding to $\bar{p}N$ is labeled $V(\bar{p}N)$. Where $\bar{p}N$ intersects $C(\bar{A})$, $n = N/\bar{A}$, so $V(\bar{p}N)$ intersects $V(\bar{A})$. In between, $n > N/\bar{A}$, so $V(\bar{p}N)$ lies under $V(\bar{A})$ as shown. Equilibrium traffic $N_5(\bar{p})$ is found at the point on $V(\bar{p}N)$ where $V_N = \bar{p}$. In the figure, $S_5(\bar{p}) > S_2$. But this is of course not necessarily the case. With ticket price fixed even higher, say at $\bar{\bar{p}}$, we find $S_5(\bar{\bar{p}}) < S_2$.

6. FIXED TICKET PRICE, TOLL ON AIRLINES

To find the optimal flight fee to impose on airlines when ticket price is fixed at $\bar{p}$, maximize

$$(5) \qquad S = V[N, D(n)] - nc[D(n)]$$

[19] Based on (1).

[20] A referee stresses that this is a consequence of my assumption that there are competing airlines. A profit maximizing monopoly airline would fly fully loaded planes.

subject to

	primal constraint	dual variable
(22)	$nc[D(n)] + nF - \bar{p}N = 0,$	$\pi,$
(23)	$V_N - \bar{p} \geqslant 0,$	$\mu,$
(6)	$\bar{A} - N/n \geqslant 0,$	$\alpha.$

In (23), we must allow for the possibility of excess demand in the passenger market; with the optimal toll in effect, airlines may schedule fewer flights than necessary to accommodate all passengers willing to pay $\bar{p}$ for a ticket.

The Kuhn-Tucker conditions are

	dual constraint	primal variable
(24)	$V_N - \pi\bar{p} + \mu V_{NN} - \alpha/n = 0,$	$N,$
(14)	$V_D D' - c - nc'D' + \pi c + \pi nc'D' + \pi F + \mu V_{ND} D' + \alpha N/n^2 = 0,$	$n,$
(16)	$\pi n = 0,$	$F.$

We must consider two cases. In the first case, there is excess demand in the passenger market; (23) is an inequality. Then $\mu = 0$, so from (24), using (16), $\alpha > 0$, so (6) holds as an equality. Eliminating α from (24) and (14) and substituting for c in (22) gives

$$F = -V_D D' + nc'D' - \frac{N}{n}(V_N - \bar{p}). \tag{25}$$

As in the flexible ticket price case, this toll may be interpreted as the (net) external cost of a marginal flight. The first two terms, as before, represent congestion costs imposed on passengers and other airlines. In this case, though, they are partially offset by an external benefit represented by the third term. A marginal flight would accommodate N/n passengers, each of whom values the trip at V_N, but pays a lesser amount $\bar{p}$ for his ticket.

With excess demand in the passenger market, some mechanism is needed to assure that the N_6 passengers are in fact the N_6 who place the highest value on the trip. One possibility is a black market in which tickets are freely traded. Another is a head tax equal to the difference between the market clearing ticket price p_3 and $\bar{p}$.[21]

If some such ticket allocating mechanism is assumed to be in operation, perfect optimality is again attained if the toll given by (25) is in effect. Excess demand in the passenger market assures that (6) is an equality. Substituting F from (25) in (22), we get

$$V_N = \frac{n}{N}(-V_D D' + nc'D' + c); \tag{9}$$

[21] In practice, one would expect an eventual upward adjustment of $\bar{p}$ to the market clearing level.

with the optimal toll, competition among airlines precisely fulfills the second condition for optimality.

This first case is illustrated in Figure 1. Starting at the competitive equilibrium with no toll at $N_5(\bar{p})$, we can trace the equilibrium path as successively higher flight fees are levied. As F is increased, airlines find it necessary to decrease n, decreasing C, in order to continue to break even. At the resulting lower level of delay, more passengers find their trip worth at least $\bar{p}$, so N increases. As long as N remains less than $\bar{A}n$, the passenger market is in equilibrium; V is along the positively sloped line through $V(\bar{p}N_5)$ that connects points on the value curves for which $V_N = \bar{p}$. Continued increases in F increase V to X and decrease C to X'; at that point, planes are full. Further increases in F continue to reduce n, and these reductions now entail proportionate reductions in N. Value V moves along $V(\bar{A})$ and C along $C(\bar{A})$ until the optimum is reached at N_1.

In the second case, there is no excess demand in the passenger market; (23) holds as an equality. Still, we have from (14), using (16), that $\alpha > 0$, so (6) also holds as an equality. F follows from (22) as

(26) $$F = \bar{p}\bar{A} - c.$$

With this flight fee in effect, competition among airlines assures that planes fly fully loaded; that is, (26) and (22) imply $N/n = \bar{A}$. The amount of transportation provided is then determined by (23) as an equality.

This case is illustrated in Figure 1 starting at $N_5(\bar{\bar{p}})$. Successively higher tolls reduce n and C, as N increases due to decreasing delay, until the second-best allocation is attained at $N_6(\bar{\bar{p}})$. At that point, airplanes fly fully loaded. Further increases in F would reduce both n and N. Since $N_6(\bar{\bar{p}}) < N_1$, this would reduce S. In this case the optimal toll is just sufficient to result in full loads.

The first case occurs when $\bar{p} < p_3 = p_4 + H_4$. An appropriate flight fee then results in fully loaded planes, and also restricts output to the optimal level. The second case occurs when $\bar{p} > p_3$. The best flight fee then insures fully loaded planes, but is powerless to expand output to the optimal level.

In summary, we have for this section $N_6 \leqslant N_1$, $n_6 = N_6/\bar{A}$, and $S_6 \leqslant S_1$.

7. FIXED TICKET PRICE, TOLL ON PASSENGERS

Now consider the alternative of levying a toll or head tax, H, on passengers to promote more efficient use of the airport, when ticket price is fixed at $\bar{p}$. To find the appropriate H, maximize

(5) $$S = V[N, D(n)] - nc[D(n)]$$

subject to

	primal constraint	dual variable
(20)	$nc[D(n)] - \bar{p}N = 0,$	$\pi,$
(27)	$V_N - \bar{p} - H = 0,$	$\mu,$
(6)	$\bar{A} - N/n \geqslant 0,$	$\alpha.$

The Kuhn-Tucker conditions are

	dual constraint	primal variable
(24)	$V_N - \pi\bar{p} + \mu V_{NN} - \alpha/n = 0,$	$N,$
(28)	$V_D D' - c - nc'D' + \pi c + \pi nc'D' + \mu V_{ND} D' + \alpha N/n^2 = 0,$	$n,$
(29)	$-\mu = 0,$	$H.$

By (20), as argued in Section 5, $\bar{A} - N/n > 0$, so $\alpha = 0$. From (24) and (28), then,

$$V_N = \pi\bar{p} = \left(1 + \frac{-V_D D'}{c + nc'D'}\right)\bar{p}. \tag{30}$$

Comparing (30) and (27), and substituting from (20) for $\bar{p}$, we find

$$H = \frac{-(n/N)V_D D'}{1 + (n/c)c'D'}. \tag{31}$$

Although it is not immediately obvious that this is the cost imposed on others by a marginal passenger, that is once again the proper interpretation. Taking total differentials in (5) and (20), we have

$$dS = V_N\, dN + V_D D'\, dn - cdn - nc'D'\, dn \tag{32}$$

and

$$dn = \frac{\bar{p}}{nc'D' + c}\, dN. \tag{33}$$

Substituting from (33) into (32) and dividing by dN we find the change in value due to a marginal passenger to be

$$\frac{dS}{dN} = V_N + \frac{V_D - c - nc'D'}{nc'D' + c}\bar{p}. \tag{34}$$

In the absence of a head tax, the passenger considers only value to himself less ticket price, $V_N - \bar{p}$. Deducting this from (34) leaves an external cost equal to $V_D D'\bar{p}/(nc'D' + c)$, which, upon substitution for $\bar{p}$ from (20), is seen to be precisely the cost measured by the optimal toll.

With H in effect, we have (20), (27), and (31) to determine equilibrium values N_7, n_7, and H_7. These do not represent an optimum; we know from (20) that $n_7 > N_7/\bar{A}$, violating one necessary condition for an optimum.

In Figure 1, we can trace the effects of successively higher tolls, starting at $N_5(\bar{p})$ with no toll in effect. Increasing H decreases N.[22] Thus as H increases, C decreases

[22] From (27), $dH = V_{NN}\, dN + V_{ND}D'\, dn$. Using (33), we find

$$\frac{dN}{dH} = \frac{nc'D' + c}{V_{NN}(nc'D' + c) + V_{ND}D'\bar{p}}.$$

This is negative by (1), (2), and (3).

along $\bar{p}N$. V moves left along the value curve corresponding to the N/n implicit in $\bar{p}N$, that is, along $V(\bar{p}N)$. V may increase or decrease, but in any case it does not initially decrease as fast as does C; from (34), using (21), $dS/dN > 0$ at $N_5(\bar{p})$. Further increases in H continue to reduce N and increase S until $\bar{p}N$ and $V(\bar{p}N)$ are farthest apart at $N_7(\bar{p})$.

Exactly the same argument applies if one starts with a higher fixed ticket price at $N_5(\bar{\bar{p}})$. Thus we have in general that $N_7 < N_5$, $n_7 < N_7/\bar{A}$, and $S_7 < S_1$.

It is apparent that, with fixed ticket price, a head tax is inferior to a flight fee as a congestion toll. For $\bar{p} \leqslant p_3$, the appropriate flight fee results in perfect optimality, and the appropriate head tax does not. For $\bar{\bar{p}} > p_3$, neither achieves perfect optimality, but the flight fee comes closer. F_6 leads to a level of transportation $N_6(\bar{\bar{p}}) > N_5(\bar{\bar{p}})$, and assures that it is provided efficiently, with $n_6(\bar{\bar{p}}) = N_6(\bar{\bar{p}})/\bar{A}$. H_7 leads to a level of transportation $N_7(\bar{\bar{p}}) < N_5(\bar{\bar{p}})$, and leaves it inefficiently provided, with $n_7(\bar{\bar{p}}) > N_7(\bar{\bar{p}})/\bar{A}$. Since both $N_6(\bar{\bar{p}})$ and $N_7(\bar{\bar{p}}) < N_1$, it follows that $S_7(\bar{\bar{p}}) < S_6(\bar{\bar{p}})$.

8. CONCLUSION

I have examined the nature and function of congestion tolls in a model that reflects one important aspect of commercial air transport: transportation services are produced and sold to passengers at a price fixed above the competitive level. Airlines, competing for passengers, schedule flights until passenger loads are forced down to break-even levels. In the absence of a toll, this involves two sorts of inefficiencies. As in previous congestion models, the level of transportation provided is non-optimal. In addition, it is produced inefficiently, using partially loaded airplanes.

We have seen that an appropriate toll on airlines can do much to correct both of these inefficiencies. It assures that planes fly fully loaded. If ticket price is not too far above the competitive level, it results in provision of the optimal level of transportation as well. Otherwise, it at least results in some increase in transportation to a more nearly optimal level, although perfect optimality is not attained. In contrast, the best toll on passengers leaves transportation inefficiently provided; in fact, it even reduces average passenger loads.[23] In the model, a toll on airlines is always superior to a toll on passengers.

The model incorporates many simplifying assumptions in order to highlight one important but little discussed function of congestion tolls at commercial airports: to increase passenger loads to more efficient levels. Among the most important assumptions are those listed at the beginning of the paper: continuous variables, a single destination, homogeneous traffic, and no time variation in demand.

Relaxing the assumptions of continuity and single destination softens the requirement that airplanes fly fully loaded. For when flights are scheduled as

[23] From (20), $d(N/n) = (1/\bar{p})c'D'dn$ and $dn = (\bar{p}/(c + nc'D'))dN$. Therefore $d(N/n)/dN = c'D'/(c + nc'D') > 0$ by (2) and (3). From Footnote 22, $dN/dH < 0$; hence $d(N/n)/dH < 0$.

discrete events to many destinations, frequency of service may be valued. Particularly on low density routes, it may be efficient to fly planes that are less than fully loaded in order to provide a satisfactory frequency of service. Casual observation of statistics on average load factors—typically on the order of fifty per cent—suggests, however, that the present situation is far from optimal.

If one relaxes the assumption of homogeneous traffic, congestion tolls assume another important function. Even at the busiest commercial airports, a significant fraction of traffic is relatively low-value general aviation. An important function of congestion tolls is to discourage such low-value use at times when it imposes high delay costs on others. Finally, from recognition of the existence of peaks in airport usage, it is apparent that the optimal toll will vary over time. For an explicit treatment of these two aspects of the problem, see Carlin and Park [**3**].

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Manuscript received September, 1969; revision received November, 1969

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