Diagnosability of Enhanced Hypercubes

Dajin Wang, *Member, IEEE*

Abstract—**An enhanced hypercube is obtained by adding 2^{n-1} more links to a regular hypercube of 2^n processors. It has been shown that enhanced hypercubes have very good improvements over regular hypercubes in many measurements such as mean internode distance, diameter and traffic density. This paper proves that in the aspect of diagnosability, enhanced hypercubes also achieve improvements. Two diagnosis strategies, both using the well-known PMC diagnostic model, are studied: the precise (one-step) strategy proposed by Preparata et al. and the pessimistic strategy proposed by Friedman. Under the precise strategy, the diagnosability is shown to be increased to $n+1$ in enhanced hypercubes (in regular hypercubes the diagnosability is n under this strategy). Under the pessimistic strategy, the diagnosability is shown to be increased to $2n$ (in regular hypercubes the diagnosability under this strategy is $2n-2$). Since the failure probability of one node is fairly low nowadays so that the increase of diagnosability by one or two will considerably enhance the system's self-diagnostic capability, and considering the fact that diagnosability does not "easily" increase as the links in networks do, these improvements are noticeable.**

Index Terms—**Diagnosability, diagnosis, fault tolerance, graph theory, hypercubes, multicomputer networks.**

I. Introduction and Preliminaries

The interconnections of a set of processors can be adequately represented by a graph $G=(V,E)$, where each node $v_i \in V$ represents a processor and each edge $\{v_i, v_j\} \in E$ represents a link between v_i and v_j. Two linked processors can directly access each other. If $\{v_i, v_j\} \in E$, without considering the faulty/fault-free status of v_i and v_j, we can test v_i with v_j, and vice versa. Therefore, we call v_i a tester of v_j, and v_j a tester of v_i. For a subset $V' \subset V$, the *tester-set* of V', denoted $\mu^{-1}V'$, is defined to be $\{v | v \in V$ and $\{v, v'\} \in E$ for some $v' \in V'\} - V'$.

There are several strategies for interconnected processors to diagnose faulty processors among themselves. The two strategies concerned in this paper were initially proposed by Preparata *et al.* [7] and Friedman [3]. These two strategies both use the diagnostic model initially introduced by Preparata, Metze, and Chien (known as the PMC model) [7]. In this model, the self-diagnosable system is represented by a directed graph $G=(V,A)$, or digraph for short, in which a node v_i can test all nodes v_j if arrow $(v_i \rightarrow v_j) \in A$. (Notice that a graph $G=(V,E)$ is just a special case of a digraph $G=(V,A)$ in which $(v_i \rightarrow v_j) \in A \Longleftrightarrow (v_j \rightarrow v_i) \in A$.) The test-result is simply a conclusion that the tested node is "faulty" or "fault-free," denoted as label 1 or 0 on the corresponding arrow. The PMC model assumes that a fault-free node should always give correct test-result, whereas the test-result given by a faulty node is unreliable. A *syndrome* is defined as a function $s: A \rightarrow \{0,1\}$. A subset $F \subseteq V$ is *consistent* with a syndrome s if s can arise from the circumstance that all nodes in F are faulty and all nodes in $V-F$ are fault-free. It is worth pointing out that for a given syndrome s, there may be more than one subset of V that are consistent with s. If this happens, the system cannot diagnose for syndrome s, because the faulty-sets that can cause s are not unique. It is clear that under the PMC model, any diagnosis strategy must have some (at least one) good processors to self-diagnose. For a certain strategy, the *diagnosability* is a number t such that if $|F| \geq t$, the diagnosis cannot be carried out effectively.

Along with their diagnostic model, Preparata *et al.* proposed a diagnosis strategy. The strategy is called *one-step diagnosis* whose target is to identify the exact set of *all* faulty nodes before their repair or replacement. We have the following definition of (one-step) diagnosability under this strategy.

Definition 1: Under the one-step diagnosis strategy, a system is said to be *t_p-diagnosable* if for any syndrome s, there is at most one faulty-subset $F \subseteq V$ that is consistent with s, given that $|F| \leq t_p$. t_p is called the diagnosability of the system under consideration.

The faulty set F identified under above strategy is *precise* in the sense that all nodes in F are truly faulty and all nodes in $V-F$ are fault-free. For this reason we can call this strategy a *precise diagnosis strategy* (in contrast with the other diagnosis strategy we study in this paper). Clearly t_p is dependent on the topology of the system, i.e., the way the nodes are linked.

Later in [3], another strategy was proposed in which the target is to locate a subset $\tilde{F}$ of nodes such that *all* faulty nodes are within $\tilde{F}$. The faulty/fault-free status of a specific node in $\tilde{F}$ can not be further identified. So the whole $\tilde{F}$ has to be replaced to repair the system. The diagnosability of a system under this strategy can be formally defined as follows.

Definition 2: A system is *t_1/t_1-diagnosable* if given any syndrome, a subset $\tilde{F}$ can be identified such that $|\tilde{F}| \leq t_1$ and

$$v \quad \textit{is faulty} \Longrightarrow v \in \tilde{F}.$$

The strategy is also called *pessimistic diagnosis strategy* because $\tilde{F}$ may contain some fault-free nodes. The motivation of this strategy is to increase the diagnosability. In most cases, for the same system, this strategy has greater diagnosability than the precise strategy, i.e., the system can allow more nodes to be faulty to correctly identify all faulty nodes (may be together with some fault-free ones). The price paid for this increase in diagnosability is that some good nodes may be

Manuscript received August 4, 1992; revised November 12, 1993.
The author is with the Department of Mathematics and Computer Science, Montclair State University, Upper Montclair, NJ 07043 USA; e-mail: wang@apollo.montclair.edu.
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unnecessarily replaced. However, it can be shown that at most one fault-free node will be included in $\tilde{F}$ [2], [11].

The *n-hypercube* is a well-known interconnection model. Graph theoretically, an n-hypercube can be viewed as a graph $G = (V, E)$ such that V consists of 2^n nodes, numbered from $\underbrace{00\cdots0}_{n}$ to $\underbrace{11\cdots1}_{n}$. $\{v_i, v_j\} \in E$ if and only if v_i and v_j have only one bit different. Thus, every node has links with exactly n other nodes. There are altogether $n2^{n-1}$ links. Two nodes v_i, v_j of an n-hypercube that have d bits different are said to have *Hamming distance d*, denoted as $H(v_i, v_j) = d$. So in an n-hypercube, a link exists between v_i and v_j if and only if $H(v_i, v_j) = 1$. Notice that the nodes can be numbered differently as long as the above link regulation is obeyed. From now on we will call n-hypercube simply n-cube. n-cube as a topology to interconnect processors has many attractive properties. It has become the most popular architecture for multiprocessor-systems. The t_p- and t_1/t_1-diagnosability of hypercubes were studied in [1] and [5], respectively. An n-cube is shown to be n-diagnosable under the precise strategy [1] and $(2n-2)/(2n-2)$-diagnosable under the pessimistic strategy [5].

This paper studies the t_p- and t_1/t_1-diagnosability of *enhanced hypercubes.* In real systems using hypercube topology, the processors are usually manufactured with the maximum allowable links. Very often not all these links are used in the regular hypercube of the system. This gives the motivation of utilizing the left-over links to get extra connections between the nodes. These hypercubes with extra links can be called enhanced hypercubes. We call these extra links *skips.* It was shown that this augment in the system's communication ability can notably improve the system performance as a whole [8]. In [8], Tzeng and Wei investigated the performance of one category of enhanced hypercubes, in which 2^{n-1} skips are added in the following way: In addition to the $n2^{n-1}$ regular links in an n-cube, a skip exists between a pair of nodes $b_n b_{n-1} \cdots b_{k+1} b_k b_{k-1} \cdots b_1$ and $b_n b_{n-1} \cdots b_{k+1} \bar{b}_k \bar{b}_{k-1} \cdots \bar{b}_1$, where k, ranging from 2 to n, is a parameter of the enhanced n-cube which is the Hamming distance between the two nodes linked by a skip. We use notation *(n, k)-cube* to denote an enhanced n-cube with parameter k. Examples of $(3, 2)$-cube and $(3, 3)$-cube are given in Fig. 1. The proposal of having extra links is a very practical one because these links are equipped as the processors are fabricated, would be unused otherwise, and the implementation does not pose difficulty. It turns out that the enhanced hypercubes can achieve noticeable improvements in many measurements such as mean internode distance, diameter and traffic density, compared to regular hypercubes [8].

In this paper, we show that for enhanced hypercubes, the diagnosability is also increased under both precise and pessimistic strategies. More specifically, under the precise strategy, an enhanced n-cube is $(n+1)$-diagnosable when $n \geq 4$, whereas a regular n-cube is n-diagnosable. Under the pessimistic strategy, an enhanced n-cube is $2n/2n$-diagnosable when $n \geq 5$ and $k \geq 4$, while a regular n-cube is $(2n-2)/(2n-2)$-diagnosable. The rest of this paper is organized as follows: In Section II, the $(n+1)$-diagnosability of enhanced

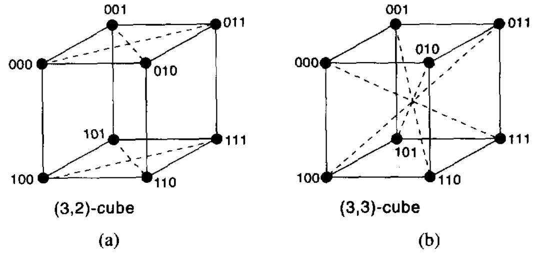

Fig. 1. (a) $(3, 2)$-cube and (b) $(3, 3)$-cube. The skips are shown with dashed links.

hypercubes under the precise strategy is proved. In Section III, we prove the $2n/2n$-diagnosability of enhanced hypercubes under the pessimistic strategy. We then give some concluding remarks in Section IV.

II. THE t_p-DIAGNOSABILITY OF ENHANCED HYPERCUBES

We need some definitions and previous results for the discussion in this section.

Definition 3: The connectivity $\kappa(G)$ of a graph $G(V, E)$ is the minimum number of nodes whose removal results in a disconnected or a trivial (one node) graph.

The following fact is well-known in graph theory.

Lemma 1 [10]: $\kappa(G) = K$, if and only if the least number of disjoint paths between any two nodes of G is K.

The particular graph concerned in this paper is an enhanced hypercube. The κ of a regular n-cube can be readily established from the following lemma.

Lemma 2 [6]: There exist exactly n disjoint paths between any two nodes in an n-cube. Furthermore, if the Hamming distance of the two nodes is d, then there exist d paths of length d, and $n-d$ paths of length $d+2$.

Therefore, by Lemma 1, an n-cube has $\kappa = n$. On the basis of Lemma 2, we can prove that for enhanced hypercubes, the connectivity is increased to $n+1$, resulting in a better t_p-diagnosability of the system.

Lemma 3: There exist $n+1$ disjoint paths between any two nodes in an enhanced (n, k)-cube.

Proof: Arbitrarily pick two nodes $v_0, v_1 \in V$ such that $H(v_0, v_1) = d$. Without loss of generality we can number $v_0 = 00..0\underbrace{00..0}_{d}$ and $v_1 = 00..0\underbrace{11..1}_{d}$. By Lemma 2, there are n disjoint paths between v_0 and v_1 *without* skips. The n disjoint paths are shown in Fig. 2. d of them have length d, and $n-d$ of them have length $d+2$.

Now with additional skips in the (n, k)-cube, there is a new link between every pair of nodes $(b_n..b_{k+1}b_k..b_1 \longleftrightarrow b_n..b_{k+1}\bar{b}_k..\bar{b}_1)$. If there is a skip between the picked nodes v_0 and v_1, then trivially there are $n+1$ disjoint paths between v_0 and v_1. In the rest of the proof we separately deal with three different cases: $k = d$, $k < d$ and $k > d$. For the case of $k = d$, we assume that no skip exists between v_0 and v_1. For the latter two cases, this property automatically holds.

Case 1: $k = d$. Let $v_0 = 00..0\underbrace{00..0}_{d}$. Since there exists a skip from $00..0\underbrace{00..0}_{d}$ to $00..0\underbrace{11..1}_{d}$, we cannot number

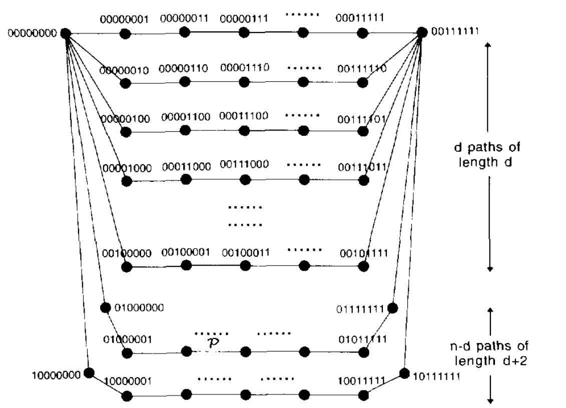

Fig. 2. n disjoint paths between v_0 and v_1. In this example, $n = 8$ and $d = 6$.

v_1 as $00..0\underbrace{11..1}_{d}$. So, without loss of generality, let $v_1 = 0..0\underbrace{1..1}_{d_1}\overbrace{0..0\underbrace{1..1}_{d_2}}^{d}$, such that $d_1 + d_2 = d$. We denote node $00..0\underbrace{11..1}_{d}$ as v_0'.

We renumber the nodes by repeatedly exchanging two bits over all nodes in such a way that under the new numbering $v_0 = 00..0\underbrace{00..0}_{d}$ and $v_1 = 00..0\underbrace{11..1}_{d}$. Notice that node $v_0' = 0..0\underbrace{1..1}_{d_1}\overbrace{0..0\underbrace{1..1}_{d_2}}^{d}$ $(d_1 + d_2 = d)$ under the new numbering. There is a skip from v_0 to v_0', and a skip from v_1 to $0..0\underbrace{1..1}_{d_1}\overbrace{1..1\underbrace{0..0}_{d_2}}^{d}$ $(d_1 + d_2 = d)$, which we denote with v_1'.

We now specify the $n+1$ disjoint paths from v_0 to v_1. $n-1$ paths are directly taken from Fig. 2. We construct the other 2 paths using 2 skips and the links in the remaining path in Fig. 2.

`Path1`

$$v_0 = 00..0\underbrace{00..0}_{d} \to 0..001\underbrace{0..0}_{d}{}^{\dagger} \to 0..011\underbrace{0..0}_{d}$$
$$\to \cdots \to 0..0\underbrace{1..1}_{d_1}\underbrace{0..0}_{d} \to 0..0\underbrace{1..1}_{d_1}\underbrace{10..0}_{d_1}\underbrace{0..0}_{d_2}$$
$$\to 0..0\underbrace{1..1}_{d_1}\underbrace{11..0}_{d_1}\underbrace{0..0}_{d_2} \to \cdots \to 0..0\underbrace{1..1}_{d_1}\underbrace{1..1}_{d_1}\underbrace{0..0}_{d_2}(= v_1')$$
$$\xrightarrow{\text{skip}} 00..0\underbrace{11..1}_{d} = v_1$$

Notice that only the node with † $(0..001\underbrace{0..0}_{d})$ in `path1` belongs to one particular path in Fig. 2 (the second from the bottom). Denote this particular path as $\mathcal{P}$. Since $d = k \geq 2$, we can always set $d_1 \geq 1$ and $d_2 \geq 1$. Then all other intermediate nodes in `path1` are disjoint with all nodes in Fig. 2.

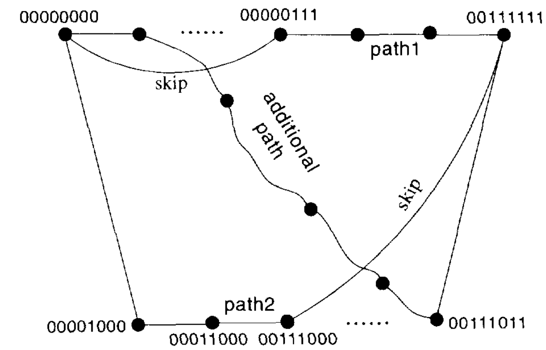

Fig. 3. Using two skips, two originalpaths can be reconstructed and one morepath is constructed. In this example, $n = 8$, $d = 6$ and $k = 3$.

`Path2`

$$v_0 = 00..0\underbrace{00..0}_{d} \xrightarrow{\text{skip}} 0..0\underbrace{1..11}_{d_1}\underbrace{0..0}_{d_1}\underbrace{1..11}_{d_2}(= v_0')$$
$$\to 0..0\underbrace{0..11}_{d_1}\underbrace{0..0}_{d_1}\underbrace{1..11}_{d_2} \to \cdots \to 0..0\underbrace{0..01}_{d_1}\underbrace{0..0}_{d_1}\underbrace{1..11}_{d_2}{}^{\dagger}$$
$$\to 0..0\underbrace{0..01}_{d_1}\underbrace{0..1}_{d_1}\underbrace{1..11}_{d_2}{}^{\dagger} \to \cdots \to 0..0\underbrace{0..01}_{d_1}\underbrace{1..1}_{d_1}\underbrace{1..11}_{d_2}{}^{\dagger}$$
$$\to 0..0\underbrace{0..00}_{d_1}\underbrace{1..1}_{d_1}\underbrace{1..11}_{d_2} = v_1$$

Notice that nodes with † all belong to the path $\mathcal{P}$. `path1` uses one node from $\mathcal{P}$, while `path2` uses $d_1 + 1$ of them. All other intermediate nodes before the first † node are disjoint with all nodes in Fig. 2, and disjoint with nodes of `path1`. Both `path1` and `path2` are of length $2d_1 + 1$.

We have specified the $n+1$ disjoint paths from v_0 to v_1: $n-1$ paths from Fig. 2, 2 paths constructed with $d_1 + 2$ nodes in the remaining path $\mathcal{P}$, two skips (emanating from v_0 and v_1, respectively), and some other nodes disjoint with nodes in Fig. 2.

Case 2: $k < d$. Let $v_0 = 00..0\underbrace{00..0}_{d}$ and $v_1 = 00..0\underbrace{11..1}_{d}$. v_0 has a skip to $v_0' = 00..0\underbrace{0..0\overbrace{1..1}^{k}}_{d}$. v_1 has a skip to $v_1' = 00..0\underbrace{1..1\overbrace{0..0}^{k}}_{d}$. The situation is shown in Fig. 3.

We show that by rearranging two original paths together with the newly added skips, we can get one more path from v_0 to v_1. Observe the two paths shown in Fig. 3. We reconstruct `path1` in Fig. 3. as

$$v_0 \xrightarrow{\text{skip}} 00..0\underbrace{0..0\overbrace{1..1}^{k}}_{d}(= v_0') \to 00..0\underbrace{0..\overbrace{11..1}^{k+1}}_{d}$$
$$\to 00..0\underbrace{0.\overbrace{111..1}^{k+2}}_{d} \to \cdots$$
$$\to 00..0\underbrace{1..11..1}_{d}(= v_1)$$

Reconstruct path2 of Fig. 3. as

$$v_0 \to 00..0\underbrace{0..001\overbrace{0..0}^{k}}_{d} \to 00..0\underbrace{0..011\overbrace{0..0}^{k}}_{d}$$

$$\to \cdots \to 00..0\underbrace{1..111\overbrace{0..0}^{k}}_{d}(=v_1') \xrightarrow{\text{skip}} v_1$$

Construct an additional path as follows:

$$v_0 \to \underbrace{0..0}_{n-d}\underbrace{0..00}_{d-k}\underbrace{0..01}_{k} \quad (1)$$

$$\to \underbrace{0..0}_{n-d}\underbrace{0..001}_{d-k}\underbrace{0..01}_{k} \to \underbrace{0..0}_{n-d}\underbrace{0..011}_{d-k}\underbrace{0..01}_{k} \to \underbrace{0..0}_{n-d}\underbrace{0..111}_{d-k}\underbrace{0..01}_{k} \quad (2)$$

$$\to \cdots \quad (3)$$

$$\to \underbrace{0..0}_{n-d}\underbrace{01..11}_{d-k}\underbrace{0..001}_{k} \to \underbrace{0..0}_{n-d}\underbrace{01..11}_{d-k}\underbrace{0..011}_{k} \to \underbrace{0..0}_{n-d}\underbrace{01..11}_{d-k}\underbrace{0..111}_{k} \quad (4)$$

$$\to \cdots \quad (5)$$

$$\underbrace{0..0}_{n-d}\underbrace{01..11}_{d-k}\underbrace{01..11}_{k} \to \underbrace{0..0}_{n-d}\underbrace{11..11}_{d-k}\underbrace{01..11}_{k} \to v_1 \quad (6)$$

Referring to Figs. 2 and 3, we can see that none of the intermediate nodes in (1)–(6) belongs to the new path1 or path2 or any one of the remaining $n-2$ paths: $\underbrace{0..0}_{n-d}\underbrace{0..00}_{d-k}\underbrace{0..01}_{k}$ and $\underbrace{0..0}_{n-d}\underbrace{11..11}_{d-k}\underbrace{01..11}_{k}$'s disjointness with other paths is obvious. For other intermediate nodes in (1)–(6), just notice that in all those nodes, bits d through 1 have the pattern of $\underbrace{0..01..10..01..1}_{d}$, i.e., a block of 0's followed by a block of 1's, then another block of 0's and another block of 1's. None of the nodes in other paths has this property.

The $n+1$ disjoint paths between v_0 and v_1 are thus established.

Case 3: $k > d$. Let $v_0 = 00..0\underbrace{00..0}_{d}$ and $v_1 = 00..0\underbrace{11..1}_{d}$. We separately deal with two different cases—$k = d+1$ and $k \geq d+2$.

Case 3.1: $k = d+1$. Observe a path of length $d+2$ in Fig. 2,

$$v_0 \to \underbrace{0..01}_{n-d}\underbrace{0..000}_{d} \to \underbrace{0..01}_{n-d}\underbrace{0..001}_{d} \to \underbrace{0..01}_{n-d}\underbrace{0..011}_{d} \to \cdots$$

$$\to \underbrace{0..01}_{n-d}\underbrace{1..111}_{d} \to \underbrace{0..00}_{n-d}\underbrace{1..111}_{d}(=v_1)$$

Now since there is a skip from v_0 to $\underbrace{0..01}_{n-d}\underbrace{1..111}_{d}$, and there is a skip from v_1 to $\underbrace{0..01}_{n-d}\underbrace{0..000}_{d}$, we can construct two paths using only two nodes from the above path:

$$v_0 \xrightarrow{\text{skip}} \underbrace{0..01}_{n-d}\underbrace{1..111}_{d} \to v_1$$

and

$$v_0 \to \underbrace{0..01}_{n-d}\underbrace{0..000}_{d} \xrightarrow{\text{skip}} v_1.$$

The above two paths are obviously disjoint with each other and disjoint with any of the remaining $n-1$ existing paths.

Case 3.2: $k \geq d+2$. We construct a new path from v_0 to v_1, i.e., we do not use any links in the n existing paths.

$$v_0 \xrightarrow{\text{skip}} \underbrace{0..00}_{n-k}\underbrace{1..111}_{k} \to \underbrace{0..00}_{n-k}\underbrace{1..110}_{k} \to \underbrace{0..00}_{n-k}\underbrace{1..100}_{k}$$

$$\to \cdots \to \underbrace{0..00}_{n-k}\underbrace{1..1\overbrace{0..00}^{d}}_{k} \xrightarrow{\text{skip}} v_1 \quad (7)$$

To see that the path in (7) is disjoint with any of the n existing paths, just notice that all intermediate nodes have 11 at bits $(d+2)$-$(d+1)$. None of the intermediate nodes in paths in Fig. 2. has this property.

So far we have shown that there are least $n+1$ disjoint paths between any pair of nodes in an enhanced (n,k)-cube. Since each node has only $n+1$ links emanating from it, there will be no more than $n+1$ disjoint paths between a pair of nodes. We come to the conclusion that there are exactly $n+1$ paths between any pair of nodes. □

Lemma 4: An enhanced (n,k)-cube has connectivity $n+1$.

Proof: Immediately from Lemma 1 and Lemma 3. □

The following two characterizations of a t_p-diagnosable system are due to Preparata *et al.* [7], and Hakimi and Amin [4], respectively.

Lemma 5: [7] Two conditions are necessary for a system S of N processors to be t_p-diagnosable:

1) $N \geq 2t_p + 1$,

2) Each processor is tested by at least t_p other processors.

Lemma 6: [4] Let $G(V,E)$ be the graph representation of a system S, with V representing S's processors and E the interconnection among them, such that $|V| = N$. The sufficient conditions for S to be t_p-diagnosable are:

- $N \geq 2t_p + 1$
- $\kappa(G) \geq t_p$.

An enhanced (n,k)-cube has 2^n nodes. And $2^n \geq 2(n+1)+1$ holds when $n \geq 4$. Each node in a (n,k)-cube is linked with (therefore tested by) exactly $n+1$ other nodes. By Lemma 4, $\kappa((n,k)\text{-cube}) = n+1$. Combining the preceding simple reasoning with Lemma 5 and Lemma 6, we immediately arrive at the following theorem:

Theorem 1: An enhanced (n,k)-cube is $(n+1)$-diagnosable, where $n \geq 4$.

III. THE t_1/t_1-DIAGNOSABILITY OF ENHANCED HYPERCUBES

There are several characterizations for t_1/t_1-diagnosable systems. For our purpose, we use the one proposed by Chwa and Hakimi [2].

Lemma 7: [2] Let $G(V,E)$ be the graph representation of a system S, with V representing S's processors and E the interconnection among them. Let $\mu^{-1}V'$ be the tester-set of V', formally defined as $\mu^{-1}V' = \{v | v \in V$ and $\{v, v'\} \in E$

for some $v' \in V'\} - V'$. Then S is t_1/t_1-diagnosable if and only if for any $V' \subset V$, such that $|V'| = 2p$ for any $1 \leq p \leq t_1$, $|\mu^{-1}V'| \geq t_1 - p + 1$.

Based on this characterization, Kavianpour and Kim [5] established that the t_1/t_1-diagnosability of a plain n-hypercube is $2n - 2$, where $n \geq 4$. In this section, we will show that in an enhanced (n, k)-cube, t_1/t_1-diagnosability is enhanced to $2n$, where $k \geq 4$ and $n \geq 5$.

Unlike the case of t_p-diagnosable systems, here to get an increased t_1/t_1-diagnosability, the skip parameter k in the enhanced (n, k)-cube must be ≥ 4, i.e., the Hamming distance of two nodes linked by a skip should be at least 4, or otherwise the t_1/t_1-diagnosability won't be improved in the enhanced hypercube. To see the necessity of this restriction, consider $p = 1$ and $k = 2$. Then $|V'| = 2$. Choose $V' = \{0..000, 0..011\}$. By Lemma 2, there are 2 paths of length 2 and $n - 2$ paths of length 4, all disjoint with each other, between 0..000 and 0..011. Therefore, V' has $2n - 2$ testers. The skip from 0..000 to 0..011 does not help increase nodes testing V'. Hence, $|\mu^{-1}V'| = 2n - 2 < 2n - p + 1$. Taking the same line, let $p = 1$, $k = 3$ and choose $V' = \{0..0000, 0..0011\}$. Again, without skips, V' would have $n - 2$ testers, which include 0..0111 and 0..0100. Now notice that the skip of 0..0000 goes to 0..0111, and the skip of 0..0011 goes to 0..0100. So the two skips contribute no new testers for V'. We have $|\mu^{-1}V'| = 2n - 2 < 2n - p + 1$. The necessity for $n \geq 5$ is shown as follows: By Lemma 7, for a system to be $2n/2n$-diagnosable, for an arbitrary V' with $|V'| = 2 \cdot 2n$, we should have $|\mu^{-1}V'| \geq 2n - 2n + 1 = 1$. When $n \leq 4$, $2 \cdot 2n \geq 2^n = |V|$, resulting in $|\mu^{-1}V'| < 1$. Therefore n must be at least 5.

For the purpose of our following discussion, we classify two types of V' according to the bit pattern of its nodes. Denote the most significant bit (i.e., the leftmost bit) of the binary representation of a node $v_i \in V'$ as $MSB(v_i)$. $MSB(v_i) \in \{0, 1\}$. We classify V' into the following two types.

Type 1: There is a numbering of V such that at least $p + 1$ nodes in V' have their $MSB = 0$;

Type 2: No matter how nodes of V are numbered, there will be exactly p nodes in V' with $MSB = 0$ and p nodes with $MSB = 1$.

It is important to point out that any V' falls into one of the above two types. To see this, suppose we end up with a numbering such that there are more nodes of $MSB = 1$ than nodes of $MSB = 0$ in V'. Then by reversing MSB ($0 \rightarrow 1$, $1 \rightarrow 0$) over all nodes of V, which clearly gives another valid numbering, V' becomes of Type 1.

Lemma 8: V' is of Type 2, if and only if at each binary position i ($i = 1, 2, \cdots, n$), exactly p nodes in V' have 1, and p nodes in V' have 0.

Proof: See Appendix.

Example: If $V' = \{00000, 11011, 10111, 01100, 11001, 00110\}$, then no matter how V is renumbered subject to the regulation of hypercube, V' will remain of Type 2.

Lemma 9: In an n-cube such that $n \geq 4$, if V' is of Type 2, then for any $v_i \in V'$, there is a $v_j \in V'$ such that $H(v_i, v_j) \geq 3$.

Proof: See [9].

We are now ready to state and prove the t_1/t_1-diagnosability for enhanced hypercubes.

Theorem 2: The t_1/t_1-diagnosability of an enhanced (n, k)-cube is $2n$, where $n \geq 5$ and $k \geq 4$.

Proof: Let $G(V, E)$ be the graph representation of an enhanced (n, k)-cube, with V representing its processors and E the interconnection among them that follows the rule of enhanced (n, k)-cube. We prove the theorem by showing that when $n \geq 5$ and $k \geq 4$, $G(V, E)$ satisfies the condition in Lemma 7, i.e., for any subset $V' \subset V$ such that $|V'| = 2p$, where $1 \leq p \leq 2n$, $|\mu^{-1}V'| \geq 2n - p + 1$.

We first show that the preceding claim is true for $p = 1$, i.e., $|V'| = 2$. Let $V' = \{v_0, v_1\}$. If $H(v_0, v_1) = 1$, then, without loss of generality, we can number $v_0 = 0..00$ and $v_1 = 0..01$. By Lemma 2, there is 1 path of length 1 (the direct link) and $n - 1$ paths of length 3, all disjoint with each other, between 0..00 and 0..01. So without skips there would be $2n - 2$ nodes testing V'. Now 0..00 has a skip to $0..0\underbrace{1..11}_{k}$ and 0..01 has a skip to $0..0\underbrace{1..10}_{k}$, where $k \geq 4$. It can be easily seen that neither $0..0\underbrace{1..11}_{k}$ nor $0..0\underbrace{1..10}_{k}$ belongs to the original testers of V'. So V' has 2 more testers. Hence, $|\mu^{-1}V'| = 2n$, justifying $|\mu^{-1}V'| \geq 2n - p + 1$. If $H(v_0, v_1) = 2$, we can number $v_0 = 0..000$ and $v_1 = 0..011$. By Lemma 2 again, V' would have $2n - 2$ testers without skips. The two skips of 0..000 and 0..011 go to $0..0\underbrace{1..111}_{k}$ and $0..0\underbrace{1..100}_{k}$, respectively. Since $k \geq 4$, $0..0\underbrace{1..111}_{k}$ and $0..0\underbrace{1..100}_{k}$ contribute as new testers of V', resulting in $|\mu^{-1}V'| = 2n$. When $H(v_0, v_1) \geq 3$, by Lemma 2 there would be already $2n$ testers for V' without skips. So $|\mu^{-1}V'| \geq 2n$ readily holds.

For $2 \leq p \leq 2n$, we will show separately for V' of Type 1 and Type 2, that any V' ($|V'| = 2p$ and $2 \leq p \leq 2n$) satisfies the condition of Lemma 7.

A. Type 1

At least $p + 1$ nodes in V' have their $MSB = 0$. Without loss of generality, let $MSB(v_i) = 0$, $i = 0, 1, \cdots, p$.

Case 1: $k < n$. We have shown that $|\mu^{-1}\{v_0, v_1\}| \geq 2n$. For $\{v_2, v_3, \cdots, v_p\}$, let $U = \{u_2, u_3, \cdots, u_p\}$ such that v_j differs with u_j only at MSB, $j = 2, 3, \cdots, p$. Then u_j tests v_j, $j = 2, 3, \cdots, p$. Notice that all nodes in $\mu^{-1}\{v_0, v_1\}$ have (Hamming) distance 1 or k (skips) to either v_0 or v_1. Since $k < n$, the two nodes in $\mu^{-1}\{v_0, v_1\}$ linked by skips have $MSB = 0$. Meanwhile, nodes in U have at least distance 2 to v_0 or v_1. We can conclude from the preceding argument that

$$\mu^{-1}\{v_0, v_1\} \cap U = \phi. \tag{8}$$

Another two facts are:

$$v_0, v_1 \notin \mu^{-1}\{v_0, v_1\} \text{ and } v_0, v_1 \notin U. \tag{9}$$

By the definition of μ^{-1}, combining (8) and (9), we have

$$\begin{aligned}|\mu^{-1}V'| &\geq |\mu^{-1}\{v_0, v_1\}| + |U| - (2p-2)\\ &\geq 2n + (p-1) - (2p-2)\\ &= 2n - p + 1.\end{aligned}$$

Case 2: $k = n$. When $k = n$, (8) does not necessarily hold. (If (8) holds, the argument for Case 1 can be directly used here.) Now suppose $\mu^{-1}\{v_0, v_1\} \cap U \neq \phi$. Without loss of generality, suppose v_0's tester $v_{0k} \in U$ and number $v_0 = 00\cdots0$. Then $v_{0k} = 11\cdots1$. Since v_{0k} is also a tester of some $v_j \in \{v_2, v_3, \cdots, v_p\}$, $v_j = 01\cdots1$ according to the definition of U. Now repartition $\{v_0, v_1, \cdots, v_p\}$ into two parts: $\{v_0, v_j\}$ and $\{v_1, \cdots, v_{j-1}, v_{j+1}, \cdots, v_p\}$, for the latter of which a U' is similarly defined as in Case 1. Notice that $|\mu^{-1}\{v_0, v_j\}| = 2n$ and the two skips (i.e., $11\cdots1$ and $10\cdots0$) of v_0, v_j just fall in $\mu^{-1}\{v_0, v_j\}$. Therefore

$$\mu^{-1}\{v_0, v_j\} \cap U' = \phi,$$

and we have the same conclusion as in Case 1. This completes the proof for V' of Type 1.

B. Type 2

Under any numbering of V, there will be exactly p nodes in V' with $MSB = 0$ and p nodes with $MSB = 1$. Without loss of generality let $V' = \{v_0, v_1, \cdots, v_{2p-1}\}$ such that $MSB(v_i) = 0$, $i = 0, 1, \cdots, p-1$, and $MSB(v_j) = 1$, $j = p, p+1, \cdots, 2p-1$.

Without loss of generality, let $v_0 = 00\cdots0 \in V'$. Then by Lemma 9, there exists a $v_1 \in V'$ such that $H(v_0, v_1) \geq 3$. We can always renumber nodes of V by repeatedly either reversing ($0 \to 1$, $1 \to 0$) a particular bit over all nodes or exchanging two bits over all nodes. (See Appendix, the proof of Lemma 8.) Therefore by doing several bit-exchanges we can get a numbering of V under which $v_0 = 00\cdots0$ and $v_1 = 0\cdots0\underbrace{1\cdots1}_{d}$, where $d \geq 3$. We will show in the rest of the proof that even without using the skips, V' of Type 2 will have a $\mu^{-1}V'$ such that $|\mu^{-1}V'| \geq 2n - p + 1$. (With skips added, $|\mu^{-1}V'|$ could only be greater or same.)

Case 1: $d \leq n-2$. Let $v_1 = 0\cdots0\underbrace{1\cdots1}_{d}$. Since $d \geq 3$, $\{v_0, v_1\}$ has $2n$ testers without skips. For $\{v_2, v_3, \cdots, v_{p-1}\}$, let $U = \{u_2, u_3, \cdots, u_{p-1}\}$ such that v_j differs with u_j only at MSB, $j = 2, 3, \cdots, p-1$. Then u_j tests v_j, $j = 2, 3, \cdots, p-1$. Notice that $\mu^{-1}\{v_0, v_1\} \cap U = \phi$. Therefore $|\mu^{-1}\{v_0, v_1\} \cup U| = 2n + (p-2)$.

As pointed out earlier, $v_0, v_1 \notin \mu^{-1}\{v_0, v_1\} \cup U$. If there is another $v_j \in \{v_2, \cdots, v_{p-1}, v_p, \cdots, v_{2p-1}\}$ such that $v_j \notin \mu^{-1}\{v_0, v_1\} \cup U$, then we have

$$\begin{aligned}|\mu^{-1}V'| &\geq |\mu^{-1}\{v_0, v_1\} \cup U| - (2p-3)\\ &= 2n + (p-2) - (2p-3)\\ &= 2n - p + 1,\end{aligned}$$

and the theorem is proved.

If there does not exist such a v_j, however, we will show that there must be a $v_q \in \{v_p, \cdots, v_{2p-1}\}$ such that at least one tester of v_q doesn't belong to $\mu^{-1}\{v_0, v_1\} \cup U$. For this sake, suppose such a v_j doesn't exist. Then, we have

$$\{v_2, \cdots, v_{p-1}, v_p, \cdots, v_{2p-1}\} \subseteq \mu^{-1}\{v_0, v_1\} \cup U. \quad (10)$$

The nodes of $\mu^{-1}\{v_0, v_1\}$ are

$$\left.\begin{array}{l}00..00000..001\\00..00000..010\\ \quad\cdots\cdots\\01..00000..000\\10..00000..000\end{array}\right\} n \text{ testers of } v_0$$
$$\left.\begin{array}{l}00..00011..110\\00..00011..101\\ \cdots\cdots\\00..00001..111\\00..001\underbrace{11..111}_{d}\\00..01011..111\\ \cdots\cdots\\01..00011..111\\10..000\underbrace{11..111}_{d}\end{array}\right\} n \text{ testers of } v_1 \quad (11)$$

Because of (10) and the fact that all the nodes in U have $MSB = 1$, we have

$$\{v_2, \cdots, v_{p-1}\} \subseteq \mu^{-1}\{v_0, v_1\} - \{10..00000..000, 10..000\underbrace{11..111}_{d}\}. \quad (12)$$

Denote v_i's numbering as $0\tilde{b}_i$, $i = 2, \cdots, p-1$. Then $U = \{1\tilde{b}_2, 1\tilde{b}_3, \cdots, 1\tilde{b}_{p-1}\}$. By (10) and (12),

$$\{v_p, \cdots, v_{2p-1}\} = \{1\tilde{b}_2, 1\tilde{b}_3, \cdots, 1\tilde{b}_{p-1}, 10..00000..000, 10..000\underbrace{11..111}_{d}\} \quad (13)$$

The nodes of V' therefore have the following pattern:

$$\left.\begin{array}{rcl}v_0 &=& 00..00000..000\\ v_1 &=& 00..000\underbrace{11..111}_{d}\\ v_2 &=& 0\tilde{b}_2\\ \cdots\cdots && \\ v_{p-1} &=& 0\tilde{b}_{p-1}\end{array}\right\} MSB = 0$$
$$\left.\begin{array}{rcl}v_p &=& 1\tilde{b}_2\\ \cdots\cdots && \\ v_{2p-3} &=& 1\tilde{b}_{p-1}\\ v_{2p-2} &=& 10..00000..000\\ v_{2p-1} &=& 10..000\underbrace{11..111}_{d}\end{array}\right\} MSB = 1$$

By Lemma 8, the nodes of V' should have the same number of 0's and 1's at all dimensions. Therefore this should hold at dimension $n-1$ (the binary position next to MSB). From the pattern shown above, there are at least four 0's at dimension $n-1$. So $0\tilde{b}_2, \cdots, 0\tilde{b}_{p-1}, 1\tilde{b}_2, \cdots, 1\tilde{b}_{p-1}$ should contribute at least four 1's at dimension $n-1$. By (11) and (12), there are *at most* two 1's at dimension $n-1$ of nodes $0\tilde{b}_2, \cdots, 0\tilde{b}_{p-1}$. Therefore there will be at most four nodes in

$0\tilde{b}_2, \cdots, 0\tilde{b}_{p-1}, 1\tilde{b}_2, \cdots, 1\tilde{b}_{p-1}$ that have 1 at dimension $n-1$. So the only possible valid pattern of V' is

$$\left.\begin{array}{l} 00..00000..000 \\ 00..11111..111 \\ 01..00000..000 (= 0\tilde{b}_2) \\ 01..11111..111 (= 0\tilde{b}_3) \end{array}\right\} MSB = 0$$

$$\left.\begin{array}{l} 11..00000..000 (= 1\tilde{b}_2) \\ 11..11111..111 (= 1\tilde{b}_3) \\ 10..00000..000 \\ 10..11111..111 \end{array}\right\} MSB = 1$$

Now pick $v_q = 11..00000..000 \in \{v_p, \cdots, v_{2p-1}\}$. Choose its tester $v_q' = 11..00000..001$. Obviously $v_q' \notin \mu^{-1}\{v_0, v_1\} \cup U$. We have

$$\begin{aligned} |\mu^{-1}V'| &\geq |\mu^{-1}\{v_0, v_1\} \cup U| + |\{v_q'\}| - (2p-2) \\ &= 2n + (p-2) + 1 - (2p-2) = 2n - p + 1, \end{aligned}$$

and the theorem is proved.

Case 2: $d = n - 1$. Similarly as in Case 1, suppose

$$\{v_2, \cdots, v_{p-1}, v_p, \cdots, v_{2p-1}\} \subseteq \mu^{-1}\{v_0, v_1\} \cup U.$$

And the nodes of $\mu^{-1}\{v_0, v_1\}$ are of the following:

$$\left.\begin{array}{l} 000..0000..001 \\ 000..0000..010 \\ \cdots\cdots \\ 010..0000..000 \\ 100..0000..000 \end{array}\right\} n \text{ testers of } v_0$$

$$\left.\begin{array}{l} 011..1111..110 \\ 011..1111..101 \\ \cdots\cdots \\ 001..1111..111 \\ 111..1111..111 \end{array}\right\} n \text{ testers of } v_1$$

Taking the same line as Case 1, we end up with the pattern of V':

$$\left.\begin{array}{l} 000..0000..000 \\ 011..1111..111 \\ 0\tilde{b}_2 \\ \cdots\cdots \\ 0\tilde{b}_{p-1} \end{array}\right\} MSB = 0$$

$$\left.\begin{array}{l} 1\tilde{b}_2 \\ \cdots\cdots \\ 1\tilde{b}_{p-1} \\ 100..0000..000 \\ 111..1111..111 \end{array}\right\} MSB = 1$$

where $\{0\tilde{b}_2, \cdots, 0\tilde{b}_{p-1}\}$ is a subset of $\mu^{-1}\{v_0, v_1\} - \{100..0000..000,\ 111..1111..111\}$.

Pick $v_q = 1\tilde{b}_2 \in \{v_p, \cdots, v_{2p-1}\}$ and consider its testers. v_q will have four different general forms.

Case 2.1: $1\tilde{b}_2 = 100..0000..001$. Then choose its tester $v_q' = 100..0000..011$. v_q' belongs to neither $\mu^{-1}\{v_0, v_1\}$ nor U.

Case 2.2: $1\tilde{b}_2 = 100..0010..000$. Choose $v_q' = 100..0010..001$. v_q' belongs to neither $\mu^{-1}\{v_0, v_1\}$ nor U.

Case 2.3: $1\tilde{b}_2 = 111..1111..110$. Choose $v_q' = 111..1111..100$. v_q' belongs to neither $\mu^{-1}\{v_0, v_1\}$ nor U.

Case 2.4: $1\tilde{b}_2 = 111..1101..111$. Choose $v_q' = 111..1101..110$. v_q' belongs to neither $\mu^{-1}\{v_0, v_1\}$ nor U.

Summarizing the above four cases, we can always find a tester of v_q that does not belong to $\mu^{-1}\{v_0, v_1\} \cup U$. So we have the same inequality as Case 1:

$$|\mu^{-1}V'| \geq |\mu^{-1}\{v_0, v_1\} \cup U| + |\{v_q'\}| - (2p-2) = 2n - p + 1.$$

Case 3: $d = n$. Then $\{v_0, v_1\} = \{000..0000..000, 111..1111..111\}$. If there is a $v_i = 000..00\underbrace{11..111}_{d} \in V'$ such that $d \geq 3$, then we could choose it as v_1 and we are back to Case 1 or Case 2. Suppose such a v_i does not exist. Then all other nodes in V' have either one or two 1's. Now flip-flop all bits for all nodes of V. 000..0000..000 becomes 111..1111..111 and 111..1111..111 becomes 000..0000..000. Since $n \geq 5$, V' now contains some nodes which have at least three 1's. We are back to either Case 1 or Case 2.

We are almost done except for one note: Thus far for the proof of Type 2, we have been assuming $p > 2$, i.e., $\{v_2, \cdots, v_{p-1}\} \neq \phi$. When $p = 2$, $V' = \{v_0, v_1, v_2, v_3\}$, which implies $U = \phi$. One can easily check that the only valid V' of Type 2 in keeping with $\{v_2, v_3\} \subseteq \mu^{-1}\{v_0, v_1\}$ can be numbered to have the following pattern:

$$\{000..0000..000, 011..1111..111, 100..0000..000, 111..1111..111\}.$$

Picking 100..0000..000's tester $100..0000..001 \notin \mu^{-1}\{v_0, v_1\}$, we arrive at the required inequality.

This completes the proof for V' of Type 2, thus completing the proof of Theorem 2. □

IV. Conclusion

Enhanced hypercubes have been shown to achieve improvements over the regular hypercubes in many aspects, and can be implemented with spare link-ports which are otherwise unused [8]. The diagnosability of enhanced hypercubes is studied in this paper. It is shown that the enhanced hypercubes have improved diagnosability under both precise (one-step) and pessimistic diagnosis strategies compared to the regular hypercubes, in addition to the already known improvements in many other measurements. It is proved that under the precise strategy, the diagnosability of an enhanced hypercube of 2^n nodes is increased to $n+1$, while in a regular hypercube the diagnosability is n. Under the pessimistic strategy, the diagnosability of an enhanced hypercube is increased to $2n$, whereas in a regular hypercube the pessimistic diagnosability is $2n-2$. As the technology progresses rapidly, the failure probability of each processor drops considerably. Therefore the increase of diagnosability by one or two will greatly extend the system's ability of self-diagnosis. Furthermore, the diagnosability of a network is usually a small number compared to the numbers of nodes and links, and does not "easily" increase as links increase. For all those reasons, the improvements in diagnosability achieved by enhanced hypercubes are noticeable.

The fault torlerance of hypercubes with respect to their self-diagnosis capabilities is a very interesting research field, in which many problems remain open. In this paper we studied the diagnosability of enhanced hypercubes, in which extra links are added in a systematic, symmetric manner. In [5], the diagnosability of a hypercube with some symmetrically reduced links was determined. However, the diagnosability of a hypercube with arbitrarily absent links is not known yet. To solve problems of these kinds, we may need some method to classify the distributions of links, and more properties of hypercubes are to be revealed.

APPENDIX

Proof of Lemma 8: We first show that given a valid numbering of V, a sequence of the following operations,

- Reversal ($0 \to 1$, $1 \to 0$) of any bit b_i, $1 \le i \le n$, over *all* nodes of V or
- Exchange of any two bits b_i, b_j over *all* nodes of V

will give another valid numbering of V. To see this, consider any edge $\{v_i, v_j\} \in E$. Then under a valid numbering we have $v_i = b_n \cdots b_k \cdots b_1$ and $v_j = b_n \cdots \bar{b}_k \cdots b_1$ for some k. It is then obvious to see that after any number of operations of reversal/exchange, v_i and v_j will have one and only one bit different. Since the operation is performed over all nodes of V, this results another valid numbering. We now prove the sufficiency and necessity of the Lemma.

($\Rightarrow$): Suppose V' is of Type 2. For the sake of contradiction suppose there is a j such that of all nodes in V', at binary position j there are more 1's than 0's (or more 0's than 1's). If $j = n$, then V' is not of Type 2. A contradiction to the assumption. If $j \neq n$, then exchanging binary positions n and j over all nodes of V, which gives another valid numbering, will violate V''s being Type 2. Again a contradiction to the assumption.

($\Leftarrow$): Suppose that of all nodes in V', at every binary position exactly p nodes in V' have 1, and p nodes in V' have 0. Clearly, after any number of either *reversal* and/or *exchange*, MSB of V' will have equal number of 1's and 0's. It remains to show that *any* valid numbering of V can be obtained by a sequence of reversals and/or exchanges. By a simple induction on n we can show that there are altogether $n!2^n$ different numberings for an n-cube. But that is exactly the number of numberings one can get by exhausting all possible reversals and/or exchanges: There are $n!$ permutations for the n bit-positions, which can be obtained by exchanges, and each permutation can have 2^n different values, which can be obtained by reversals. □

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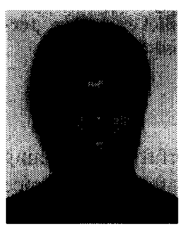

D. Wang (M'90) received the B.Eng. degree from Shanghai University of Science and Technology, Shanghai, People's Republic of China, in 1982, and the M.S. and Ph.D. degrees in computer science from Stevens Institute of Technology, Hoboken, NJ, USA, in 1986 and 1990, respectively.

Since 1990, he has been an Assistant Professor in the Department of Mathematics and Computer Science, Montclair State University, Upper Montclair, NJ, USA. His research interests include fault-tolerant computing, algorithmic robotics, and parallel processing.